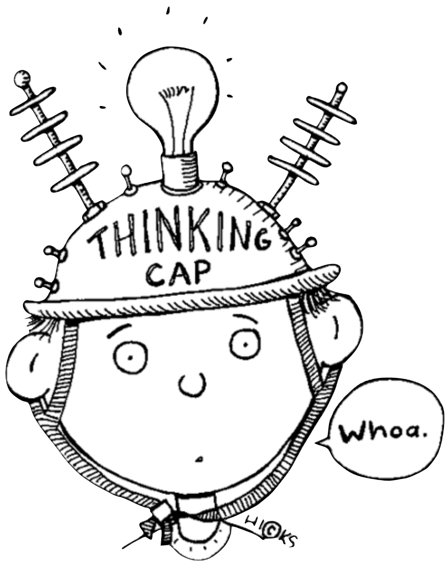

Hundreds Magic

By Brad Fulton
Educator of the Year
(Voted by the California League of Schools – 2005)
brad@tttpress.com

- ☑ High interest discovery teaching
- ☑ Generalizing arithmetic
- ☑ Finding patterns
- ☑ Using variables
- ☑ Writing algebraic proofs
- ☑ Practicing operations with integers

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Brad Fulton

Educator of the Year

- ♦ Consultant
- ♦ Educator
- ♦ Author
- ♦ Keynote presenter
- ♦ Teacher trainer
- ♦ Conference speaker

Known throughout the country for motivating and engaging teachers and students, Brad has co-authored over a dozen books that provide easy-to-teach yet mathematically rich activities for busy teachers while teaching full time for over 30 years. In addition, he has co-authored over 40 teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

Seminar leader and trainer of mathematics teachers

- 2005 California League of Middle Schools Educator of the Year
- California Math Council and NCTM national featured presenter
- Lead trainer for summer teacher training institutes
- Trainer/consultant for district, county, regional, and national workshops

Author and co-author of mathematics curriculum

- Simply Great Math Activities series: six books covering all major strands
- Angle On Geometry Program: over 400 pages of research-based geometry instruction
- Math Discoveries series: bringing math alive for students in middle schools
- Teacher training seminar materials handbooks for elementary, middle, and secondary school

Available for workshops, keynote addresses, and conferences

All workshops provide participants with complete, ready-to-use activities that require minimal preparation and give clear and specific directions. Participants also receive journal prompts, homework suggestions, and ideas for extensions and assessment.

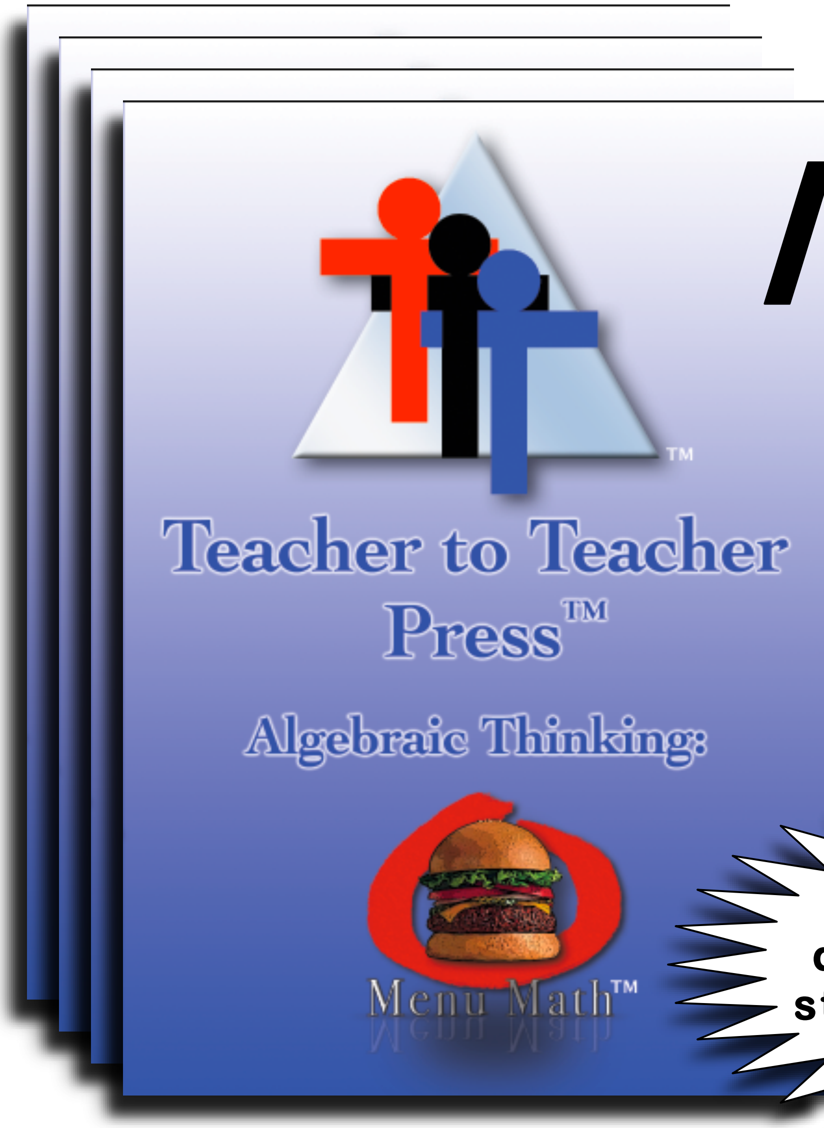
Brad's math activities are the best I've seen in 38 years of teaching!
Wayne Dequer, 7th grade math teacher, Arcadia, CA

