

Establishing & Evaluating the High Quality Math Classroom

by

Bill Lombard

Here are tips, tricks, activities, journal prompts, and goodies collected from many years of teaching and from many great teachers. Included are lesson ideas for providing success for all students. While providing for struggling students, these activities will make even your brightest students think about math in new ways.

This presentation is ready to use with your students. Complete with overhead transparencies and activities, it is available in its entirety ready to download at the author's website:

www.tttpress.com

Abstract

We want teachers to be models of mathematical reasoning and problem solving. By providing teachers with the proper tools, they can build the confidence and abilities of their students. They can check for understanding, provide for collective and collaborative thinking, problem solving, and reflections.

Using simple activities that can be modified for use in any grade, teachers can develop deep concepts of mathematics in number sense, algebraic thinking, and geometry. The activities lead naturally into extensions, providing use of these activities over several months, and allowing students to become competent and confident in their skills.

The comprehensive handout will enable all participants to become experts at communicating essential skills and will enable math leaders and trainers to inservice their staffs.

Statement of Purpose

Teacher to Teacher Press produces curriculum written by teachers, to teachers, and for students. We know how busy you are. In addition to the literally hundreds of duties and responsibilities you already have, you may be teaching a class without a good textbook or other source of curriculum. You simply don't have the time to find all the material you need. That's where we hope to help.

Our books are based on specific and important goals that we think you'll appreciate. First of all, we are schoolteachers. We know first hand that many texts and units are either too symbolic or solely concrete and manipulatively based. Students are in transition as they learn. They learn from concrete experiences, but they are capable of moving into the abstract. All of our materials are designed to help them make a successful transition.

Secondly, we believe that the N.C.T.M. Standards are filled with great ideas. Now it is up to those of us in the field to develop curriculum to meet these great standards.

Thirdly, we know that your time is precious. You have much to teach and only 180 days or so to do the job. Your students need more experience with a new subject than the two-page spread offered by the typical textbook. At the same time, you can't afford to spend six weeks on a unit that only covers a single topic. That is why our activities are designed to take shorter periods of time or to be broken up into smaller increments of time. Sure, you can spend more time on an activity or unit if you have some spare days, and your students want it—we offer plenty of extensions—but our books are designed to be finished before your students become bored.

Lastly, you didn't go to college to spend your life in front of a copy machine, or to chase down the obscure supplies required for teaching some units. That is why our units and their materials lists are simple. We teach what we write, and if it isn't simple and effective, we find something that is.

TESTIMONIALS FROM TEACHERS

“Brad and Bill's math activities are the best I've seen in 30 years of teaching middle school! These activities are engaging and meaningful but simple to use. They are invaluable in infusing algebraic concepts and processes into lower grades in preparing all students for the demands of formal algebra. Many other math teachers at my school site are finding them equally useful.”

– Wayne Dequer, 7th grade math teacher, Arcadia, CA

“The most informational workshop I have ever attended. This is a workshop everyone should attend. There is information available for use in any grade or classroom.”

– Melba VanPelt, RSP Special Education, Palmdale, CA

“A fun and innovative way to teach math. I have used Brad and Bill's activities with middle school and high school students with great success. I have also presented them to teachers in workshops and in an algebra standards course at the college level as an excellent way to actively involve students in learning important algebra concepts.”

– Terry L. Nevin, Algebra Teacher/El Dorado High School, Placentia, CA

“If we want students to engage in mathematics, we need to have engaging, rigorous curriculum to share with them. These units do that better than anything else I have ever used. The activities can be modified to use at many grade levels to introduce or review important mathematical concepts. Students love the activities, and because of the connections to the world outside of school, parents are impressed with what their students are learning to do.”

– Judy Ferrarri, 4th/5th grade teacher, Springfield, OR

“Your methods should be taught to all math teachers and in all math books!! What a wonderful way to teach outside the box.”

– Nancy Heim, Adult Education, Dana Point, CA

“The content was great! Very captivating and energetic. The whole staff was very involved with the activities.”

– Michelle Wehrhan, Title 1 Teacher, Santa Ana, CA

“Absolutely loved the step-by-step process of leading the students to complex algebraic processes while providing a deep understanding for the students at every step.”

– Kathy Meiser, 4th grade/GATE, Palmdale, CA

TESTIMONIALS FROM ADMINISTRATORS

“You made the connection of how to present algebra to a young child and the same concept to a high school student using different language and strategies. The “Language of Math” (algebra) is not a mystery.”

– Dr. Jean Cantor, Principal, El Segundo, CA

“Superb presentation! Brad & Bill are excellent presenters.”

– Richard Dixon, Assistant Principal, Compton, CA

“The presentation was outstanding and easy to follow.”

– John Cotton, Assistant Principal, LaVerne, CA

“The approach was non-threatening; it was fun and I learned many strategies that will help me. I would like to attend another workshop in math like this one.”

– Artie Davis, Assistant Principal, Central Juvenile Hall, Los Angeles, CA

“Well done presentation. We need more of this type of inservice that emphasizes real-world approach and practical examples.”

– Vincent Chugho, Assistant Principal, 9 – 12

“We need all of our teachers to go through this training. I learned that algebra can be fun! Math and algebra is systematic as shown by your examples. Now it is easier to get the system. It is no longer “magic”, only for those who are mathematicians.”

– Jeff Heller, Principal, Norwalk, CA

“The book is great. I’m excited about getting to read the book and follow up with the activities.”

– Judith Fenders, Principal

“I’m not a math person; however, the light bulb lit up as you both broke down the concepts to patterns, then equating them to equations. Your attitude towards learning is an essential characteristic to model for educators (teachers and administrators). I like the ability you both demonstrated to demystify math.”

– Elsa Martinez, Principal, 9-12, International Polytechnic H.S., Pomona, CA

“I learned so many clear ways to explain how to solve math problems that I know would make it easier for my students to learn if they were provided with the material presented today. Thank you for your enjoyable and valuable presentation!”

– Valerie Huber, Assistant Principal, Los Angeles County High School for the Arts

“This has been one of the most instantly useful workshops I have ever been to!”

– Alissa Levy, 9th grad math teacher, International Polytechnic H.S., Pomona, CA

“My experience in this workshop is that “math is fun; I can learn math too.”

– Marjorie Rudy, Assistant Principal, Palos Verdes Estates, CA

Over the years I have been a student, teacher, and trainer of math teachers and supervisors.

My former teachers, students, and colleagues have enriched my life. It is my turn to pass along this wealth of knowledge to others.

Please visit our website, www.tttpress.com to download the free presentation materials as well as many other resources to enrich your life and the lives of others.

– Bill

Qualities of a Good Math Teacher

Project work

Have the room be colorful


Posters on the walls and papers that have things to do with what we do in class

Someone who plays math games

 Gives help when asked

 Make sure you're organized

 A math teacher should really love teaching math so it's not boring

 A smile, a sense of humor, and a big box of chocolate


 Pass out candy for rewards

 Go over material multiple times

 One-on-one help

 Always have a review before a test

 Do examples on the board and leave them up there for reference

 Patience – take time to make sure each student understands the material

 Make math fun

 Be as clear as possible

(These responses are from Bill's 2004-05 Trig/Precalculus students. They are good consumers of mathematics, quite articulate about what they like and what they don't.)

How to Create a Great Math Classroom to Promote Great Math Students

Give your students a small piece of paper, and ask two simple questions:

1 – “What am I doing that helps you learn math?”

and

2 – “What needs improvement?”, or

“What can I do better?”

Ask students to take some time answering these questions. Students don't need to sign their names, and it may be best if they don't.

