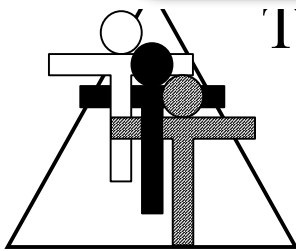


The Language of Math

Helping Students Speak, Write,
and Think Mathematically

By Brad Fulton



Teacher to Teacher Press

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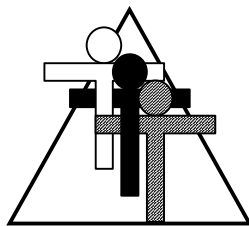
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Brad Fulton Educator of the Year

- ◆ Consultant
- ◆ Educator
- ◆ Author
- ◆ Keynote presenter
- ◆ Teacher trainer
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Known throughout the country for motivating and engaging teachers and students, Brad has co-authored over a dozen books that provide easy-to-teach yet mathematically rich activities for busy teachers while teaching full time for over 30 years. In addition, he has co-authored over 40 teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

Seminar leader and trainer of mathematics teachers

- ◆ 2005 California League of Middle Schools Educator of the Year
- ◆ California Math Council and NCTM national featured presenter
- ◆ Lead trainer for summer teacher training institutes
- ◆ Trainer/consultant for district, county, regional, and national workshops

Author and co-author of mathematics curriculum

- ◆ Simply Great Math Activities series: six books covering all major strands
- ◆ Angle On Geometry Program: over 400 pages of research-based geometry instruction
- ◆ Math Discoveries series: bringing math alive for students in middle schools
- ◆ Teacher training seminar materials handbooks for elementary, middle, and secondary school

Available for workshops, keynote addresses, and conferences

All workshops provide participants with complete, ready-to-use activities that require minimal preparation and give clear and specific directions. Participants also receive journal prompts, homework suggestions, and ideas for extensions and assessment.

Brad's math activities are the best I've seen in 38 years of teaching!

Wayne Dequer, 7th grade math teacher, Arcadia, CA

"I can't begin to tell you how much you have inspired me!"

Sue Bonesteel, Math Dept. Chair, Phoenix, AZ

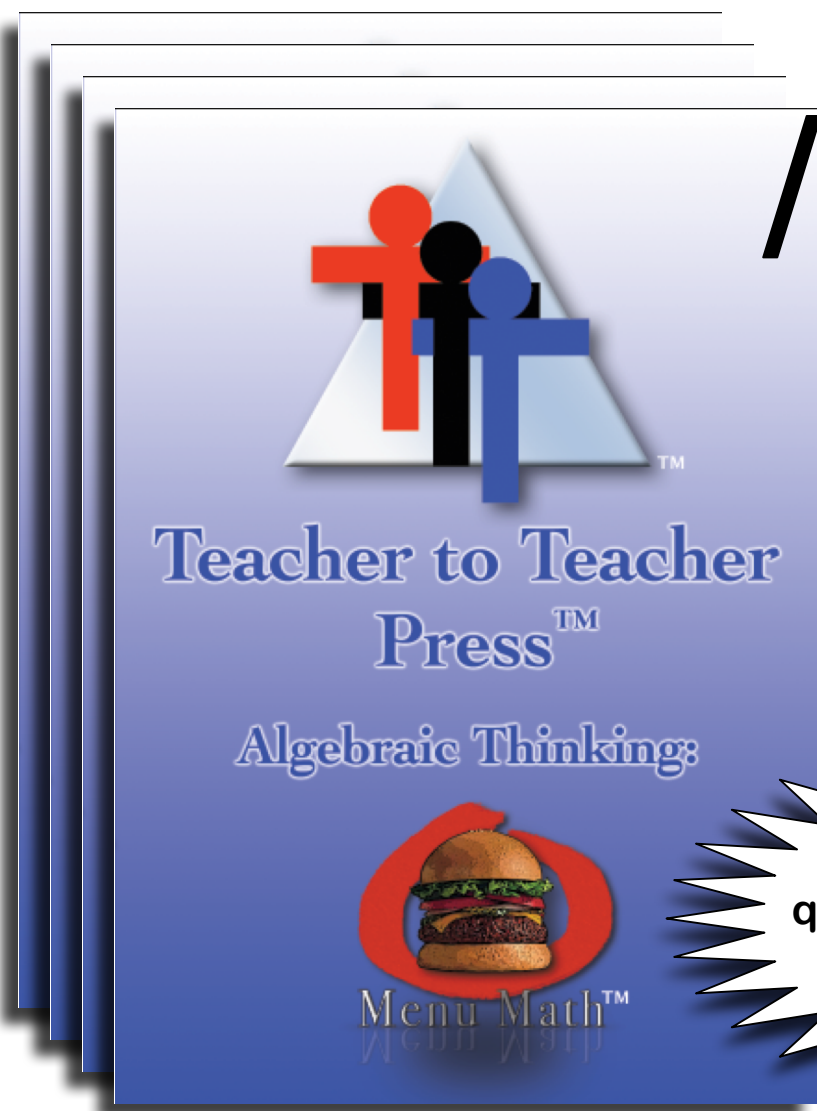
"Your entire audience was fully involved in math!! When they chatted, they chatted math. Real thinking!"