"I can't begin to tell you how much you have inspired me!"
Sue Bonesteel, Math Dept. Chair, Phoenix, AZ

"Your entire audience was fully involved in math!! When they chatted, they chatted math. Real thinking!"
Brenda McGaffigan, principal, Santa Ana, CA

"Absolutely engaging. I can teach algebra to second graders!"
Lisa Fellers, teacher

References available upon request



I want...

- a) effective staff development
- b) affordable staff development
- c) ongoing staff development

(pick three)

14 new DVD presentations offer quality mathematics staff development at a fraction of the cost!

Fast Facts and Fractions: Help students master their multiplication facts and learn simple strategies for taming fractions.
Hundreds Magic: An engaging exploration of arithmetic, number sense, algebra, and mathematical reasoning.
Number Line: Help students compare and order fractions, decimals, and percents while developing reasoning skills.
Safely Navigating Social Networks: Help your students stay safer at home and school. Great for parents and staff
Solving Linear Equations: Simple steps and strategies to help your students find success with equations.
Multiplying and Factoring Polynomials: Help students learn the seamless way to tackle polynomials.
Take Your Places: A rich and engaging activity integrating number sense, operations, probability, and algebra.
Integer Strategies: Help students overcome integer operations with these classroom-tested strategies.
Teaching 2-Digit Multiplication: Use *Conceptual Layering* to maximize mathematical skill and reasoning.
Leo's Pattern: Learn how to use *conceptual layering* to help students transition from simple addition to algebra.
Math Projects: Emancipate the intelligence of your students with an authentic assessment and teaching strategy that will amaze both you and your students.
Menu Math: Students from grades two through college have finally made sense of algebra with this clever approach.
The Power of Two: Finally students understand exponents, the zero power, and even *negative* exponents!
X Marks the Spot: Practice with the four operations should be engaging, enriching, and empowering. Find out how to maximize the effectiveness of drill work with this easy approach.

ACTIVITY 5

Hundreds Magic

Materials:

- ☒ paper
- ☐ transparency master
- ☐ activity master

Overview: This powerful activity will keep your students engaged as they explore patterns, practice mathematical operations, and transition from inductive reasoning based on patterns to deductive reasoning supported by algebraic proofs.

Vocabulary: sum, product, adjacent, induction, deduction

PROCEDURE

Skills:

- Multiplying two-digit numbers
- Mental math
- Estimation
- Multiplying polynomials

1. The students will need to make a hundreds chart like the activity master. Alternately, you may wish to provide grid paper for this or simply distribute copies of the transparency master while you use the transparency on the overhead projector.

2. Ask the students to circle any four adjacent numbers which form a square. We will use 7, 8, 17, and 18 as an example. Tell them to add the two diagonals of the square and compare the results. they will notice that $7 + 18 = 8 + 17$. Have them try the same process with a different set of four numbers.