Then read each student's replies, and list them on a transparency, on the board, or on a piece of paper.

Pay attention to comments that occur often. Then talk about these responses the next day so students see that you value what they have to say. Modify what you can to meet their needs, and explain why you may not be able to do some of the items.

Do this twice a year and your students will appreciate you and their math class!

Mathemagics and the Teacher

Effective teachers perform mathematical feats of strength, skill, and magic on a daily basis. They:

- ✓ Are enthusiastic
- ✓ Control their students
- ✓ Are skilled presenters
- ✓ Have a sense of humor
- ✓ Think on their feet and improvise

This helps them:

- ✓ Reduce math anxiety
- ✓ Teach problem-solving skills
- ✓ Include recreational mathematics
- ✓ Help kids appreciate and enjoy math
- ✓ Illustrate math concepts in exciting ways

Problem Solving Signs

To assist students in solving problems, it's helpful to have methods available for them to view. Here are some that have worked for many students in many situations.

- Use Past Experience
- Just Graph It!
- Guess & Check
- Solve a Simpler Problem
- Find A Pattern
- Find a Formula
- Work sdrawkcaB
- Make a Diagram
- Be Physical
- Make an Organized List
- Venn Diagram

Integrating Problem Solving Strategies into Daily Curriculum

- Introduce a problem solving strategy and show a simple application of the strategy
- Provide another problem for a warmup or homework
- Continue adding variations of the strategy by changing the numbers or context of the problem

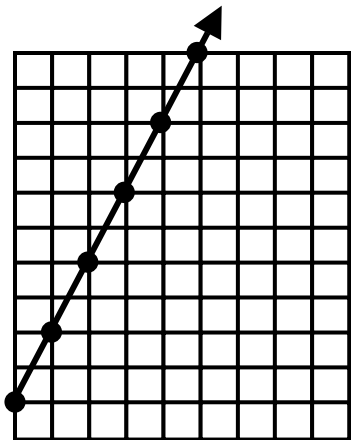
Move to another strategy when students seem to master the first strategy

Find Your
Favorite
Method

Use It to
Solve
Problems!

Use Past Experience

Just Graph

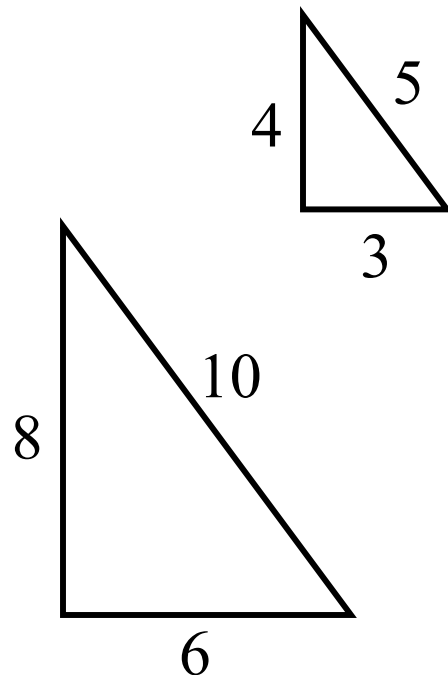


it!

Guess &

? Check

Solve a Simpler Problem

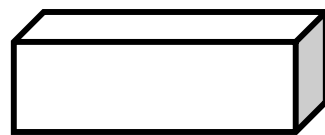
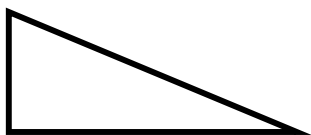


Find
a Pattern
a a b a a c a a d a a _

Find a
Formula
 $f(n) = 2n + 1$

Work
Backwards

Make a
Diagram



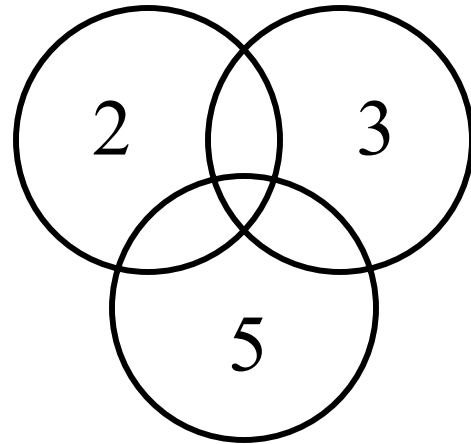
Be Physical

Make an

Nickels	0	2	4	6
Dimes	3	2	1	0

Organized List

Venn



Diagram

Bill's

Method

Hundreds Magic in Your Classroom

- ✓ High interest discovery teaching
- ✓ Generalizing arithmetic
- ✓ Strengthens number sense
- ✓ Provides base for explorations
- ✓ Relates arithmetic, algebra, and geometry
- ✓ Finding and extending patterns
- ✓ Using variables
- ✓ Source for projects
- ✓ Writing algebraic proofs
- ✓ Practicing integer operations

Explorations and Journal Prompts:

For the square pattern of numbers we started with, are the differences of the diagonals the same? Why or why not? Justify algebraically.

For the horizontal three in a row pattern, multiply the largest and smallest numbers. Compare this product with the square of the middle number. Make a conclusion and justify algebraically.

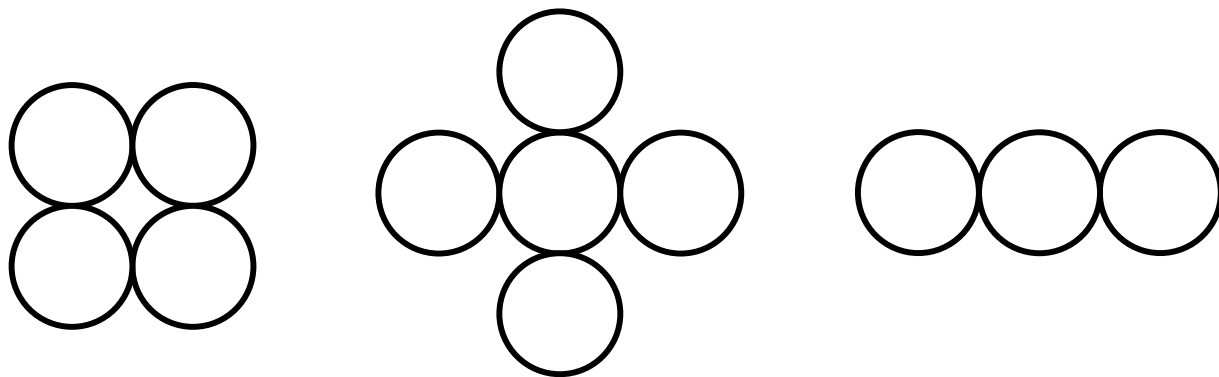
Make up an arrangement of numbers other than those on the Hundreds Magic charts, and explore the patterns you find there. Justify your conclusions.

Do the patterns you have found still work when you have negative integers? Explore relationships using the Hundreds Magic chart $-49 - 50$.

Using a page from a calendar, explore these same relationships. What similarities and differences are there? Can you justify these?

