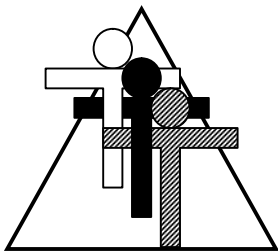


# Building Number Sense Through Engaging Instruction

RESOURCE HANDOUT

By  
Brad Fulton and Bill Lombard



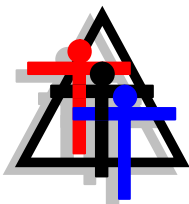
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Brad Fulton and Bill Lombard

***Teacher to Teacher Press***

*"Building Mathematical Skill on a Foundation of Understanding"*



Brad Fulton

- ◆ **Consultants**
- ◆ **Educators**
- ◆ **Authors**
- ◆ **Seminar leaders**
- ◆ **Teacher trainers**
- ◆ **Conference speakers**



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Known throughout the country for motivating and engaging teachers and students, Brad and Bill have authored over ten books that provide easy-to-teach yet mathematically-rich activities for busy teachers. In addition, they have co-authored six teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

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- ◆ California Math Council and NCTM presenters
- ◆ Lead trainers for summer teacher training institutes
- ◆ Trainers/consultants for district, county, regional, and national workshops

Authors and co-authors of mathematics curriculum

- ◆ *Simply Great Math Activities* series: five books covering all major strands
- ◆ *Math Discoveries* series: bringing math alive for students in middle schools
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Available for workshops, keynote addresses, and conference sessions.

All workshops provide participants with complete and ready-to-use activities. These activities require minimal preparation, use materials commonly found in classrooms, and give clear and specific directions and format. Participants will also receive journal prompts, homework suggestions, and ideas for extensions and assessment.

*Brad and Bill's math activities are the best I've seen in 30 years of teaching!*

Wayne Dequer, 7th grade math teacher

*"The high-energy, easy-to-follow handouts were clear. The instructors were great!"*

DeLinda Van Dyke, middle school teacher

***References available upon request.***

## ACTIVITY 4

# Take Your Places

### Materials:

- paper
- activity master

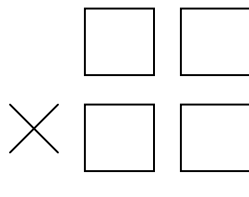
**Overview:** “Drill without kill!” is the motto. Here is a way to practice new skills without boring your students to death. They can even increase their number sense as they explore the possible solutions, or use the activities to study probability. What a bargain!

**Vocabulary:** sum, difference, product, quotient, divisor, dividend, numerator, denominator, operation

## PROCEDURE

### Skills:

- Understanding place value
- Building number sense in all operations
- Using probability



- 1 Have the students copy the diagram on the left as you write it on the board.
- 2 Number ten chips zero through nine and place them in a container. Alternately you may use a die or a spinner to generate random numbers.
- 3 Choose one number from the container. Each student must decide where to place that number to achieve the highest possible product.
- 4 When each student has written the number in a box, choose another number and repeat step three. Do not replace the numbers that have been chosen. If you are using a die or spinner, ignore a number if it occurs a second time. Do this until all boxes are filled.
- 5 Have students compute their product. Who has the largest?
- 6 The power of this game comes when students play it repeatedly. Where should the four numbers be placed to achieve the largest possible product?
- 7 For variations, choose the game format from the activity master that best suits the current needs of your students, or create your own problem.
- 8 On game C, try to get the greatest or least possible quotient. Then try to achieve the greatest or least possible remainder.

### Good Tip!



Have students write with ink and sit with a partner to avoid changing their placement of the numbers. Allowing students to change an answer circumvents the learning of probability in this activity.

9 On game D, decide in advance whether this is an addition, subtraction, multiplication, or division problem.

10 To extend number sense even more, pose this question:

Four different numbers,  $a$ ,  $b$ ,  $c$ , and  $d$ , are selected such that  $a > b > c > d$ . They are paired to form two-digit numbers. Two of the numbers could be  $ac$  and  $bd$ . Would the number  $ab$  be between these two numbers? How many numbers could you form that would fall between  $ac$  and  $bd$ ?



### Journal Prompts:



What patterns or rules did you learn that helped you decide where to place a number?

If the first number chosen is a six, where would you place it? Why?

### Homework:



Have students show all the possible arrangements for a given set of numbers in a problem and the answers that can be achieved.

For example, if you are multiplying a two-digit number times a two-digit number on problem A, there are 24 possible problems that can be created with four given numbers. However, they do not generate 24 different answers.

### Taking a Closer Look:



Have students find which arrangements minimize a solution.

Have them try to hit a target number.

Using a set of chips numbered zero through nine encourages students to think about the probability as they make their decisions. For example, if a seven is drawn first, where should it be placed considering there are only two higher numbers?

### Assessment:



Ask students to tell their solutions as you list them on the board.

Then ask if another student has a greater answer. Students can also compare papers to check their results.

For four different numbers  $a$ ,  $b$ ,  $c$ , and  $d$ , such that  $a > b > c > d$ , the numbers should be placed in one of the two arrangements shown on the right to maximize the product. You are trying to create two numbers that have the least difference as possible.

$$\begin{array}{r} \begin{array}{|c|} \hline a \\ \hline \end{array} \begin{array}{|c|} \hline d \\ \hline \end{array} \\ \times \begin{array}{|c|} \hline b \\ \hline \end{array} \begin{array}{|c|} \hline c \\ \hline \end{array} \\ \hline \end{array}$$

$$\begin{array}{r} \begin{array}{|c|} \hline b \\ \hline \end{array} \begin{array}{|c|} \hline c \\ \hline \end{array} \\ \times \begin{array}{|c|} \hline a \\ \hline \end{array} \begin{array}{|c|} \hline d \\ \hline \end{array} \\ \hline \end{array}$$

# Take Your Places A

Name \_\_\_\_\_

Your teacher will select four numbers, one at a time. Place them in the boxes to try to maximize the product.

