Fostering The Common Core Mathematical Practices

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✓ Three simple strategies for fostering the Common Core Mathematical Practices

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A colleague of mine, Dave Hill\footnote{Dave Hill was the California League of Schools Region 2 nominee for Educator of the Year in 2006. A former 5\textsuperscript{th} grade teacher, he currently teaches middle school physical education and intervention classes at Mistletoe Elementary in Redding, CA.}, had finished teaching his 5\textsuperscript{th} graders a unit on the digestive system. He asked them, “How long is the esophagus?”

“I have no idea!” responded one student. Dave hadn’t told them this fact.

“You have no idea?” Dave questioned.

“None!” the student replied.

“Well then, could it be a mile long?” Dave proposed.

“Of course not,” the student responded.

“How about an inch long?” Dave suggested.

“No way,” the student easily replied.

“Then you do have some idea about how long an esophagus is,” Dave reminded them. “Where is the esophagus?” he asked.

“It runs from your throat to your stomach,” the student quickly answered.

“How long is that?” Dave asked.

“About a foot and a half.”

This anecdote illustrates what needs to change not just in education in general, but in mathematics education in particular. We are moving away from an era of testing students on recall of procedures and facts and moving toward measuring how well students can think mathematically. It is this mathematical thinking that is the target of the eight Mathematical Practices of the Common Core Math Standards.

While the CCSS-Math will bring some major shifts in the mathematical content we teach, it will usher in an era of even greater change in how math is taught and tested. Students will need to develop these eight habits as they work in the mathematics classroom.

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

The student at the beginning of this article illustrates how the use of skills such as perseverance, critical reasoning, and other practices can facilitate learning. This
guide will outline three key strategies that we can implement today to move students toward greater use of these eight practices.

**Strategy 1: Math Talks**

Traditionally math classes start with a warm up. A student once asked me why we do this. “Are you afraid we will pull our math muscle if we don’t warm up?” he joked. That got me to thinking about the practice, and I came up with these three observations:

1. Students who are already good at math and tend to be engaged tend to do well at the warm ups.
2. Students who are not as skilled or who tend to disengage from the math class experience are usually doing the warms ups wrong and are disengaging from them.
3. Warm ups provide one more set of papers for me to grade.

Although I believe warm ups can be used successfully, I began to wonder if there was a better way to utilize these crucial early minutes of class. That’s when I began using math talks.

These five to ten minute introductions help students think mathematically at deep levels. They also encourage students to express their thinking so that they think more clearly and so that other students may benefit also. Here is one example.

The first day of school, I asked my 8th graders if they thought there were more inches in a mile or seconds in a day?

They were not allowed to use pencil or paper or calculator. I asked them to think about that without talking for about a minute. I did this because if I allowed immediate feedback, a clever student might propose an answer that the other students would assume was correct. At that point they would stop thinking. This 60-second pause allowed each student to approach the problem in his or her own way.

Next I asked them to discuss their thoughts with their table group. After about a minute, I asked them to stand up on one side of the class if they thought there were more inches in a mile and the other side if they thought there were more seconds in a day. They could stand in the middle if they thought the two quantities were equal.

This strategy of physical engagement is crucial. I knew from experience that if I asked for a show of hands that three students would vote for the first option, two for the second, and one for the third. This would indicate that either I have only six students in class, that many of them were born without hands, or that a majority of them don’t want to mentally engage in the task. By asking them to leave their seats I had 100% physical engagement, and this was because 100% of them had mentally engaged in the task.

Next I questioned some of them:

- Did anyone change their mind after talking with their table group? Why?
Over the course of the first few weeks of school I had to instruct them on how to listen to one another, how to speak in complete sentences, and how to provide constructive arguments. Eventually I saw students begin to use strategies they had learned from other students in previous days.

During these sessions students became more comfortable and more fluent with thinking mathematically. Though they often were discouraged initially, they eventually became used to the process of persevering mathematically. Since they were doing the problem mentally, they were reasoning abstractly. In their discussions, they made viable arguments. Although they didn’t model their mathematics or use tools constructively in this mental exercise, they did gain precision in their thinking. They also attended to the structure and patterning in a problem instead of implementing a paper-and-pencil procedure. They had demonstrated six of the eight mathematical practices.

To help them measure how well they were progressing, I often solved the problem on a calculator afterward so they could gage their accuracy. For example, I asked them this question one day.

The 8th grade class needs to raise $9,250 for our graduation ceremony, dance, and end of year trip. We currently have $2,918. What percent have we raised?

Some students said it was less than 50%. Others rounded the numbers to $3,000 and $9,000 and said we had raised about a third or approximately 33%. Some got even greater accuracy by saying it had to be less than 33% since the numerator, $2,918, is less than $3,000, and the denominator, $9,250, is more than $9,000. Since the actual answer is about 31.5%, all of these estimates show a degree of accuracy. The student who quickly calculated 33% is off by only 1.5%. Comparing their estimates to the actual answer gave them confidence as they tackled future problems.