Brenda McGaffigan, principal, Santa Ana, CA

"Absolutely engaging. I can teach algebra to second graders!"

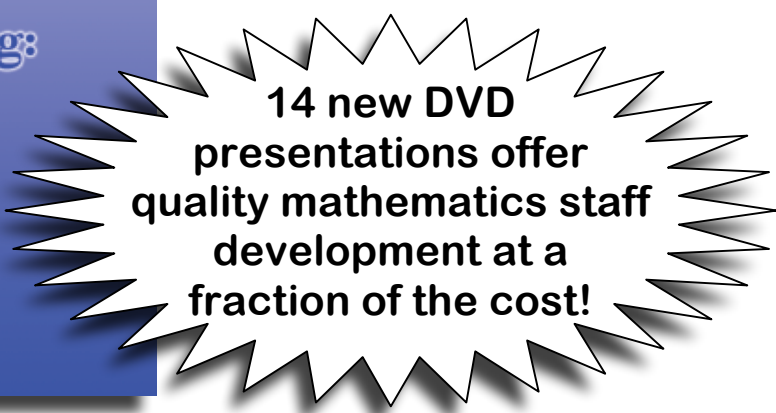
Lisa Fellers, teacher

References available upon request



- a) effective staff development
- b) affordable staff development
- c) ongoing staff development

(pick three)



- Fast Facts and Fractions:** Help students master their multiplication facts and learn simple strategies for taming fractions.
- Hundreds Magic:** An engaging exploration of arithmetic, number sense, algebra, and mathematical reasoning.
- Number Line:** Help students compare and order fractions, decimals, and percents while developing reasoning skills.
- Safely Navigating Social Networks:** Help your students stay safer at home and school. Great for parents and staff
- Solving Linear Equations:** Simple steps and strategies to help your students find success with equations.
- Multiplying and Factoring Polynomials:** Help students learn the seamless way to tackle polynomials.
- Take Your Places:** A rich and engaging activity integrating number sense, operations, probability, and algebra.
- Integer Strategies:** Help students overcome integer operations with these classroom-tested strategies.
- Teaching 2-Digit Multiplication:** Use *Conceptual Layering* to maximize mathematical skill and reasoning.
- Leo's Pattern:** Learn how to use *conceptual layering* to help students transition from simple addition to algebra.
- Math Projects:** Emancipate the intelligence of your students with an authentic assessment and teaching strategy that will amaze both you and your students.
- Menu Math:** Students from grades two through college have finally made sense of algebra with this clever approach.
- The Power of Two:** Finally students understand exponents, the zero power, and even *negative* exponents!
- X Marks the Spot:** Practice with the four operations should be engaging, enriching, and empowering. Find out how to maximize the effectiveness of drill work with this easy approach.

Brad Fulton is a mathematics teacher in Redding, California. He is a contributor to the California Math Communicator and a frequent presenter at math conferences throughout the United States.

He has authored and co-authored over a dozen books and staff development DVDs. For ordering information, visit us online at www.tttpress.com.

In this resource manual you will find quotes from research and articles about mathematics. Quotes by Larry Bushman, Margaret E. McIntosh, Karen S. Norwood, and Glenda Carter were taken from *Activities for Junior High School and Middle School Mathematics*, published by the National Council of the Teachers of Mathematics. The quote by Renate Nummela Caine and Geoffrey Caine is taken from *Making Connections: Teaching and the Human Brain*, published by Addison Wesley Innovative Learning Publications. All other quotations were taken from various sites on the internet.