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# Take Your Places B

Name \_\_\_\_\_

Your teacher will select four numbers, one at a time. Place them in the boxes to try to maximize the difference.

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# Take Your Places C

Name \_\_\_\_\_

Your teacher will select four numbers, one at a time. Place them in the boxes to try to maximize the quotient.

□		□	□	□
---	--	---	---	---

□		□	□	□
---	--	---	---	---

□		□	□	□
---	--	---	---	---

□		□	□	□
---	--	---	---	---

# Take Your Places D

Name \_\_\_\_\_

Your teacher will select four numbers, one at a time. Place them in the boxes to try to maximize the result.

$$\frac{\square}{\square} ? \frac{\square}{\square} =$$

$$\frac{\square}{\square} ? \frac{\square}{\square} =$$

$$\frac{\square}{\square} ? \frac{\square}{\square} =$$

$$\frac{\square}{\square} ? \frac{\square}{\square} =$$

$$\frac{\square}{\square} ? \frac{\square}{\square} =$$

$$\frac{\square}{\square} ? \frac{\square}{\square} =$$

# Take Your Places E

Name \_\_\_\_\_

Your teacher will select three numbers, one at a time. Place them in the boxes to try to maximize the result.

$$\frac{\square - \square}{\square} =$$

$$\frac{\square - \square}{\square} =$$

$$\frac{\square - \square}{\square} =$$

$$\frac{\square - \square}{\square} =$$

$$\frac{\square - \square}{\square} =$$

$$\frac{\square - \square}{\square} =$$

# Take Your Places F

Name \_\_\_\_\_

Your teacher will select three numbers, one at a time. Place them in the boxes to try to maximize the result.

$$\square \left( \square - \square \right) =$$

$$\square \left( \square - \square \right) =$$

$$\square \left( \square - \square \right) =$$

$$\square \left( \square - \square \right) =$$

$$\square \left( \square - \square \right) =$$

$$\square \left( \square - \square \right) =$$

# Take Your Places G

Name \_\_\_\_\_

Your teacher will select four numbers, one at a time. Place them in the boxes to try to maximize the result.

$$\frac{\square \left( \square - \square \right)}{\square} =$$

$$\frac{\square \left( \square - \square \right)}{\square} =$$

$$\frac{\square \left( \square - \square \right)}{\square} =$$

# Take Your Places H

Name \_\_\_\_\_

Your teacher will select four numbers, one at a time. Place them in the boxes to try to maximize the result.

$$\frac{\square (\square + \square)}{\square} =$$

$$\frac{\square (\square + \square)}{\square} =$$

$$\frac{\square (\square + \square)}{\square} =$$

## ACTIVITY 1

### Materials:

- paper
- transparency master
- activity master

# Pyramid Math

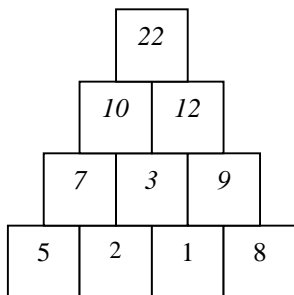
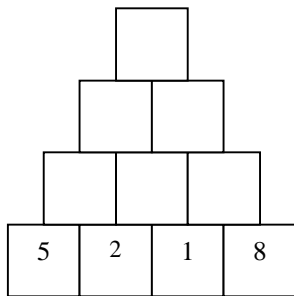
**Overview:** This creative practice activity facilitates discovery of number patterns and develops number sense. The activity works great with both positive and negative numbers and decimals and fractions. Because it can be designed to be self-checking, it is easy for the teacher and engaging for the students.

**Vocabulary:** commutative property

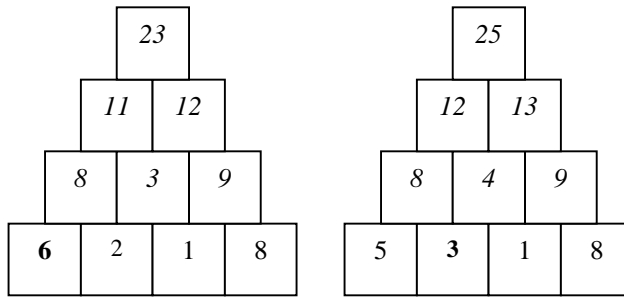
## PROCEDURE

### Skills:

- Addition and subtraction of integers, decimals, and fractions
- Finding patterns

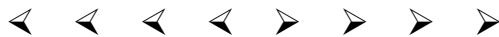


- 1 Display the transparency master and enter four numbers in the cells of the bottom row as shown. (Use single-digit whole numbers at first so students can focus on the structure of the problem instead of stumbling on the computation at this point.) To solve the pyramid, an adjacent number pair is added. The sum is written in the box above the number pair. This is repeated for the other number pairs in the bottom row. Then this process is repeated for the second row to fill the third row. Finally the number pair in the third row is added to get the final top number as shown.
- 2 Since each sum is based on the sums below, all students should get the same answer in the top cell. Thus they only need to check the top answer. If that is correct, all other cells are likely correct too.
- 3 Now try another pyramid using new numbers. Students will catch on to the process quickly and will be eager to check their answer with those of their classmates. (No more correcting papers!)
- 4 As students understand how the problems work, introduce appropriate numbers. If you are studying decimals, throw in a few decimal points. If you have covered integers, use some negative numbers. Fractions make these problems much more difficult. Try one yourself before asking the students to do one. We suggest beginning with like denominators. Or you could use fractions that have a fairly small common denominator. For example,  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{1}{2}$ , and  $\frac{3}{4}$  can all use fourths for a common denominator.
- 5 Try to make slight variations in the arrangement and values of the numbers to help children focus on the number sense involved. For example, in the first problem if we increase the five by one, making it a six, the top number also increases by one, but if we change the two to a three, the top number increases by three.



