ACTIVITY 2

The Power of Two

Materials:

- ☒ any type of paper
- ☐ napkins, tissue, newspaper
- ☐ calculators

Overview:
At last — a truly tangible way to introduce exponents! Astonish your students with a model of exponential growth. They will see the value of exponential notation as they put it to use. They will even be able to tell you why $2^0 = 1$!

Vocabulary: exponent, exponential growth, base, power

PROCEDURE

Skills:

- Modeling exponential growth
- Writing exponential notation

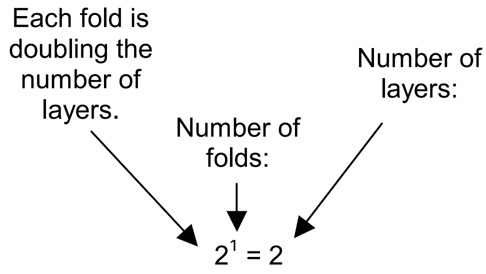
1. Ask students to get out any piece of paper. This can be lined or blank, new or used, standard-size or less. In fact, it is best if students use different types and sizes of paper.

2. Ask them to predict how many times they could fold the piece of paper in half. Take time to get the students to announce their prediction. You will likely hear many estimates as high as fifteen or twenty. Record them on the board. You may wish to analyze the range, mean, median, and mode of their answers.

3. Now ask them to try the experiment. Soon they will be trying to stomp and bite their paper to make it live up to their estimates! You can tell them that if you find footprints or teeth marks, it's not a fold, it's a stomp or a chomp. All students will end up with six or seven folds if they have followed instructions correctly. You may have occasional students who claim to have a dozen or more folds. By unfolding their papers, you can show these student that they have not truly folded the paper in half each time.

4. Record their results on the board next to the original estimates. The students will be intrigued by the differences. Ask them why most students didn't get as many layers as they predicted. Some student will usually say that the paper got too thick. Ask for an explanation. Someone will suggest that this occurs because the number of layers is *doubling* each time the paper is folded. This is a critical observation and should be brought to the attention of the entire class.

5. This is the time to introduce exponential notation. Ask the students to make a chart recording the number of folds



compared to the number of layers. The chart should look like the one on the left. Explain that the large "2" (the "base") represents that the number of layers is doubling each time. The small number (the exponent) represents the number of folds. The number on the right of the equation is the number of layers.

6. Ask them to complete their charts to show how many layers were achieved on the sixth and seventh folds. Record the chart on the board. The chart will then look like the one on the right.

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$

7. Ask them to take their chart into the "theoretical world" and explore some of the early predictions. How many layers of paper would you get if you folded a paper in half ten times or twenty times? How far can their calculator go? A good way to find out is to type "1 x 2 =" in the calculator. Then repeatedly type the equal key. This will repeat the "times two" operation on most calculators.

8. Lastly, ask the students to complete the first line of the chart on the right.

2^0 = ?
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$

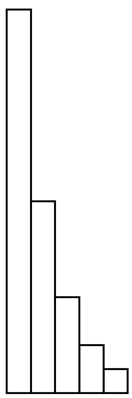
Although many students will think the answer should be zero, others will realize that when you have zero folds in your paper, you have one layer! Thus $2^0 = 1$. They can further justify the logic of this by saying that half of eight is four, half of four is two, and half of two is one.

9. You can even get the students to explore negative exponents by cutting one in half repeatedly to get the final chart.

$2^{-3} = \frac{1}{8}$
$2^{-2} = \frac{1}{4}$
$2^{-1} = \frac{1}{2}$
$2^0 = 1$
$2^1 = 2$
$2^2 = 4$

Good Tip!

As an additional assignment, ask students to find how many times they can cut a long strip of paper in half. The pieces can be taped side by side to create a bar graph showing exponential decay.



Journal Prompts:

How could you increase the number of folds you could achieve?
Why is $2^0 = 1$?
How does 2^5 compare to 5^2?

Homework:

Ask the students to go home and try to “break the record” by folding different materials. They should record their results using a t-chart. They may wish to try using larger paper or thinner paper. Encourage them to use other materials such as aluminum foil or plastic wrap. As of the date of this book’s publication, the record is twelve folds, but your students will have to figure out how it was done!

Taking a Closer Look:

Students can explore what happens when you triple or quadruple the number of layers each time you fold. This is difficult to do with paper, but students will find the t-chart helpful.

Scientific calculators can be used to explore these patterns. For example, if you could fold a paper in half ten times you would have 2^{10} layers of paper. On a calculator with an exponent key, the student should push “2”, “x^y”, “10”, “=”. The calculator shows 1024. Ask them to find n so that $2^n = 100$. They will see that 2^6 is only 64 and 2^7 is 128. They will have to use a decimal value for the exponent.

Assessment:

Check to see that students are using the notation correctly in their t-charts. That is one of the main purposes of this lesson. The other is to see that n^0 is always equal to 1 for all non-zero values of n. Ask them what 3^0 would be.

Name______________________________

Try to break your folding record from school. You many fold any material you wish. Before each attempt, write the name of the material, and then estimate the number of folds. Record your results in the t-charts below.

1. Material ____________________

Estimate: __________

fold	layers
2^0	
2^1	
2^2	
2^3	

2. Material ____________________

Estimate: __________

fold	layers
2^0	
2^1	
2^2	
2^3	

3. Material ____________________

Estimate: __________

fold	layers
2^0	
2^1	
2^2	
2^3	

4. Material ____________________

Estimate: __________

fold	layers
2^0	
2^1	
2^2	
2^3	