

Mathematical “Rules” - Fun with Algorithms



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Mathematical Rules

What is 25 divided by 5 ?

<http://video.google.com/videosearch?q=mathematics&hl=en&page=2&so=0&lr=>



Mathematical Rules

What is 25 divided by 5 ?

Would you believe the answer is 14 ?

Let's take a look.



Mathematical Rules

What is 25 divided by 5 ?

$$\begin{array}{r} 1 \\ 5 \overline{)25} \end{array}$$



Mathematical Rules

What is 25 divided by 5 ?

First division step

$$\begin{array}{r} 1 \\ \hline 5 \overline{)25} \\ \underline{-5} \\ 20 \end{array}$$



Mathematical Rules

What is 25 divided by 5 ?

Second division step

$$\begin{array}{r} 14 \\ \hline 5 \overline{) 25} \\ \underline{-5} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$



Mathematical Rules

What is 25 divided by 5 ?

$$\begin{array}{r} 14 \\ \times 5 \\ \hline \end{array}$$

Verifying by multiplication

(Remember: the teacher
always asks you to check
your work!)



Mathematical Rules

What is 25 divided by 5 ?

Verifying by multiplication

$$\begin{array}{r} 14 \\ \times 5 \\ \hline 20 \\ + 5 \\ \hline 25 \end{array}$$



Mathematical Rules

What is 25 divided by 5 ?

$$\begin{array}{r} 14 \\ 14 \\ 14 \\ 14 \\ + 14 \\ \hline \end{array}$$

Verifying by Addition

(another check - we're really being careful!)



Mathematical Rules

What is 25 divided by 5 ?

Verifying by Addition

$$\begin{array}{r} 14 \\ 14 \\ 14 \\ 14 \\ + 14 \\ \hline 20 \\ 5 \\ \hline 25 \end{array}$$



Mathematical Rules

What's the message here?



Mathematical Rules

What's the message here?

No algorithm, improperly applied, will give a correct answer.

We must be firmly grounded on concepts and understanding.



Did You Know that $1 = 2$?

Seriously, this just can't be true, can it ?

Well, let's take a look at this claim.



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Suppose $a = b$



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Then $a \cdot a = ab$

multiply by a



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subtract b^2



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$(a + b)(a - b) = b(a - b)$

factor



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$a + a = a$

since $a = b$



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factor

$a + b = b$

divide by $(a - b)$

$a + a = a$

since $a = b$

$2 = 1$

WOWEE !!



Why Can't We Divide by Zero ?

We can divide by any other number.

So what makes zero so special ?



Why Can't We Divide by Zero ?

We know that subtraction means adding the opposite. So subtraction is just a different way to add.



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In the same way, division by a number can be defined as multiplying by the reciprocal of the number.

In other words, any division problem can be changed into a multiplication problem.



Why Can't We Divide by Zero ?

An example:

$8 \div 4 = 2$ is a division



Why Can't We Divide by Zero ?

An example:

$8 \div 4 = 2$ is a division, and

$4 \cdot 2 = 8$ is the related multiplication

This works for all non-zero divisors.



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Suppose you try the following, and let n be your answer.

$$8 \div 0 = n \quad \text{for some value of } n$$



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But we have a major problem here.

No value of n will work in this case.



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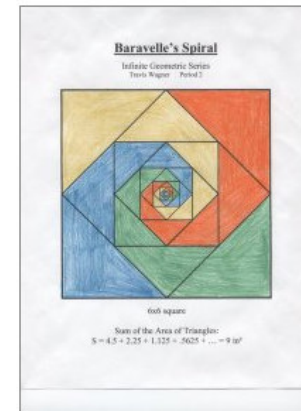
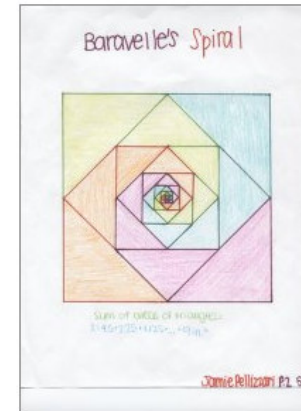
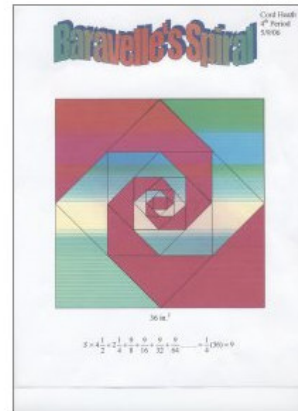
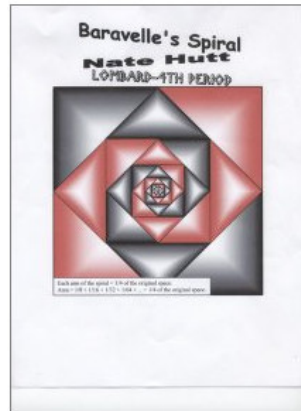
No value of n will work in this case.



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