Is this always true when we add one to a cell? What would happen if we added two to the first or second cell of the bottom row? What happens when we do this to a five-row pyramid? As students answer these questions they will develop number sense

- 6 Ask students to change the order of the numbers in the bottom row of a pyramid. How does this affect the top cell? Is the result always the same? How does the commutative property affect this result?
- 7 If everyone puts the same number in the top cell of a blank pyramid, will everyone get the same bottom row by working backwards? Why or why not?
- 8 Introduce subtraction by using Pyramid Math 6 in which other cells are filled in. You can create one of your own easily, or have students create them for their classmates to solve.
- 9 Explore what happens when all odd numbers or all even numbers are used. What if all four cells in the bottom row contain the same number? What patterns occur



**Journal Prompts:**



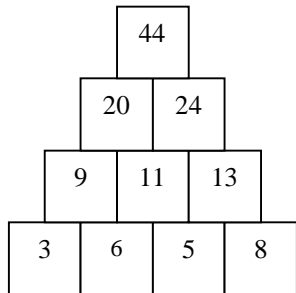
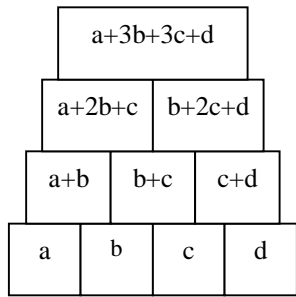
If you rearrange the numbers on the bottom row of a pyramid, will you always get the same numbers on top? Why or why not?

What can you predict about the number on the top of a four-row pyramid if all the starting numbers are equal? Does the number of rows in the pyramid affect this? In what way?

**Homework:**



Use one of the accompanying activity masters or tailor one to your students' needs using one of the blank masters.



### Taking a Closer Look:

There is a way of predicting the top of the pyramid without solving all the rows. This leads students into the algebra involved in the process. For example, let's assume that we are going to solve a four-row pyramid. The bottom cells contain four numbers called a, b, c, and d as shown. It follows that the second row contains three sums which are a + b, b + c, and c + d respectively. The third row contains these two sums:

$$(a + b) + (b + c) \text{ and } (b + c) \text{ and } (c + d)$$

These simplify into a + 2b + c and b + 2b + c. Adding these to get the top row gives a + 3b + 3c + d.

Now let's start with four numbers: a = 3, b = 6, c = 5, and d = 8.

Using the formula, the top answer should be:

$$\begin{aligned} a + 3b + 3c + d &= \\ 3 + 3(6) + 3(5) + 8 &= \\ 3 + 18 + 15 + 8 &= \\ 44 & \end{aligned}$$



### Assessment:

These activities can be made self-assessing by writing the answers at the bottom of the page. As students solve each pyramid, they can cross off the answers. If they get an answer that is not listed, they know they have made a mistake and can try again.

**Answer Key:**

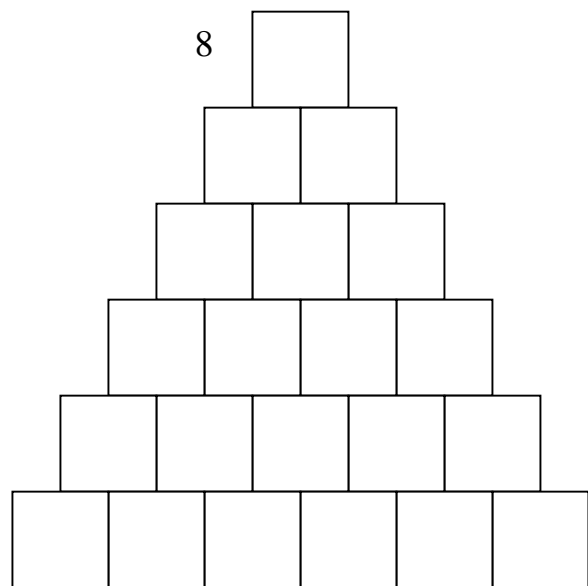
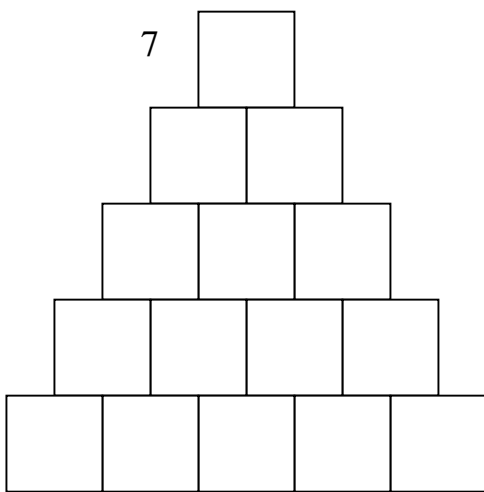
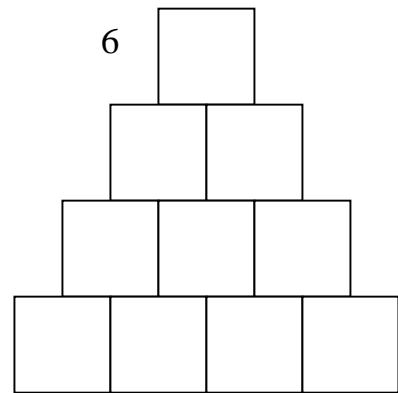
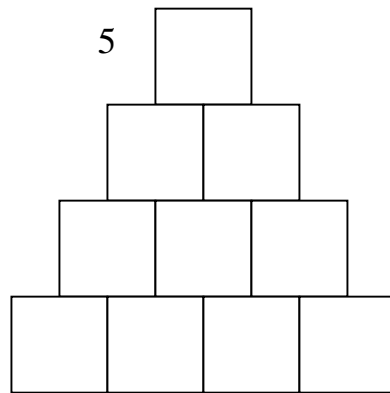
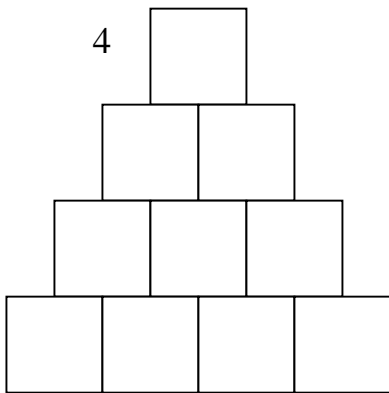
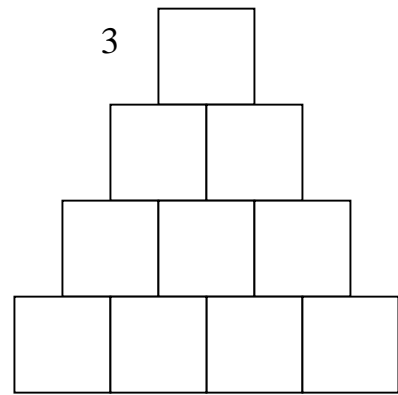
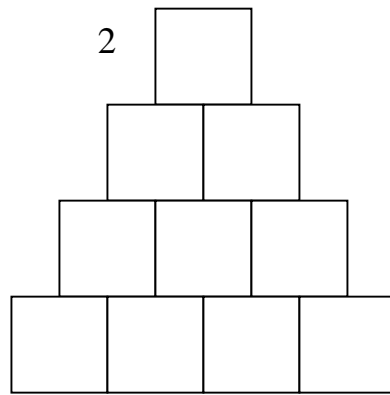
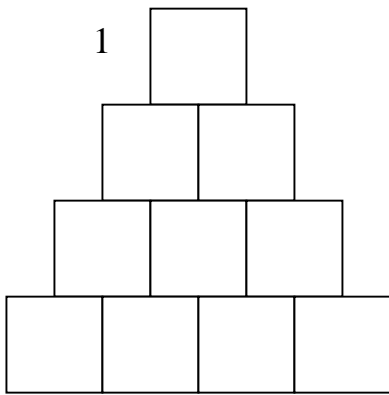
Problem	Pyramid Math Worksheet Number				
	1	2	3	4	5
1	33	16.5	21	267	$4 \frac{2}{5}$
2	24	10.2	-21	224	$3 \frac{3}{7}$
3	40	4	11	377	
4	32	3.2	-20	264	
5	60	2.05	-8	312	
6	68	23.86	0	332	
7	80	6.84	24	222	
8	23	16	-34	308	
9	23	21.81	0	332	
10	100	3.08	100	363	
11	106	1	1	528	
12	188	1.76	9	2000	