*“Reading makes a full man.
Speaking makes a ready man.
Writing makes an exact man.”*

Francis Bacon

Why Language Matters in Math

Three key reasons to incorporate language into your math classroom.

- Language *shows* us what a student is thinking.
- Language takes a student to a deeper level of understanding.
- To speak or to write well is to think well.

Oral and written language offers other benefits:

Language offers realistic and accurate assessment.

Students need time for communicating mathematical understanding and problems they have with it.

Promoting language helps prevent and avert classroom management problems.

When you grade homework, is it truly the student's work? This is never an issue with language.

Suppose a child wrote:

$$2^4 = 8$$

List what this tells you about this child's knowledge and understanding.
What does the child know and not know?

“As we move into the era of an information-based world, the value of mathematics as a former of our culture is going to become greater and greater.”

Kevin Kelly

Now suppose the child had been asked to write or explain the meaning of 2^4 , and had written this response:

2^4 is an example of an exponent. The two is called the “base” and the four is the “exponent.” The exponent tells us how many times to multiply the base by itself. It means the two is multiplied four times like this:

$$2 \times 2 \times 2 = 8$$

Now take a minute to explain what this child knows and understands.

“Answers alone often fail to reveal the nature of a student’s thinking, the strategies used in the problem-solving process, or the level of understanding.”

Larry Buschman

Oral or Written Language?

Written Language:

- More formal
- Higher-level thinking
- Deeper understanding
- Audience of one
- Time intensive
- Can be refined
- More precise
- Management issues arise

Oral Language

- Less formal
- High-level thinking
- Deep understanding
- Audience of many
- Requires less time
- Usually not edited
- Less accurate
- More easily managed

Of course, you don't have to use only one format. You can incorporate both oral and written language to suit the needs of the task at hand. Use oral language for quick work that benefits the whole class. Use written language when time allows for the greater degree of sophistication and understanding it offers.

“Brain-based education involves ensuring that students process experiences in such a way as to increase the extraction of meaning.”

*Renate Nummela Caine and
Geoffrey Caine*

Creating a Climate For Mathematically Rich Discussions:

The strategies that a teacher employs to encourage students to *speak* mathematically will do more for improving the quality of mathematical *thinking* than any other single factor. There are a few strategies and a few specific questions that can be easily learned and quickly applied.

- Use wait time. After posing a question, wait at least five seconds before calling on a student.
- Call on all students. Try to avoid calling only on those who raise their hands.
- Avoid repeating a student's words. When necessary, ask another student to paraphrase the explanation of the first student.
- Encourage students to respond to one another.
- Always ask for alternative ways to solve a problem.
- Give credit to a student's solution by writing their name by it. Refer to the method as "Jill's way."
- Provide independent and collaborative think time. Let students explore problems for up to a minute or so before they discuss their thinking with others.
- Avoid calling on students to see if they are paying attention.
- Validate a student's thinking by asking, "Who else solve the problem the same way Antonio did?"

Remember: the teacher's goal is to communicate respect for student thinking.

10 Quick Questions For Good Discussions:

- How did you solve the problem?
- What are the advantages of solving the problem that way?
- What patterns do you notice?
- Will that work for every problem like this one?
- How did your brain get that answer?
- Can you paraphrase what that student said?

“When students communicate mathematical information, they remember it, understand it, and use it to uncover and find even more information. (Perkins 1992)”

Larry Buschman

A Good Question Gets a Good Answer

How to Create the Perfect Writing Prompt

Asking a well-worded question goes a long way to eliciting well-worded responses, and consequently, well-organized mathematical thinking.

Simply asking a student to write about math will invite rambling generalizations that use lots of ink to say very little mathematically. Writing questions the right way is not a difficult skill to learn. Just make sure each question uses all four of these key components.

- Audience: To whom is the student addressing the writing?
- Voice: What character is writing? From what point of view?
- Format: What kind of writing is it: a note, a poem, a list, a paragraph?
- Specific Topic: What *exactly* should the student address: fractions in general, or how to find a common denominator?

Here are a few examples:

- As a worker bee, write a letter to the queen explaining why hexagons should be used for the hive.
- Pretend you are the teacher. Write a note to an absent student explaining how to do today's assignment.
- As a tour guide, write directions from your classroom to the cafeteria.

How to Survive the Paperwork

Students will only write their best if they know you value their work. We show this by monitoring their writing. How do we do this without burying ourselves in work? Fortunately, there are ways to make this possible.