3. Younger students will enjoy simply exploring the patterns in the chart without generalizing the relationships with formulas. More advanced students may be able to explain why the patterns occur, without using formal algebra. If your students are ready for the proof, this is the time to demonstrate it. Most students should be able to follow the explanation after their exploration of the chart. Notice that for any beginning number, the next number is $n + 1$. The numbers below these are $n + 10$ and $n + 11$. Thus the sums of the diagonals are:

$(n) + (n + 11)$ and $(n + 1) + (n + 10)$

Combining terms gives us:

$2n + 11 = 2n + 11$.

4. Next, ask students to multiply diagonals and compare the results. They will see that the products are not equal. In our example, we get $7 \times 18 = 126$ and $8 \times 17 = 136$. However, when they try other locations, they will see that the second answer is always ten more than the first.

5. Once again, the reason can be explained fairly simply:
   $n(n + 11) = n^2 + 11n$ $\qquad$ $(n + 1)(n + 10) = n^2 + 11n + 10$

➤ ➤ ➤ ➤ ➤ ➤ ➤ ➤

Journal Prompts: ✍

Are the differences of the diagonals always equal? Explain why this is or is not true.

Make up an arrangement of numbers other than the four number square. Describe any patterns or relationships that you find.

Homework:

Ask students to explore patterns found in other arrangements of numbers other than the four number square explained above. Some examples are shown in the pattern key on the following page. However, there are many more patterns and proofs for the students to discover.

Taking a Closer Look:

Ask students to explore these same relationships and others on any calendar page. An activity master is provided for this. What similarities and differences occur?

Advanced students can incorporate practice with negative numbers using the second activity master.

Assessment:

Allowing students to work in small groups will provide the opportunity for self assessment. Since all the patterns can be generalized, a single formula should result when students explore a given arrangement of numbers. Some sample patterns and proofs are offered on the following page.

Pattern Key:

Three-in-a-Row:

Pattern 1: The sum of the three numbers equals three times the middle number.

Proof: If "n" is the center then the left number is $n - 1$, and the right number is $n + 1$. Thus their sum is:

$(n - 1) + n + (n + 1) = 3n - 1 + 1 = 3n$

Pattern 2: The product of the left and right number is one less than the square of the center..

Proof: Their product can be written:

$(n - 1)(n + 1) = n2 + n - n + 1 = n2 - 1$

Five-Point: The average of the four corners is equal to the center.

Proof: If "n" is the center then the corners are $n - 11$, $n - 9$, $n + 9$, and $n + 11$. Thus the average is:

$[(n - 11) + (n - 9) + (n + 9) + (n + 11)] \div 4 = (4n) \div 4 = n$

Cross: The product of the top and bottom number is 99 less than the product of the left and right numbers.

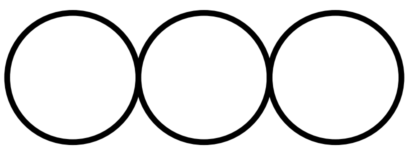
Proof: If “n” is the center number, then the product of the top and bottom numbers is:

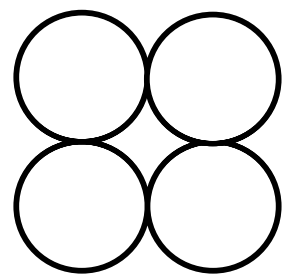
$(n - 10)(n + 10) = n^2 - 100$

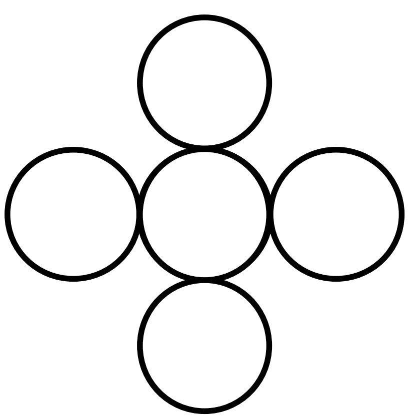
The product of the left and right numbers is:

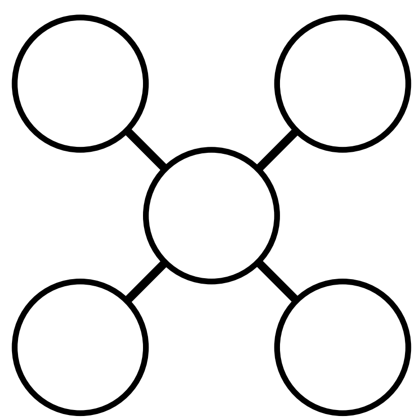
$(n - 1)(n + 1) = n^2 - 1$

and $(n^2 - 100) = (n^2 - 1) - 99$









Hundreds Magic: 1-100

Name_______________________________

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Hundreds Magic: -49-50

Name_______________________________

-49	-48	-47	-46	-45	-44	-43	-42	-41	-40
-39	-38	-37	-36	-35	-34	-33	-32	-31	-30
-29	-28	-27	-26	-25	-24	-23	-22	-21	-20
-19	-18	-17	-16	-15	-14	-13	-12	-11	-10
-9	-8	-7	-6	-5	-4	-3	-2	-1	0
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

SUN	MON	TUE	WED	THU	FRI	SAT

Algebra Man, and His Trek Through the Hundred's Chart

Grade Level: 6–8

By Brad Fulton
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www.tttpress.com

My 8th grade students enjoy discovering the algebraic patterns hidden within the hundreds chart. They are all familiar with this chart that they have seen since Kindergarten. One of our activities involves "Algebra Man." I display a transparency of the hundreds chart and a second transparency of Algebra Man on top of it as shown.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	72	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

I move the upper transparency around until a student yells, "Stop." Then the students race me to find the total of the numbers inside Algebra Man. The total for the placement shown is 580. I always win this contest and boast that it is because I am so talented at math. The students insist there is a trick and demand a rematch. After a few times, some

of the students see a pattern and discover the trick. (Turns out I'm not a rocket scientist after all!) Once a few discover my secret, we explore the problem using algebra.
The students notice that for any number on the chart, the number to the right is one more and the number to the left is one less. Also, the number beneath the given number is ten more and the number above it is ten less. This is always true no matter where Algebra Man runs. If we think of the number in his waist as n, then the following diagram shows the values of all ten numbers inside Algebra Man:

	n-20	
n-11	n-10	n-9
	n	
n+9	n+10	n+11
n+19		n+21

If we add the terms inside him, we get $10n+20$. It's a simple matter to look at the number in his waist (56), multiply it by ten (560), and add 20 (580). Many of my students say that they got the answer a different way. Often they simply look two spaces to the right of his waist and put a zero after the number. I express this algebraically on the board. The number two spaces to the right of his waist would be called $n+2$. Putting a zero after it is the same as multiplying by ten. Thus their method is $10(n+2)$. I show them that these are equivalent being the distributed and factored forms of the same expression:

$$10n+20 = 10(n+2)$$

Algebra Man works on any hundreds chart, even one that begins with -49 and ends with 50. He also works on a calendar, but the formula for his sum is slightly different. Challenge your students to find the formula when Algebra Man runs around on a calendar. This is just one of many algebraic explorations that can be made on the hundreds chart. If your students are like mine, they will want to explore all the "what if's" they suggest. One time my students wanted to find out what happens to Algebra Man's formula when he is standing on his head. Now *I* was curious.

Algebra Man Project Instructions

Getting ready:

1. Design a shape of **ten** cells on the hundreds chart.
2. Move it to three locations on your hundreds chart and calculate the sum. What pattern do you notice?
3. Find a centrally located cell (your key cell) and write a variable such as *x* or n in it.
4. Write the expressions for the other nine cells of your shape as shown in the top figure.
5. Combine like terms to find the total as shown in the second figure. You are now ready to begin your final project.
6. Show the results to your teacher

	n-20	
n-11	n-10	n-9
	n	
n+9	n+10	n+11
n+19		n+21

Designing your project:

7. Write the expressions **neatly** in the large grid cells and cut out the shape.
8. Neatly write the algebraic expressions in a column and total them. Simplify if necessary.
9. Neatly glue a hundreds chart onto your final paper.
10. Highlight your design on the hundreds chart.
11. Highlight your key cell.
12. Substitute the value of your key cell into the formula and calculate the sum.
13. Center your title and name at the top of the paper.
14. Your project should then look like the sample on the next pages.