(For sets six through eight, these are the answers to the bottom row of each pyramid.)

Problem	Pyramid Math Worksheet Number		
	6	7	8
1	8, 3, 2, 2	.3, .1, .7, .6	6, -4, 5, -2
2	3, 4, 4, 2	.5, .6, .2, .5	8, -12, 6, -8
3	11, 3, 2, 10	.6, .2, .5, .5	8, 6, -12, -8
4	1, 0, 11, 23	3, .6, .9, 1	-3, -6, -9, -14
5	7, 12, 10, 11	.7, .9, .9, 2	-8, 0, 19, -20
6	19, 0, 6, 26	.9, 0, .5, .9	-7, -6, 16, -16
7	27, 0, 0, 27	1.2, 6.3, 2.7, 3.1	21, -9, -19, -8
8	8, 19, 10, 9	17, 1.9, .8, 6.4	11, 30, -16, -14
9	31, 26, 19, 17	8.7, 7.6, 6.5, 5.4	11, -30, -16, 14
10	18, 2, 0, 27	.29, .16, .5, 1.2	-17, -3, 3, 17
11	5, 38, 19, 31	7.1, .99, .09, .5	-15, -5, 6, 13
12	13, 6, 34, 9	7.7, .77, .55, 5	18, -9, 6, -3