Hundreds Magic: 1 – 100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

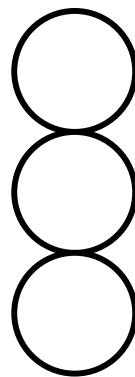
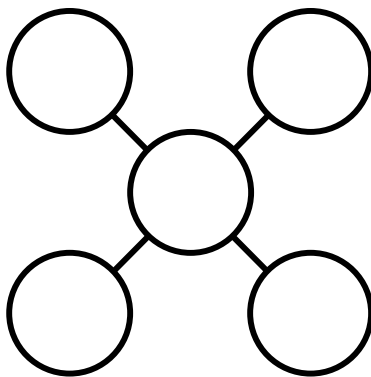
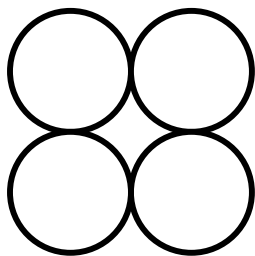


adapted from *Simply Great Math Activities; Algebra Readiness*

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Hundreds Magic: -49 – 50

-49	-48	-47	-46	-45	-44	-43	-42	-41	-40
-39	-38	-37	-36	-35	-34	-33	-32	-31	-30
-29	-28	-27	-26	-25	-24	-23	-22	-21	-20
-19	-18	-17	-16	-15	-14	-13	-12	-11	-10
-9	-8	-7	-6	-5	-4	-3	-2	-1	0
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50



adapted from *Simply Great Math Activities; Algebra Readiness*

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Time, Modeling, and Development

When math lessons begin with

- ✓ Concrete/manipulative pictorial models

Continue through the

- ✓ Linguistic, tabular, graphical model

Finishing with the

- ✓ Formula/abstract model

Then you can move children from the concrete to the abstract in a seamless fashion and not lose them along the way.

* * * * *

“Mathematics understandings are not mastered, but are developed, elaborated, deepened, and made more complete over time.”

– Joan Akers

Mathematical Understanding - Connecting the Pieces

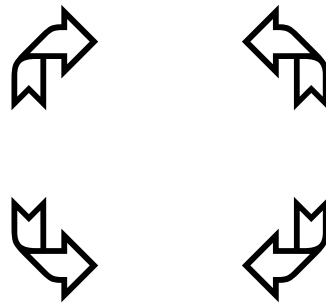
“The critical instructional problem is not one of teaching additional information, but rather one of helping students see connections between pieces of information that they already possess.”

– J. Hiebert, *Children’s Mathematics Learning: The struggle to link form and understanding.*

Arithmetic vs. Algebra

Arithmetic Likes Answers

Algebra Likes Structure



Look for arithmetic problems that have similar properties. Find the pattern and order in the problems (generalization) and you will find the algebra that rules them all.

Customizing Activities

The activities supply the structure; you supply the content/ingredients.

- ✓ Positive / negative numbers
- ✓ Fractions, decimals, percents
- ✓ Group work / individual work
- ✓ Warmups, homework, quizzes
- ✓ Journal writing, projects
- ✓ Where are your students going?
- ✓ Where have your students been?

Activities as long term content of your math class

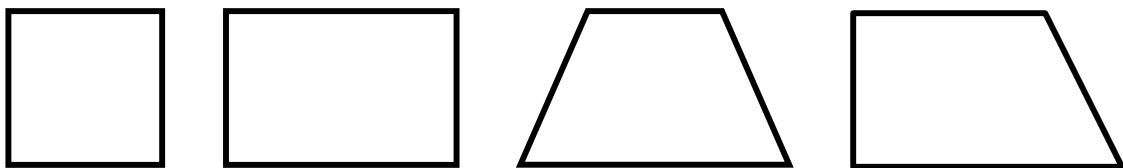
Same and/or Different

Research shows that some of the biggest gains in understanding (and test scores) occur when students see teaching examples in terms of what is the same, and/or what is different.

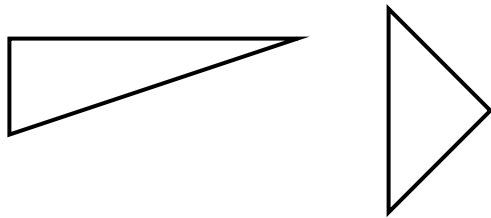
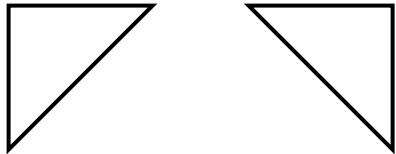
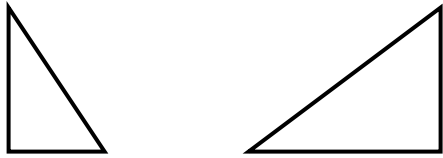
$$x^2 \quad (-x)^2 \quad -(x^2) \quad -x^2$$

$$2x^2 \quad (2x)^2$$

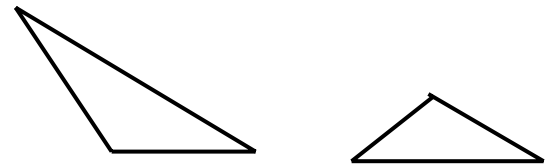
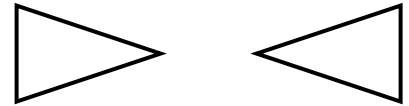
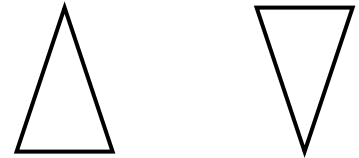
$$3 \text{ sq. ft.} \quad (3 \text{ ft.}) \text{ sq.}$$



These are
right triangles



These are not
right triangles



Line of Understanding

Research shows that using this technique allows students to clearly see where the line of separation is, and results in large gains in comprehension and computation skills.

Teaching Skill

Good teachers need more than just content knowledge and an ability to manage a classroom (a.k.a. keeping kids quiet and orderly).

They must be able to find out why their students make mistakes on problems, and then be able to explain WHY their method is incorrect.

So the teacher must know mathematics at a deeper level than just the steps of an algorithm to impart this knowledge to students.

“If you’re going to get good at golf, tennis, or algebra, it is a simple fact that it will take time and effort. Students who are on task the longest have the greatest chance for success.”

“School should not be a place where children come and watch adults work !”

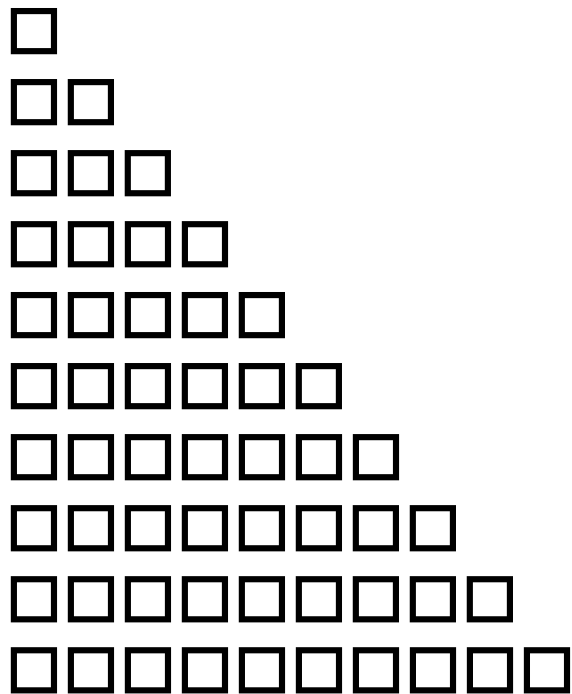
– Ed Koscik

“Teachers are more than any other the guardians of civilization.”

– Bertrand Russell

The Pyramid Problem

You have a triangular arrangement of 10 rows of boxes. Find the total number of boxes as many different ways as you can.



Ducks and Cows

Farmer Brown raises ducks and cows.

He forgot how many of each animal he has, but there are 54 animals with a total of 122 feet.

How many of each animal does he have?

Solve this problem as many ways as you can.