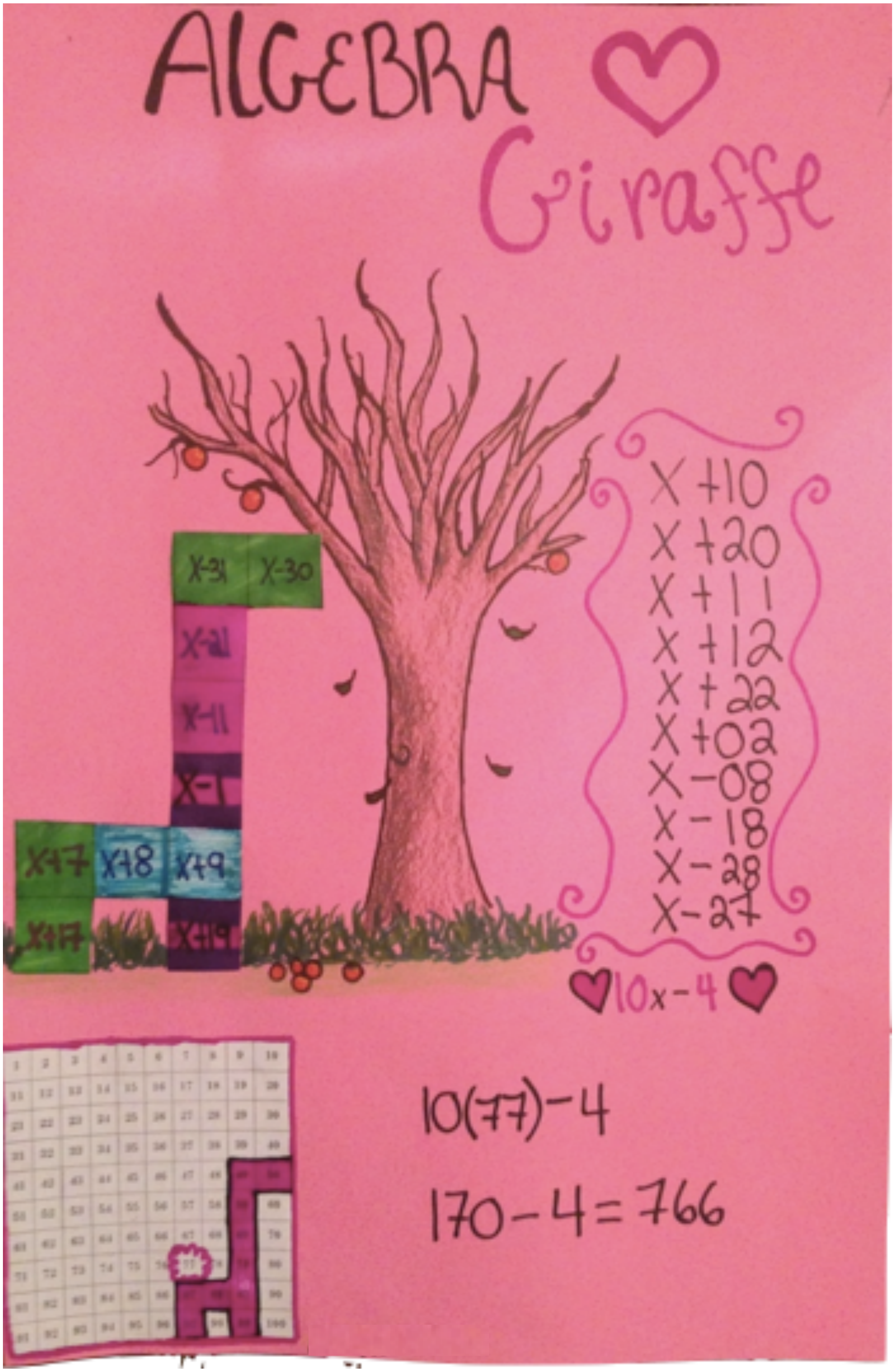
$$\begin{array}{r} n \\ n+10 \\ n-10 \\ n+11 \\ n-11 \\ n+9 \\ n-9 \\ n+21 \\ n-20 \\ +\ n+19 \\ \hline 10n+20 \\ 10(n+2) \end{array}$$

Grade Sheet (cut out and tape to the back of your project)