# Pyramid Math

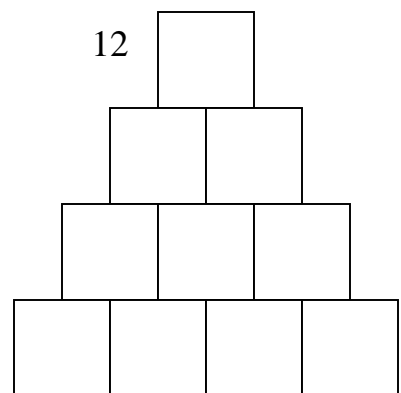
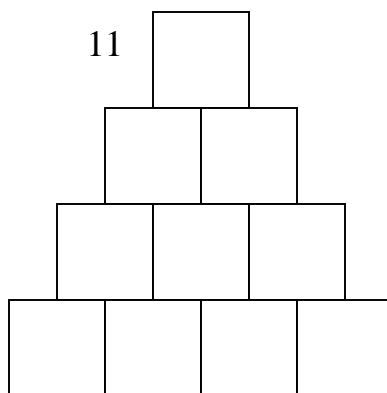
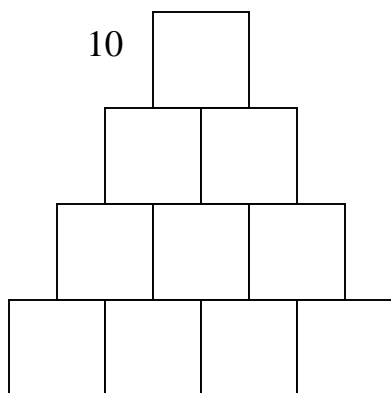
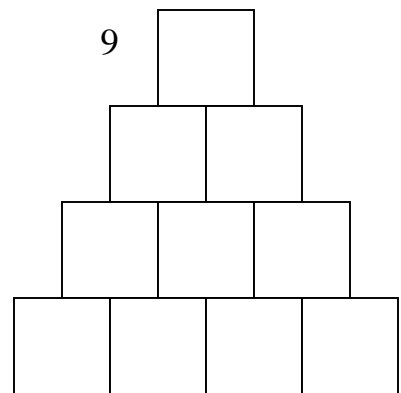
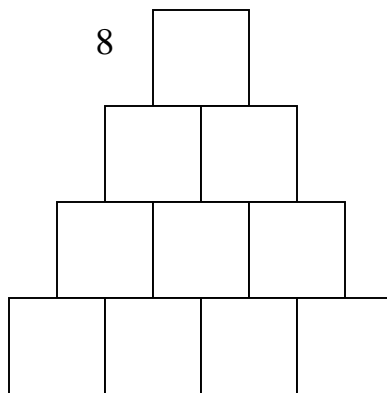
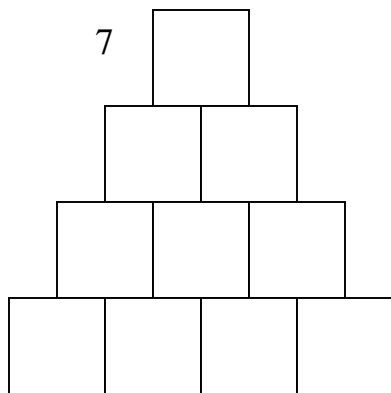
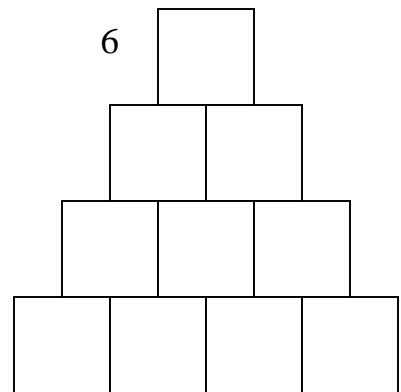
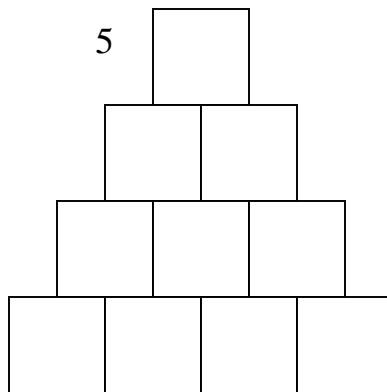
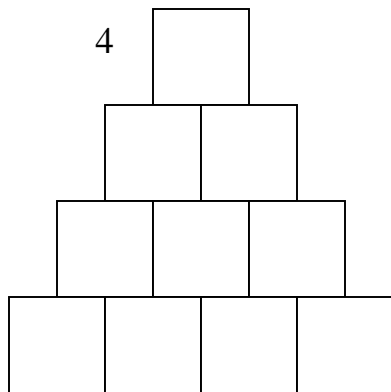
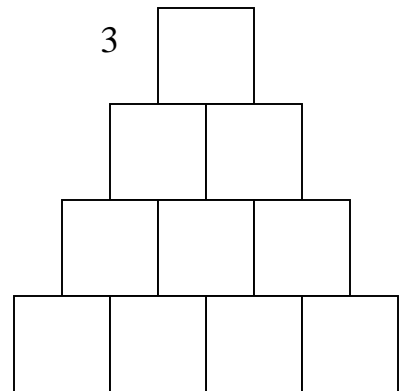
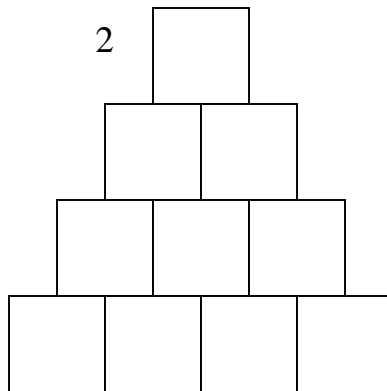
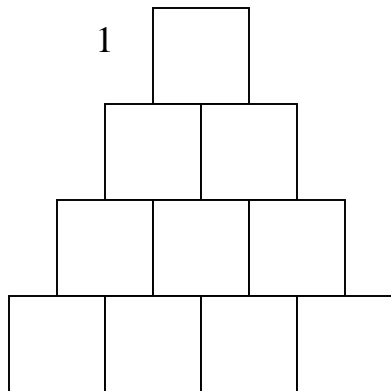
Add pairs of adjacent numbers and write their sums above them. Keep going until you reach the top of the pyramid.