Deeper mathematical understanding will occur when you see the interrelationships among different methods of solutions.

Some possible methods of solving this problem are:

Draw a Diagram

Look for a Pattern

Solve an Easier Problem

Work Backwards

Graphing

Make a Systematic List

Guess and Check

Physical Representation

Algebra

Find a Formula

Lemons and Grapefruit

Fred's Fruit Pharmacy sells lemons and grapefruit, and Fred has only one size box in which to ship his orders. He can ship either 50 lemons or 20 grapefruit per box.

He can also ship combinations of the two fruits. If Fred gets an order for 20 lemons, and he must fill the rest of the box with grapefruit, how many grapefruit does he need to fill the box?

Solve this problem as many ways as you can.

Deeper mathematical understanding will occur when you see the interrelationships among different methods of solutions.

Some possible methods of solving this problem are:

Draw a Diagram

Look for a Pattern

Solve an Easier Problem

Work Backwards

Graphing

Make a Systematic List

Guess and Check

Physical Representation

Algebra

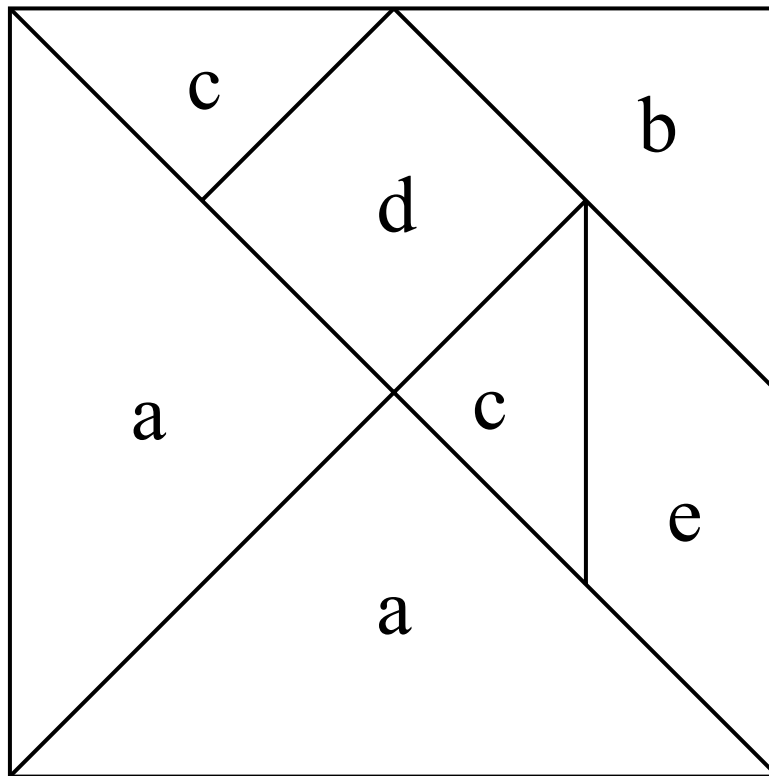
Find a Formula

– from *ComMuniCator Math Classics*, April 1996, by Bill Lombard

Tangrams

- ✓ Helps students *see* fractional representations
- ✓ Works with common fractions, decimals, and percents
- ✓ Encourages connections among fraction representations
- ✓ Fosters connections to geometry and algebra
- ✓ Helps students compare fractions
- ✓ Leads to vocabulary development for polygons
- ✓ Engages both skilled and unskilled learners
- ✓ Helps students understand congruence

Tangram 1



Student-generated equations

$2b = a$	$a = b/2$	$2c = b$
$b = d$	$2c = d$	$2c = e$
$d/2 = c$	$b = e$	$4b = 2a$
$4c = a$	$2c + d = a$	$a - b = b$
$c + d = c + e$	$8b = 4a$	$d - c = c$

Research from South America

Children who are street vendors were given an assessment of their mathematics abilities. It was found that:

They could solve 94% of context-sensitive problems, . . .

but only 37% of the same problems when given them inside the classroom.

Message: Context-sensitive math is more meaningful and easier to understand, to work with, and to solve problems.

“A mind once stretched by a new idea never regains its original dimensions.”

– source unknown

“Help people reach their full potential; catch them doing something right.”

– *The One Minute Manager*

“One merit of mathematics few will deny: it says more in fewer words than any other science.”

– David Eugene Smith

Reasons to Incorporate Language in a Mathematics Class

Just as computers use an operating system to function, language is the operating system of our brains.

Simply stated, to speak or write well is to think well.

Both oral and written language are effective in increasing depth of understanding.

It is probably a much better investment of a teacher's time to look at a writing sample as an assessment than to look at a homework paper.

Good Questions That Get Good Answers

Students love to talk. They talk between classes; they talk during class. The trick is to get them to talk *about* class. Nowhere is this more difficult than during math. Although students may offer hypotheses in science, comments during literature, or explanations during history, their dialogue in a math class is usually limited to guessing a number when the teacher calls upon them. In this month's issue, we take a look at strategies that will elicit good mathematical thought from students.

Procedure:

The trick to getting good answers is asking good questions. And the trick to asking good questions is simple. Here are ten quick questions you can ask at any time that will promote better student thinking and good student responses:

- How did you solve the problem?
- Can you solve the problem a different way?
- Is this similar to another problem you have solved?
- Can you suggest a rule or formula for solving problems like this one?
- What patterns do you notice?
- Does that answer seem reasonable?
- What information is relevant in this problem?
- How could you convince me that you are right?
- Do you agree or disagree with what that student said? Why?
- Why does that work?

1 *How* we ask our questions is as important as *what* we ask. Typically we teachers get in the habit of calling on students who raise their hands. However this sends a signal to the other students that if you don't raise your hand, you don't have to participate. A more effective strategy is to ask the question, wait for five seconds, then call on a student whether his or her hand is raised or not. Five seconds will seem like an eternity, but students will get used to this critical think time. It is important in this situation to make sure you have asked a question that any student should be able to answer. Calling on students who are not paying attention or who don't know how to answer the question will usually embarrass them. This tends to inhibit the class discussion instead of encouraging it.

Another strategy is to say, "I'm going to ask a question, and for 30 seconds, I don't want you to answer or discuss it with anyone else. Just think about the problem and how you would respond." After 30 seconds say, "Now explain your answer to the person sitting next to you." After this, you can call on individual students to share. This 30-second delay is very important for high-level questioning. It also gives plenty of time to shy or reluctant students who may hide behind the responses to students who reply quickly.

adapted from *Language of Math*

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2 For even more detailed answers, generate questions by filling in the blanks of this simple template:

As a (voice), explain to (audience) about (specific topic).

Here are samples of how this looks:

- As a teacher, explain to a fourth grader why you need to divide by two to find the area of a triangle.
- As a worker bee, explain to the queen why the hexagon is the best shape for the hive cells.
- As a fellow student explain what we learned yesterday to a student who has been absent.
- As a teacher, explain to a parent what we are working on in class.

Giving students a specific voice and audience helps them think more clearly about the subject at hand. Narrowing the focus of the question is important too. It is better to ask a student to explain why it is necessary to divide by two to find the area of a triangle than it is to ask a student to explain area or triangles.

Voice:

Teacher
Younger student
Older student
Alien
Parent
Principal
News reporter

Audience:

Younger student
Sibling
The President
Employee
Customer
Boss
Teacher

3 The last component of good questioning involves the *types* of questions we use. Although it is important to ask students to tell the answers to specific math problems, this is not where we will generate good math talk. Mathematics that lends itself to good questioning *looks* different than the simple math problems found in most texts. For this reason, we have included four transparency masters of problems that can be the basis for good questions. Simply display them for the students, allow them sufficient time to think about and work on the problems, and then ask them the suggested questions in section 1 above.