- Have students share their writing orally with one another.
- Read only selected entries each week.
- Read only one row of students or one class of students each day.
- Read entries during commercials as you watch television
- Read entries instead of correcting homework. (See the following page.)

“Clearly explain your expectations to your students, and give them details concerning how their writing will be evaluated.

Respond to the students’ writing on a frequent basis.

Join the students in writing. Address the topic from a teacher’s point of view. Share your writing with your students.”

Karen S. Norwood and Glenda Carter

“A better understanding of students’ thinking in mathematics is needed if mathematics education is to be improved.”

*Karen S. Norwood and
Glenda Carter*

Grading Math Journals versus Correcting Homework

Let's face it, correcting homework papers is boring. There are even more reasons why this task is not the best use of our time.

- A wrong answer doesn't tell me what they don't understand.
- A correct answer doesn't tell what they do understand.
- Some students use calculators, and others don't.
- Some parents help with homework, others don't.
- Students often copy homework.
- Dogs eat homework.

However, dogs never eat math journals. In addition, it's pretty obvious if it is copied or written by a parent. Calculators are of little use in writing mathematically. More importantly, we often know exactly what a child does and does not know when we read their writing.

It is still *vitaly important to grade homework*. However, that does not mean we have to *correct* each problem. We can correct representative problems or give a grade for completion. We can also assign practice that is self-assessing. The point is that to determine understanding and assign it a grade, we are better off looking at a student's explanation of the process than at answers to drill work.

The following pages contain journal prompts separated into four areas: classroom management, self-esteem, general mathematics, and specific mathematics. Our book, *The Language of Math*, available on our website: www.tttpress.com offers many more..

Journal Prompts

Class Management

1. List ten rules you think are important in this class.
2. What question would you like to ask your teacher? Why?
3. What could parents do to get their child to study more?
4. From this room of students, who would be in your ideal group of four? Why?
5. Are you satisfied with your grade? Why?
6. What did you do to earn your grade?
7. What advice would you give to someone new to this class?
8. How would you answer a student who says “I don’t get it?”
9. What did you think of the test? How do you think you did? Why?
10. What qualities are important in a teacher? In a student?

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Self Esteem/General

1. What do you think I think of you?
2. How can you help yourself be a better student?
3. If you can have anyone at your Thanksgiving table, who would you have and why?
4. What is one thing you want to do for your family?
5. Which is more important in school, to be smart or to be a hard worker?
6. How have you grown this year?
7. Rank ten events from your life that were very important to you. Begin with the most important.
8. Pretend you are a pencil and tell me about your day.
9. What words do your friends use when they describe you?
10. Give yourself a sincere compliment. Now give a sincere compliment to the person on your left.

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Curriculum: General

1. What would you say was the main idea of today’s lesson? What one question would you like to ask about today’s lesson?
2. How would you explain today’s lesson to somebody in the fourth grade?
3. Does having a test make you study harder? Why or why not?
4. What do you think of today’s math test?
5. Name ten things you have done that required math but were not part of this class. Choose things that happened as recently as possible.
6. Which would take more energy, to run up a short, steep hill, or to run up a long gradual hill?
7. Of what we have studied so far this year, what will be the most important to you in getting a job?
8. Explain if you agree or disagree with this statement, “The more math you know, the more money you will earn.”
9. Write a challenging word problem on today’s topic, then solve it.
10. If “M.A.T.H.” were an acronym, what might it stand for?

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Curriculum: Specific

1. What does “fair” mean in a contest? Give an example of something that is fair.
2. What would you do if you were 5 cm tall?
3. What would you do if you were 5 meters tall?
4. Why is a common denominator necessary?
5. What are surveys? Why do we have them?
6. Give examples in life of how positives cancel out negatives.
7. How are percents, fractions, and decimals similar? How are they different?
8. What does “fraction” mean?
9. Which will be greater: twice your age in four years, or three times your age in two years?
10. How would you measure the distance to a planet or to the sun?

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The most important mode for students' writing about mathematics is...as a follow-up to a lesson. Writing is especially effective when it follows hands-on activities, since in those exercises the most active learning takes place.

*Learning to write about
mathematics
Arithmetic Teacher,
S. Wilde*

Mr. Infinity's math class is so popular everyone is trying to get in. Here is his seating chart. If you were student 100, explain how you could find the row and column of your seat.