# Pyramid Math

Name \_\_\_\_\_

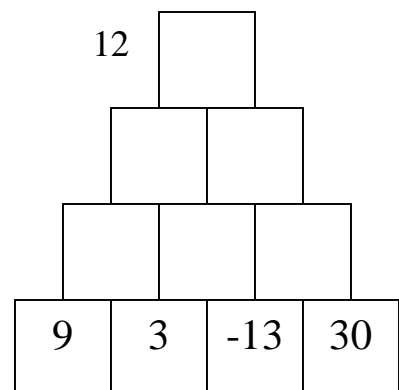
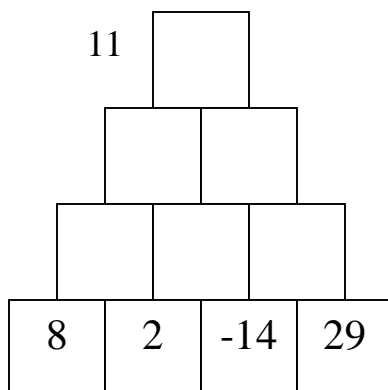
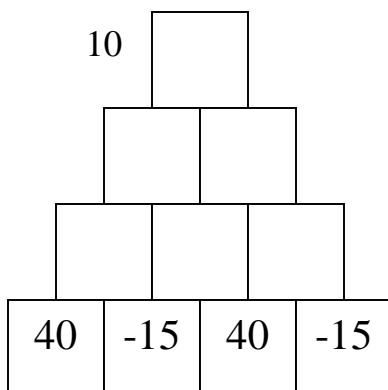
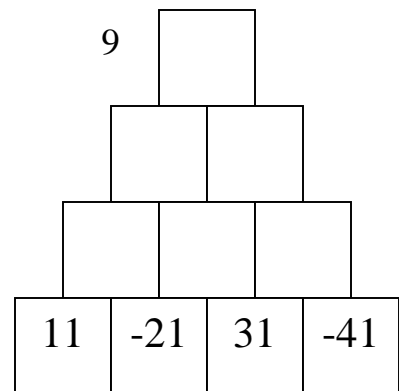
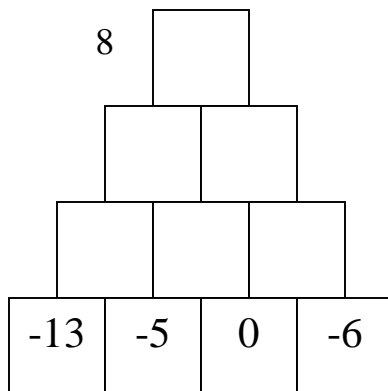
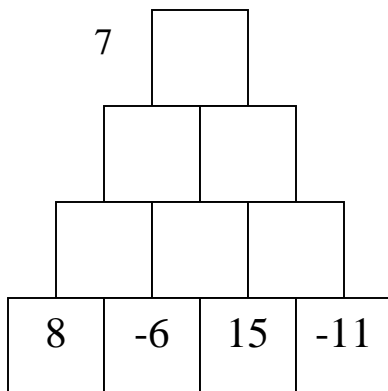
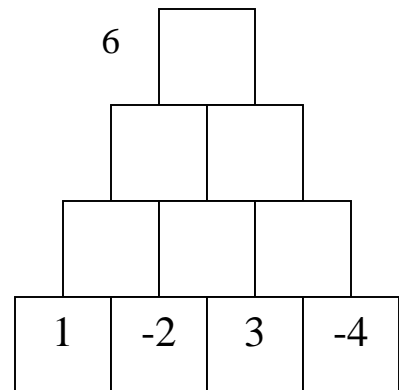
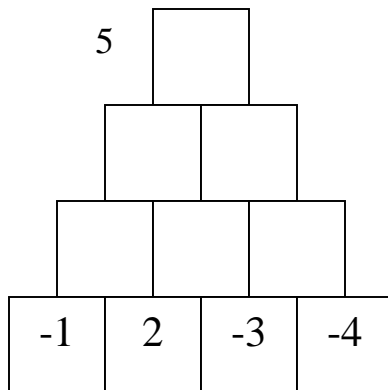
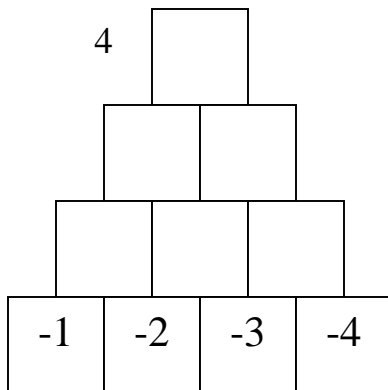
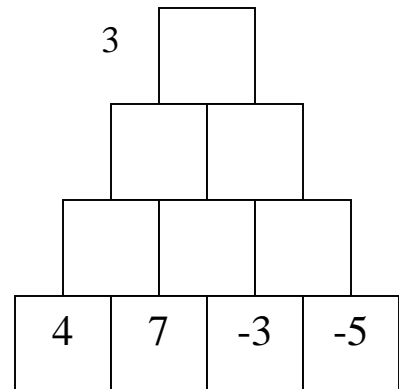
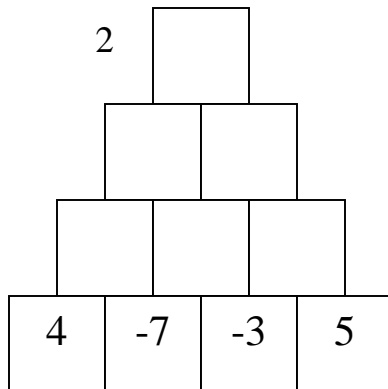
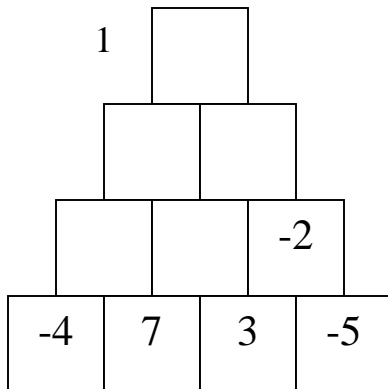
Add pairs of adjacent numbers and write their sums in the box above them. Keep going until you reach the top of the pyramid.