Building a community of mathematical thinkers who value good discussion and who respect the opinions of others can be accomplished with simple steps, but it is not accomplished quickly. Like any good skill, it develops over time. Be patient, and you will be rewarded with good results.

adapted from *Language of Math*

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Format, Voice, Audience Ideas

Write a (format) from (voice) to
(audience) about (specific topic)

<u>Format</u>	<u>Voice</u>	<u>Audience</u>
note	teacher	younger student
paragraph	younger student	sibling
song	older student	the President
list	alien	employee
award	architect	customer
dialogue	parent	boss
speech	principal	teacher
poem	news reporter	the PTA

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Examples - Four Key Elements

“As a worker bee, write a letter to the queen explaining why the hexagon is the best shape for the hive cells.”

“Pretend you are the teacher and write a note to a parent explaining what we are covering in class this week.”

“Write a dialog between two students one of whom was absent explaining what we learned in class today.”

“Write an instruction manual for young children telling them how to draw a parallelogram.”

Asking a Question in the Right Way

Four Key Elements of a Good Question

- ✓ Audience
- ✓ Voice
- ✓ Format
- ✓ Specific Topic

“Tell me what you know about area.”

or

“As an architect, write an note to a client explaining how to calculate the square feet in the bedrooms of their new house.”

Simply including these four elements in your question is all that is needed to promote effective responses.

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Fraction Reasoning

$$\frac{2}{3} \quad \frac{7}{10} \quad \frac{4}{5} \quad \frac{7}{11} \quad \frac{3}{5}$$

How do you position these on the number line?

What is the conceptual reasoning behind the positions?

Dialog with your neighbor about what your students could say to demonstrate understanding of:

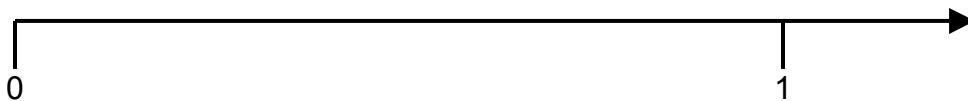
- ➔ Fraction
- ➔ Less than
- ➔ Greater than

Number Line Fractions

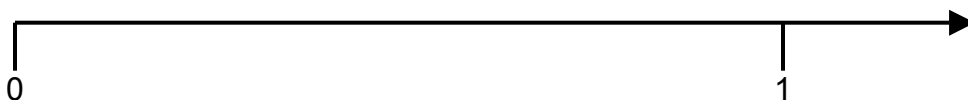
Name _____

Put the following fractions, decimals or percents on their number lines in the correct locations.

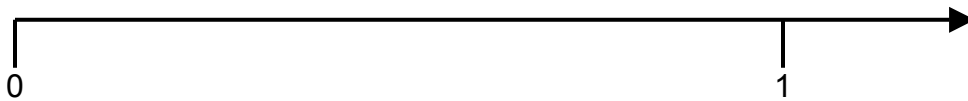
Set 1: $\frac{3}{8}$ $\frac{5}{6}$ $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$



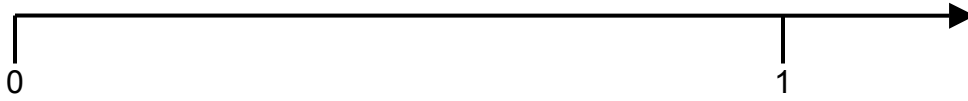
Set 2: $\frac{2}{8}$ $\frac{2}{3}$ $\frac{9}{15}$ $\frac{7}{12}$ $\frac{1}{4}$ $\frac{3}{5}$



Set 3: $\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{2}$ $\frac{5}{6}$ $\frac{4}{5}$



Set 4: $\frac{1}{3}$ $\frac{1}{5}$.2 $\frac{1}{4}$ 30% .35



Set 5: 100% .05 $\frac{1}{10}$ 50% .5% $\frac{1}{4}$



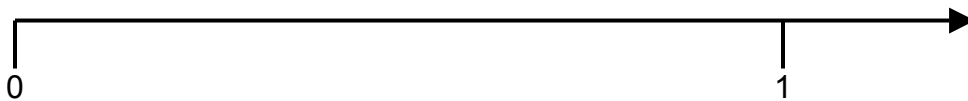
from *Simply Great Math Activities: Fractions, Decimals, & Percents*,
Fulton and Lombard, Teacher to Teacher Press

Number Line Algebra

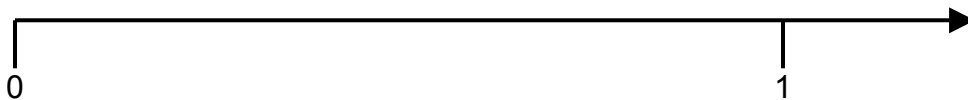
Name _____

Put the following algebra expressions on their number lines in the correct locations.

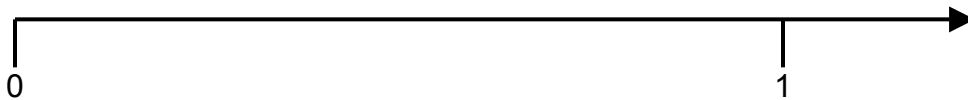
Set 1: If $x = 4$, $\frac{x}{8}$ $\frac{x}{6}$ $\frac{x}{4}$ $\frac{x}{12}$ $\frac{x}{3}$



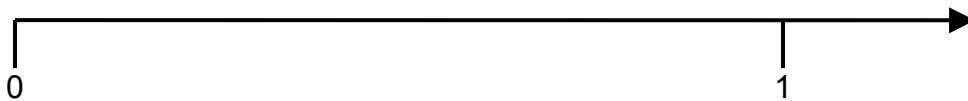
Set 2: If $x = 5$, $\frac{2}{x}$ $\frac{2}{3x}$ $\frac{x}{15}$ $\frac{7}{2x}$ $\frac{x}{4}$



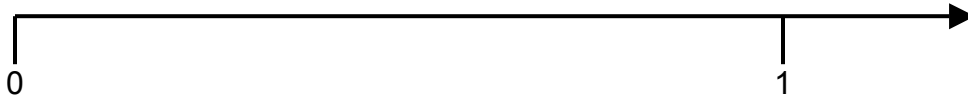
Set 3: If $x = \frac{1}{4}$, $3x$ $x + \frac{1}{3}$ $\frac{x}{2}$ $\frac{x}{3}$ x^2



Set 4: If $x = \frac{2}{3}$, $1-x$ x^2 $x(1-x)$ $\frac{1}{x}$ $x(x-1)$



Set 5: If $x = .4$, $.5x$ $\frac{x}{3}$ $x(1-x)$ $x(x+1)$ x^3



adapted from *Simply Great Math Activities: Fractions, Decimals, & Percents*,
Fulton and Lombard, Teacher to Teacher Press

1. Mr. Infinity's math class is so popular everyone is trying to get in. Here is his seating chart. If you were student 100, explain how you could find the row and column of your seat.

<i>Column</i>	<i>1</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>	<i>7</i>	<i>8</i>
<i>Row 1</i>	1	2	3	4	5	6	7	8
<i>Row 2</i>	9	10	11	12	13	14	15	16
<i>Row 3</i>	17	18	19	20	21	22	23	24
<i>Row 4</i>	25	26	27	28	29	30	31	32

adapted from *Language of Math*

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Reasons to Incorporate Language in a Mathematics Class

Just as computers use an operating system to function, language is the operating system of our brains.