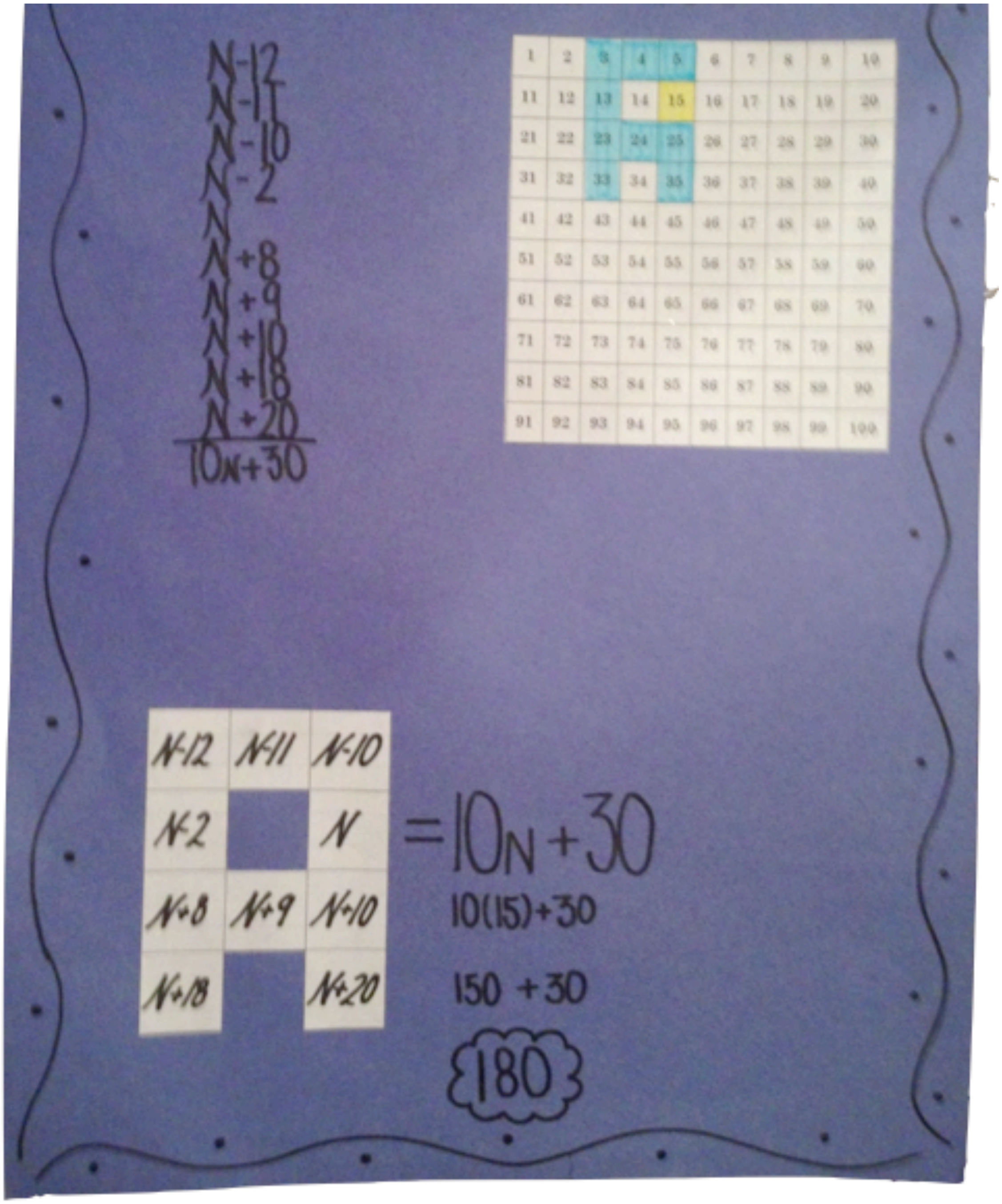
Mathematics .... _____/25
formula, combining like terms, example
Measurement .... _____/15
centered title, alignment of terms, parallel and perpendicular edges
Presentation .... _____/10
lettering, spelling, erasures creativity, coloring

Other ____

Total... _____/50



Student name withheld for privacy.



Student name withheld for privacy.

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