# Pyramid Math 3

Name \_\_\_\_\_

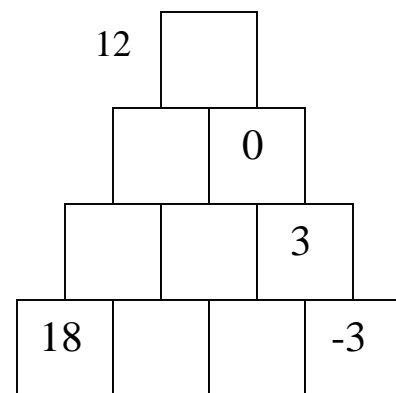
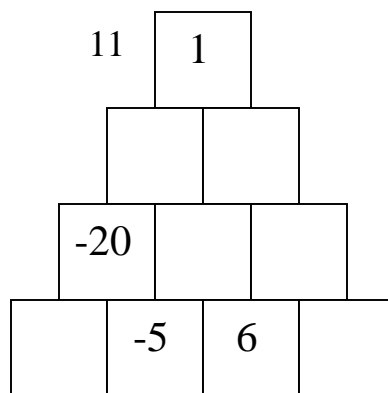
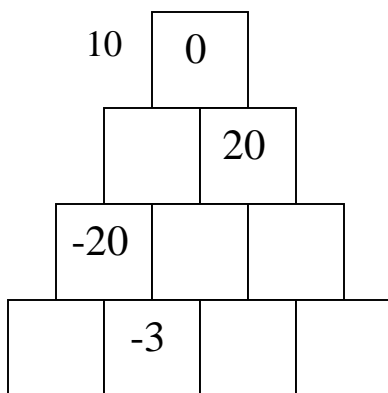
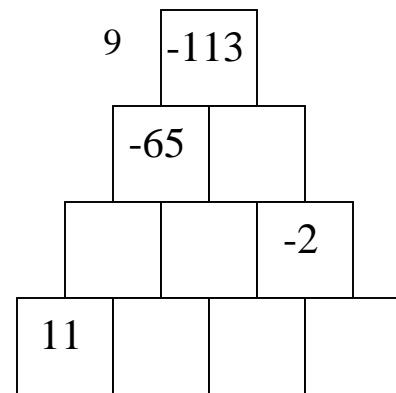
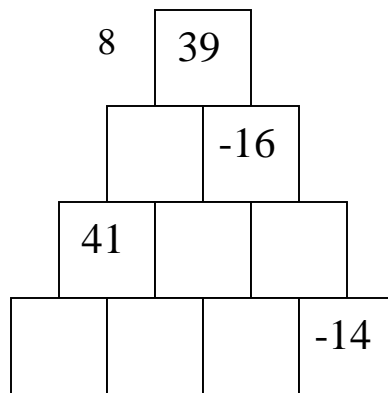
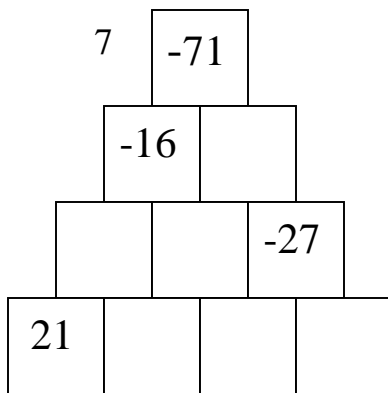
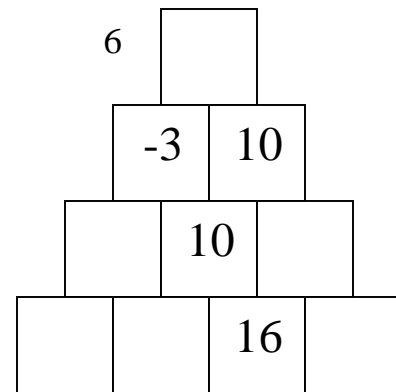
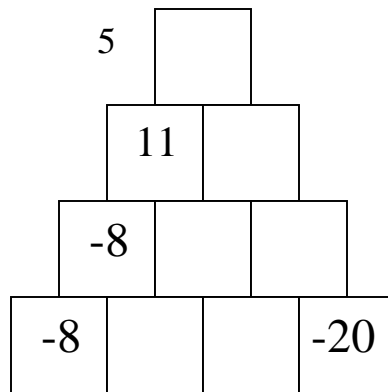
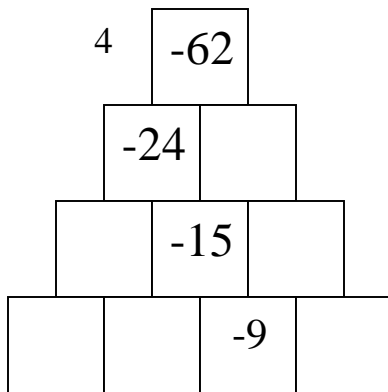
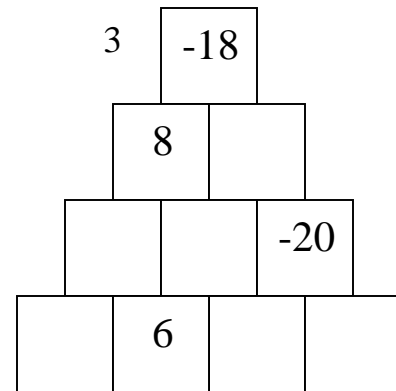
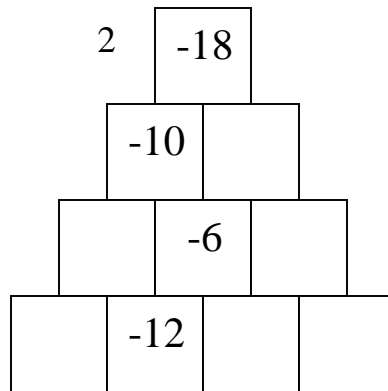
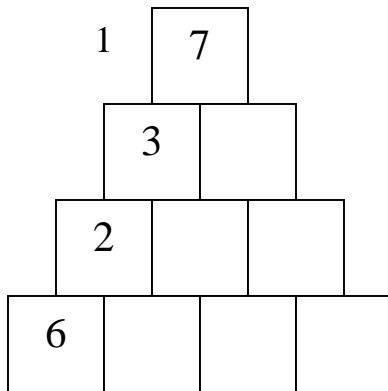
Add pairs of adjacent numbers and write their sums in the box above them as in the first example. Keep going until you reach the top of the pyramid.



# Pyramid Math 8

Name \_\_\_\_\_

Each number is the sum of the two numbers below it. Work backward to fill in the bottom row.