Simply stated, to speak or write well is to think well.

Both oral and written language are effective in increasing depth of understanding.

It is probably a much better investment of a teacher's time to look at a writing sample as an assessment than to look at a homework paper.

The Power of the D.A.E.O.P. Due At the End Of Period

When I want to make sure all students understand how to perform a process, I will ask them to solve an example problem that is placed on the board.

Students must show on their paper all the needed steps to arrive at the correct solution.

I am interested in the process they used as much as in the actual answer.

Before class is over, all students must come to see me with their solutions.

If correct, I stamp their paper.

If incorrect, the student must find the mistake.

This results in students having to examine their own work, causing them to focus better and providing self assessment.

Common Themes – Words

What do these items have in common?

Raise your hand, but do not say the answer,
as soon as you know it.

1. Apple, orange, ash, pine, oak, cedar . . .
2. Mars, Milky Way, Twix, Crackle . . .
3. Lincoln, Ford, Polk, Harrison, Adams . . .
4. Coffee, dining, end, multiplication . . .
5. Water, flower, double, trundle, bunk, futon . . .
6. Fish, traffic, college, elementary, middle, . .
7. I, it, we, you, them, she, . . .
8. Curling, cricket, soccer, track, volleyball . . .
9. Mustang, Colt, Avalon, Corolla, Accord . . .
10. Bears, Rams, Eagles, Panthers, Lions . . .

Ring a Bell Numbers

What is the significance of these numbers?

1. 3.141592 . . .

2. 90

3. 365

4. 24

5. 186,000

6. 3, 4, 5

7. 1.414 . . .

8. 1492

9. 32

10. 2.54 . . .

pi, number of degrees in a right angle, days in a year, hours in a day, speed of light in miles per second, integer sides of a right triangle, square root of 2, year Columbus discovered America, degrees at which water freezes in Fahrenheit scale, number of centimeters in an inch

Arithmetic Algorithms

"The depressing thing about arithmetic badly taught is that it destroys a child's intellect and, to some extent, his integrity. Before they are taught arithmetic, children will not give their assent to utter nonsense; afterwards they will."

– W.W. Sawyer, *Mathematician's Delight*

"Ours is not to reason why; just invert and multiply."

-- rhyme used to remember the algorithm for dividing fractions

It's possible to teach arithmetic by rote, but this isn't a good way.

Students can remember the operations easier if they know the reasons why they work.

Students that successfully move on to higher mathematics are those who can reason and think for themselves, not just follow teacher directions.

Take Your Places

- ✓ Helps children understand place value
- ✓ Helps develop an understanding of probability
- ✓ Builds number sense in all operations
- ✓ Leads students towards algebraic generalizations
- ✓ Promotes a sense of number magnitude
- ✓ Promotes higher-level mathematical thinking and discussion
- ✓ Engages all levels of learners

Take Your Places A

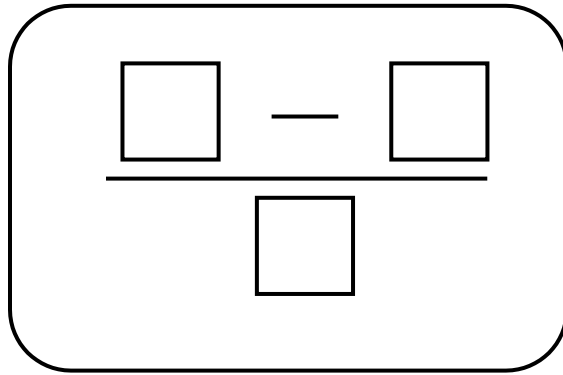
Roll a die and place your number in one of the squares. Do this three more times. Try to get the smallest number possible.

adapted from *Simply Great Math Activities: Number Sense*

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Take Your Places B

Roll a die and place your number in one of the squares. Do this two more times. Try to get the largest number possible.

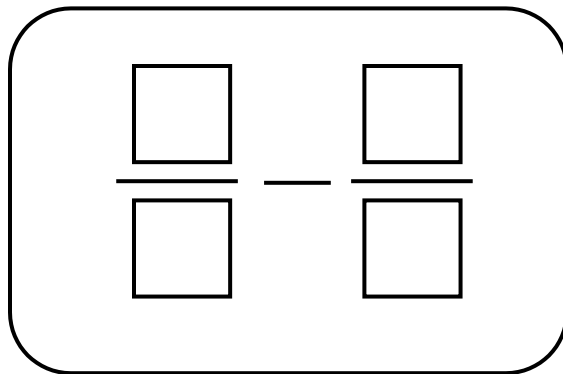


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Take Your Places C

Roll a die and place your number in one of the squares. Do this three more times. Try to get the smallest number possible.



Teachers: Emphasize Math Concepts

Research comparing teaching methods of U.S. teachers with those from other countries suggests that American teachers don't spend as much time talking about and teaching the basic concepts of mathematics. More time in the U.S. is spent on getting "the answer" and less time on "the why and how" of the problem.

Many of our teachers need assistance in ways to engage children in problems that deal with conceptual thinking. Teachers in general teach the way they were taught, and thus don't have a strong background in asking questions that probe children's thinking about math.

There is no single teaching method that works for all teachers and all students. But if students are provided with problems and activities that provide access to multiple paths to their solution, we will have come a long way towards powerful mathematical thinking.

“Do you have any ideas about why more people don’t understand math any better than they do? The problem seems to begin in school, and the struggle- for too many- lasts a lifetime.”

- Jean Acerra, Newton, MA

“I believe that much of the problem lies in the lack of logic and reasoning skills. Math is just logic with numbers and symbols attached, and success with it requires the ability to reason effectively. But children usually are taught *what* to think, not *how* to think. That’s why so many adults live in a state of perpetual misunderstanding about the world.”

- Marilyn vos Savant

Parade Magazine, 13 October, 2002

“Research shows that a self-assessing activity fosters learning better than one in which a student merely receives a grade from the teacher.”

– Brad Fulton

* * * * *

Leo’s Pattern 1

These patterns all follow the same rule:
Add two successive terms to get the next term.

A1 1, 1, 2, 3, 5, 8, 13, . . .

A2 1, 3, 4, 7, 11, 18, . . .

B1 2, _____, 6, _____, _____, _____, _____

B2 3, _____, _____, 15, _____, _____, _____

B3 _____, _____, 9, _____, _____, 41

B4 1, _____, _____, _____, _____, 28

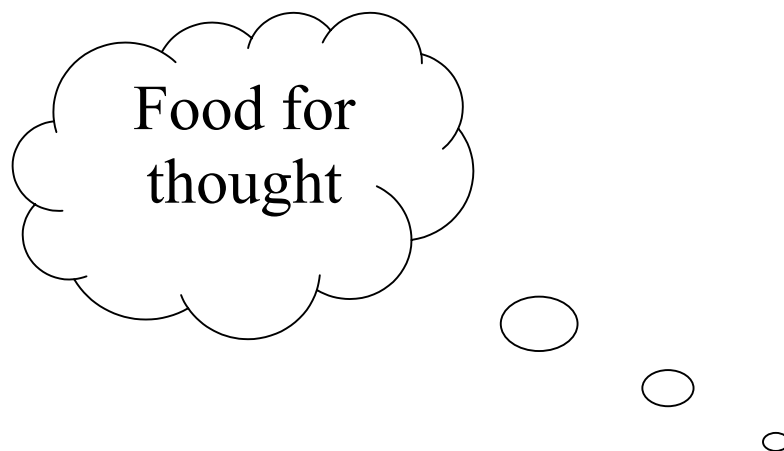
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“Learners construct understanding for themselves.
To understand is to know relationships.
Knowing relationships depends on having prior
knowledge.”