Column	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>
<i>Row 1</i>	1	2	3	4	5	6	7
Row 2	8	9	10	11	12	13	14
Row 3	15	16	17	18	19	20	21
Row 4	22	23	24	25	26	27	28

Mr. Infinity has gotten creative. Here is the seating chart for his popular math class. Explain in what row and seat student 100 would sit.

Row 1

1	2	3	4	5	6	7
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Row 2

13	12	11	10	9	8
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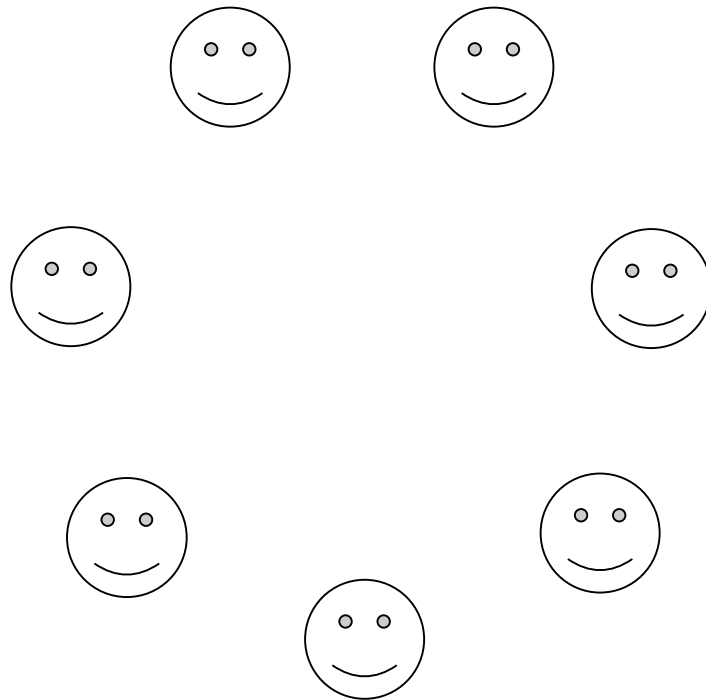
Row 3

14	15	16	17	18	19	20
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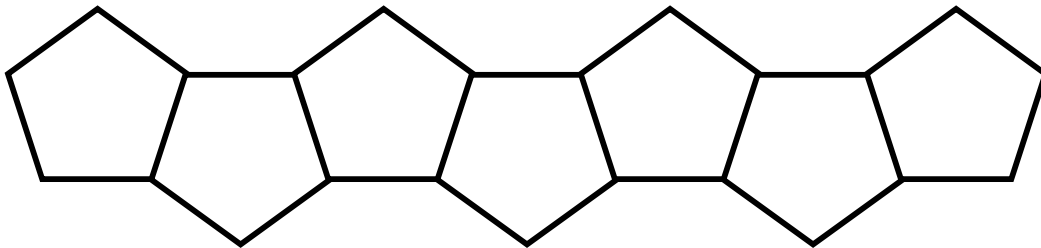
Row 4

26	25	24	23	22	21
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How many handshakes would be required for all these friends to introduce themselves to one another?



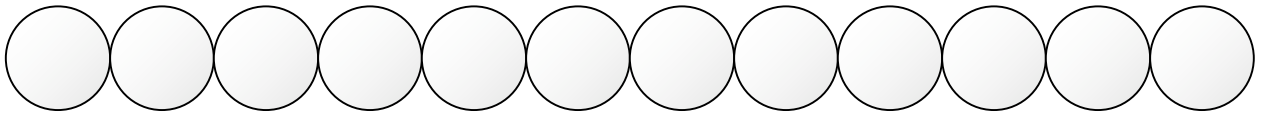
How many people can be seated at this arrangement of tables? People can sit on each exposed side or end. What if eleven tables are used. What if 100 tables were used?



“The development of a student’s power to use mathematics...is best accomplished in problem situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural.”

Margaret E. McIntosh

Twelve marbles are in a row. One fourth of them are yellow. There is one more of red than blue. There is one less green marble than blue. Explain how to find out how many of each color of marble are in the row.



Which of these fractions is greater? Explain your reasoning.

$$\frac{7}{13} \quad \frac{5}{9}$$

Make a reasonable estimate for this product.
How do you know if your estimate is
accurate? How close is it? Is it too high or
too low?

$$54 \times 82$$

*“Mathematics is the only
universal language there is,
Senator.”*

*Jodie Foster
in the film Contact*