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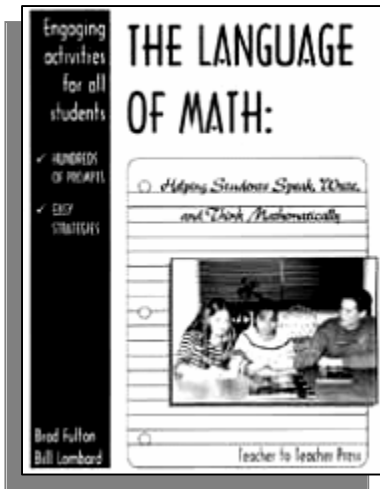
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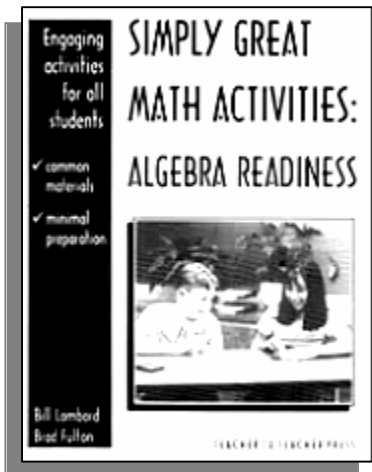
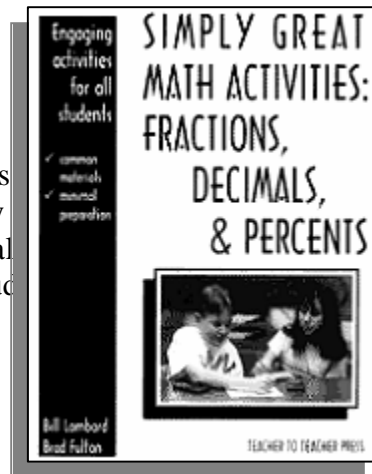
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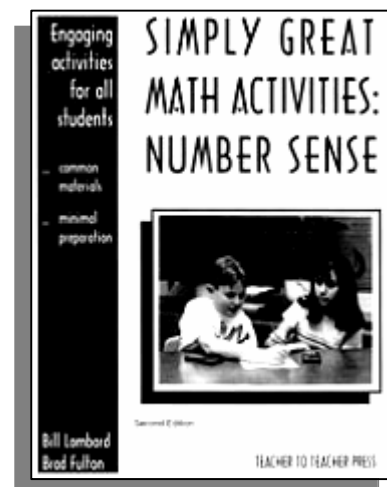
The Language of Math helps teachers create a classroom environment rich in mathematical thinking by showing them how to easily incorporate oral and written language into their math classes. Over 100 journal and discussion starters are included along with extensive instructions for making the most of your math time.

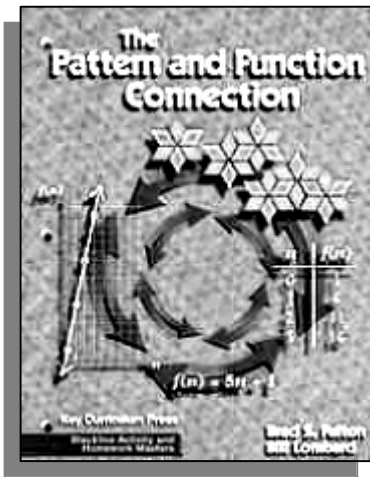
Here are a dozen unique and conceptual activities will help your students add, subtract, multiply divide fractions as well as connect them to decimal percent representations. Both you and your student will love the novel and creative approach.



Teachers are raving about how effective these activities have been in their classrooms. Children as young as fourth grade and college students alike say that algebra is easy and makes sense because of this incredible approach.

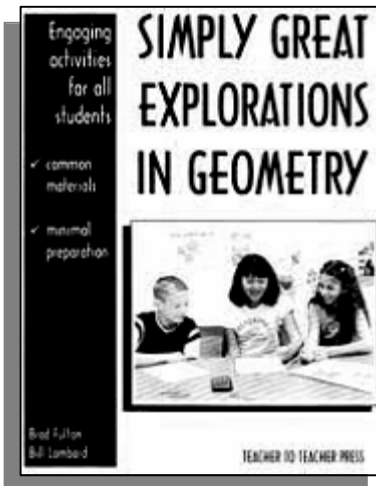
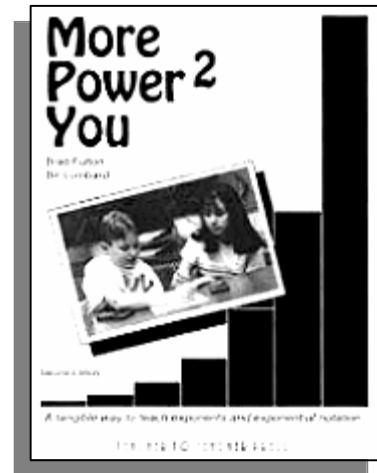
Students don't even think they are doing math sometimes because these activities are so fun and engaging, but they are developing rich and valuable number sense as they explore these eleven creative activities.





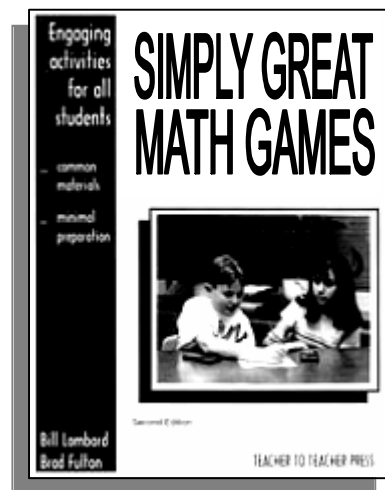
Our first book is still one of our most popular. Every teacher we talk to who has tried this approach to functions has been amazed at what their students have learned and accomplished. Over 150 pages of multiple representations of functions cover such concepts as slope, intercept, and function notation. Even elementary students have developed an understanding of functions with this book.

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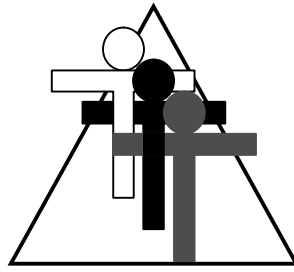
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