– Lawrence Lowery

Placing new knowledge properly assists student learning – too far away and students can’t grasp it. Too familiar and it leads to boredom. Information nearby, easily accessible, but not completely mastered, leads the brain into active exploration, emotion, and enthusiasm.



Real World Number Stumpers #2

24 =	H in a D
12 =	S of the Z
53 =	C in a D (with the J)
90 =	D in a R A
8 =	S on a S S
1001 =	A N
1 =	W on a U
11 =	P on a F T
29 =	D in F in a L Y
64 =	S on a C B
40 =	D and N of the G F
100 =	S in the U S S
- 40 =	N of D C = N of D F

hours in a day, signs of the zodiac, cards in a deck (with the joker), degrees in a right angle, sides on a stop sign, Arabian Nights, wheels on a unicycle, players on a football team, days in February in a leap year, squares on a chess board, days and nights of the great flood, senators in the United States Senate, number of degrees Celsius = number of degrees Fahrenheit

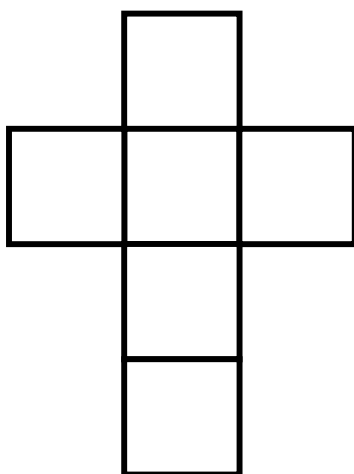
Puzzles and Problem Solving

Puzzles help students with spatial visualization. Spatial visualization, in turn, assists students in becoming better problem solvers. So puzzles are great teachers. Try this one with your students.

Here is one way to make a “cube net”, a set of six linked squares that can be folded to make a cube.

This is the usual way a cube is represented to children, and it may be the only way they ever see.

Ask them how many **other** ways they can find. There are at least eleven distinct ways, not counting rotations or reflections.



Ways to Reduce Math Anxiety

* * * * *

- ✓ Show students problem solving strategies

Find ways to help students visualize things

- ✓ Relate mathematics to the real world

There are entire college courses with the focus on “Reducing Math Anxiety”.

What Studies Say about Teaching

"Students whose teachers conduct hands-on learning activities outperform their peers by more than 70% of a grade level in math and 40% of a grade level in science."

"In math, students whose teachers emphasize higher-order thinking skills outperform their peers by about 40% of a grade level."

One of the recommendations to policy makers:

"... teachers should be encouraged to convey higher-order thinking skills [and] conduct hands-on learning activities ... "

Despite what some critics think, teaching *does* matter. And teaching based on the recommendations of the mathematics reform movement *does* make a positive difference.

From *How Teaching Matters: Bringing the Classroom Back Into Discussions of Teacher Quality*. A Policy Information Center Report from the Educational Testing Service Report written by Harold Wenglinsky, October 2000.

Splendid Company

They ask me why I teach
and I look around my classroom and reply:
Where else would I find such splendid company?
There sits a statesman to be,
Strong, unbiased, wise, another Daniel Webster,
Silver-tongued.
And there a doctor whose quick steady hand
will mend a bone or stem a life blood's flow.
A builder sits beside.
Upward rise the arches of the church wherein...
That minister will speak the word of God.
And all about, a gathering of farmers, merchants and laborers
who work and vote and build and plan and pray
Into a great tomorrow.

And I say,
I may not see the church, or hear the words, or eat the food
their hands will grow.
And yet, I may.
And later I may say,
“I knew them
And they were strong
Or weak, or kind, or proud, or bold.
I knew them once.
But they were young.”

They ask me why I teach, and I reply,
“Where else would I find such splendid company?”

– Excerpt from a presentation by Robert L. Crowe

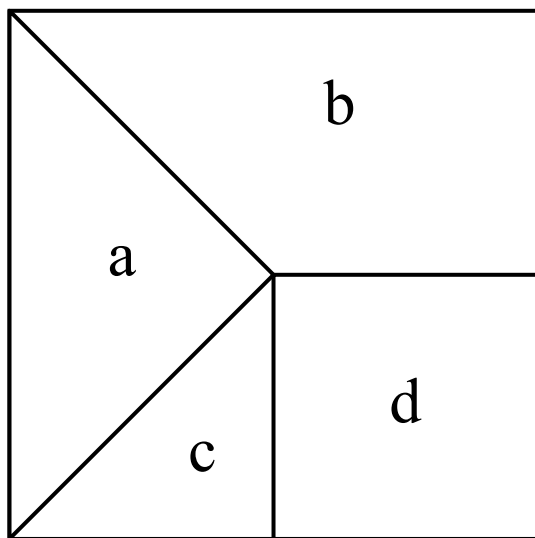
7. How many people can be seated at this arrangement of tables? People can sit on each side and both ends. What if eleven tables were used? What if 100 tables were used?



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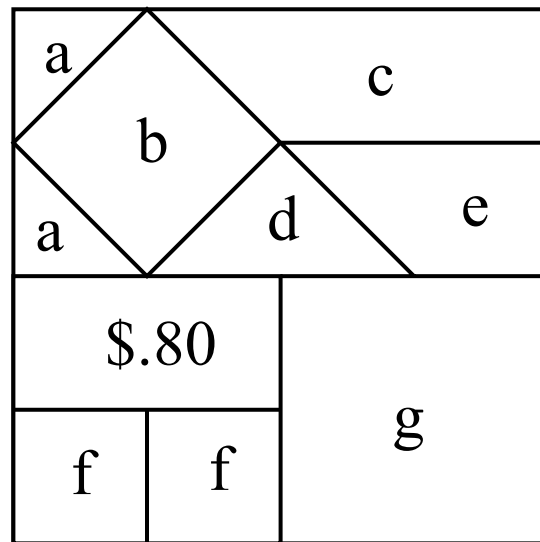
22. This tile costs \$2 when it is whole. Explain how to find a reasonable price for each of the pieces.



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23. One piece of this tile has a price. Based upon this, find a reasonable price for the other pieces and the whole tile.



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26. Which of these numbers does not belong?
Explain your reasoning.

Can you find a reason why a different number might not belong?

616

828

111

822

27. Which of these numbers does not belong?

Explain your reasoning.

Can you find a reason why a different number might not belong?

1904

8060

905

3030

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42. Juan uses this trick to add.
“I begin by adding the hundreds, then the tens, and finally the ones. Then I add those answers together.”
Explain why the method works.

$$\begin{array}{r} 358 \\ 267 \\ + 649 \\ \hline 1100 \\ 150 \\ + 24 \\ \hline 1274 \end{array}$$

44. Lashawna uses this trick to multiply numbers with two digits.

Explain how her method works and why it works.

Do you think this is a good way to multiply?

Why or why not?

$$\begin{array}{r} 42 \\ \times 26 \\ \hline 800 \\ 240 \\ 40 \\ + 12 \\ \hline 1092 \end{array}$$

75. Sasha and Josh are at the food court in the mall.

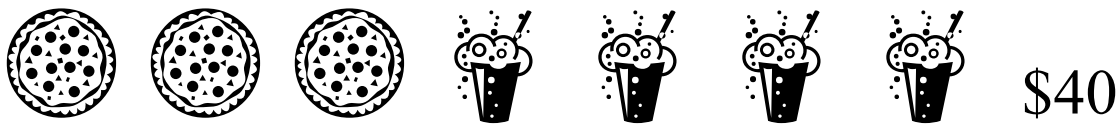
They noticed these prices.

Explain how to find the price for the burger and for the taco.



76. Renaldo and Alex are at the food court at the mall and notice these prices.

Explain how to find the price for the pizza and for the soda.



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Why use Brad and Bill for inservice training?

Four simple reasons:

- ❖ They have a passion for mathematics that is catching
- ❖ They allow teachers/students to participate in math in new and exciting ways
- ❖ They make math fun ! !
- ❖ They make math real; they show new relationships between mathematics and real life

Call Brad or Bill for an exciting, fun-filled, math-filled day for you and your staff.

Find out what thousands of other math teachers and trainers know.

Workshop List for Bill Lombard and Brad Fulton

Here is a partial list of workshops we've done in the past. These are available as conferences or day-long sessions.

Fractions, Decimals, and Percents – Creative Connections that Work

“Population, when unchecked, increases in a geometrical ratio.” – Thomas Malthus, *An Essay on the Principle of Population*

Number Sense – The Key to Bridging the Gap to Higher Mathematical Thinking

“In today's world, ‘innumeracy’ is an even greater danger than illiteracy, and is perhaps even more common.” – Arthur C. Clarke

Developing Algebraic Thinking with Understanding Instead of Tears

“. . . children usually are taught what to think, not how to think. That's why so many adults live in a state of perpetual misunderstanding about the world.” – Marilyn vos Savant

The Pattern & Function Connection – Relating Algebra to Their World

“Our biggest failure is our failure to see patterns.” – Marilyn Ferguson

“It's remarkable to discover how many connections mathematics makes, how many lightbulbs it suddenly turns on.” – Sylvia Spengler, biologist

The Language of Math: Helping Students Speak, Write, and Think Mathematically

“Mathematics is the only universal language there is, Senator.” – line spoken by Jodie Foster in the film *Contact*

Standards-Aligned Activities and Games that Motivate and Engage Students

“Puzzles, like virtue, are their own reward.” – Henry E. Dudeney, English puzzle maker

Algebra Thinking Activities for All Your Low Achievers

“Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country.” – David Hilbert

Questioning Strategies: A Key Teacher Skill that Promotes Deep and Precise Mathematical Understanding and Concept Development

“I'm absolutely convinced that mathematics is the most naturally human thing to be interested in, but it's taught with a language that's alien to many people.”

– Jaron Lanier, concert pianist and computer scientist

Establishing and Evaluating the High Quality Mathematics Classroom

“If you're going to get good at golf, tennis, or algebra, it is a simple fact that it will take time and effort. Students who are on task the longest have the greatest chance for success.”

“School should not be a place where children come and watch adults work !” – Ed Kosciak

“Teachers are more than any other the guardians of civilization.” – Bertrand Russell

Algebra in the Real World: Find It in Amazing Places!

“To those who do not know mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty, of nature.” – Richard Feynman

“There is no branch of mathematics, however abstract, which may not some day be applied to phenomena of the real world.” – Nikolai Lobachevsky

Geometry: A Mathematical Reflection of the Physical World

“And since geometry is the right foundation of all painting, I have decided to teach its rudiments and principles to all youngsters eager for art . . .” – Albrecht Dürer

“There is a geometry of art as there is a geometry of life, and, as the Greeks had guessed, they happen to be the same.” – Matila Ghyka

Data, Probability, and Statistics . . . Literacy for a Mathematical World

“Statistics is numbers that are part of a story.” – George Cobb, statistician

“Probability, like logic, is not just for mathematicians anymore. It permeates our lives.”

– John Allen Paulos, *Innumeracy*

Teacher to Teacher Press

“Building Mathematical Skill on a Foundation of Understanding”

Simply Great Math Activities: Number Sense

Eleven extensive ready-to-teach and mathematically rich activities will captivate your students' interest. Many of the activities can be extended into week-long explorations. The book includes homework masters, transparency masters, journal prompts and simple directions.

Simply Great Math Activities: Fractions, Decimals, and Percents

A dozen incredible and innovative activities will captivate and educate your students. They will learn creative and clever tricks that make fractions less frightening. The book includes homework and transparency masters, journal topics, easy-to-follow directions, and much more.

Simply Great Math Activities: Algebra Readiness

These motivating activities will work for young students just beginning to work on algebra concepts, while ideas for extensions make them just as appropriate for older students in formal algebra classes. The book includes homework ideas, transparency masters, journal prompts and simple directions.

Simply Great Math Activities: Geometry Explorations

Students use geometry as a tool to explore unique mathematical situations. Area formulas, volume, surface area, compass and straightedge constructions, angle measurement, the Pythagorean Theorem and more are covered in unique ways that promote deeper understanding. Algebraic principles and fraction concepts are embedded.

More Power² You!

A unit that actually makes the concept of exponents and exponential growth tangible! Lead your students on a journey traveling from physical and manipulative models to graphical and symbolic formats. They will even be able to explain why $n^0 = 1$. The book includes student worksheets, homework, teacher lessons, journal prompts, and ideas for extensions.

The Language of Math: Helping Students Speak, Write, and Think Mathematically

Everything you need to incorporate oral and written language into your classroom lessons, complete with easy tips for leading rich mathematical discussions, and great ideas for **easily** managing written work. There are over 70 transparency masters, 100 journal prompts, and masters for making your own math journals. Writing and speaking mathematically has never been easier!

The Pattern and Function Connection

This three-week unit is the easy and effective way to introduce students to linear functions. Students will move from physical and manipulative models to pictorial and graphical representations, then finally to symbolic expressions for linear equations. The book includes student worksheets, homework, teacher lessons, journal prompts, and ideas for extensions.

24 Pattern Cards

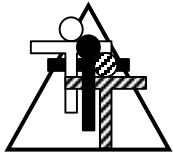
A fantastic supplement and great time saver! These full-color 11” by 17” cards are printed on heavy matte-finish card stock that is easy on the eyes and will stand up to years of student use. Students will enjoy working with the patterns so much, they won't notice how much they are learning about functions and algebra. They are a great companion to go with either of our titles: *The Pattern and Function Connection* or *Simply Great Math Activities: Algebra Readiness*.

Transparencies of Pattern Cards

Another great addition to *The Pattern and Function Connection* or *Simply Great Math Activities: Algebra Readiness* family of products. These brilliant transparencies are full-color representations of the Pattern Cards listed above. Use them for whole-class instruction to introduce the powerful mathematics of either book. The bright and clear colors will capture your students' interest, introduce them to the beauty of patterns, and send them well on their way to algebraic learning. Can be ordered separately or at a package discount with other Pattern and Function products.

A Blueprint for Geometry

In the two- to three-week project presented here, students read and draw floor plans and elevations and compute building costs. As they solve the problems inherent in drawing house plans, students learn measurement, scale, and architectural symbols. Students also use their problem-solving skills and creativity. The book contains blackline masters for 17 plans and homework sheets, as well as student outcomes, vocabulary, and procedure lists.



Teacher to Teacher Press

“Building Mathematical Skill On a Foundation of Understanding”

ORDER FORM: (All prices include sales tax. Prices valid until December 31, 2005)

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_____	Simply Great Math Activities: Number Sense	\$16.50	_____
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_____	The Language of Math	\$16.50	_____
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	Subtotal		_____
	Please add 10% shipping and handling per order		_____
		Tax included	<u>\$0.00</u>
		TOTAL	_____

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