In Research Ideas for the Classroom: Middle Grades Mathematics, editor Douglas T. Owens states, “Research has shown that for students to become highly skilled at estimation, it had to be incorporated into their regular instruction over several years.”

This tells me that the acquisition of number sense and mathematical thinking is like exercise. It is not the subject of a chapter reviewed at the beginning of the year. It is a daily endeavor. It is truly a warm up.

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2 Owens, Douglas T., Research Ideas for the Classroom: Middle Grades Mathematics, National Council of Teachers of Mathematics, page 45
The following page gives examples of other questions we can pose to our students to foster math talks.
Estimate each answer:

1. Which is greater 86 x 38 or 88 x 36?
2. What is 0.52 x 789?
3. Which is greater, $\frac{12}{17}$ or $\frac{5}{8}$?
4. 4953 ÷ 68
5. 0.4 x 58.6 x 5 x 3
1. 86x38=3,268 while 88x36=3,168. When two sets of numbers have the same sum, the pair with the lesser difference will produce a greater product. I saw a video once of a student who stated:

\[
\begin{align*}
86 \times 38 &= 86 \times 36 + 86 \times 2 \\
88 \times 36 &= 86 \times 36 + 2 \times 36 \\
\text{So} \ 86 \times 38 \text{ is greater.}
\end{align*}
\]

That student was in the third grade.

2. 410.28

3. The first fraction is approximately 0.706, while the second is 0.625. Converting \( \frac{5}{8} \) to \( \frac{10}{16} \) makes the comparison easier.

4. Approximately 72.8

5. 351.6
Strategy 2: Thinking Ahead

This strategy occurred to me prior to teaching a challenging algebra lesson one day. The pacing guide called for students to learn how to solve the famous “A train leaves Baltimore” problems. The worksheet had ten problems on it, and included a bank of the ten answers. From the experiences of past years, I knew that students rarely engaged in the lesson due to their frustration and did poorly on the assignment. Just before they arrived, I decided to try an experiment.

I explained that I had a challenging lesson to teach that day, one with which students often struggled. I told them that I wanted to see how well they could estimate the answers without any instruction. I put them in pairs, passed out the ten-question assignment with the answers cut off, and then encouraged them to think about each problem and write their best estimate for an answer. The language and strategies that ensued amazed me in their accuracy and sophistication. Some of them made tables showing how far the trains had traveled after one hour, two hours, and so on. Some drew diagrams, and some just discussed their ideas. A few even used algebra to model their thinking. After about 15 or 20 minutes, I had them put away the assignment and take notes as I explained how to approach the problems algebraically. Because of the previous activity, they were much more engaged than in previous years.

Then I passed out the answer bank and asked them to see if they could match their previous conjectures to the correct answers. Many were not only very close on their estimates, there were some students who had solved all ten problems exactly prior to my instruction by simply using mathematical thinking.

Now that they knew the answers, I asked them to write out the problems algebraically and solve them. The showing of the work became an afterthought instead of a goal in itself.

Again they had demonstrated the eight mathematical practices. They had:

- Made sense of problems and persevere in solving them.
- Reasoned abstractly and quantitatively.
- Constructed viable arguments and critiqued the reasoning of others.
- Modeled with mathematics.
- Used appropriate tools strategically.
- Attended to precision.
- Looked for and made use of structure.
- Looked for and expressed regularity in repeated reasoning.

It surprised me how incredibly astute the students were at thinking mathematically when freed from the constraints of mimicking a procedure. Though there is nothing wrong with using procedures in mathematics—they are an efficient problem-solving tool—the procedure must be built upon a solid foundation of mathematical thinking in order to stand the test of time. Procedures without meaning are soon forgotten.
Strategy 3: Promoting the Use of Student Discourse

Our brain is a language processor. Whether we are engaged in a language arts class, a science class, or even a math class, our brains utilize language to process and store learning. For this reason, students must be able to use oral and written language to build meaning and understanding when they learn.

You will notice two things. First of all, both of the previous two sections illustrate the need for language in developing meaning while learning mathematics. In “Math Talks” students literally talked about their ideas concerning the problem posed that day. They commented on one another’s thinking and verbalized their own thought processes.

Secondly, most of the eight Mathematical Practices are language intensive and language dependent:

- **Make sense of problems** and persevere in solving them.
- **Reason abstractly and quantitatively.**
- **Construct viable arguments** and critique the reasoning of others.
- Model with mathematics. (**Models can include verbal descriptions.**)
- Use appropriate tools strategically. (**Including but not limited to language.**)
- Attend to precision. (**Frequently students proceeded to more precise models due to the discussions they had with peers.**)
- Look for and express regularity in repeated reasoning.

For these reasons, one of the changes we will need to encourage in the Common Core classroom is to help students engage in mathematical discourse. This may be a major shift in instruction in some classrooms. A mathematics teacher has likely had little training in how to foster oral or written language. Fortunately, some simple strategies can help.

The questions I posed to students in the first section, “Math Talks”, will provide some guidelines. I also encourage students to speak in complete sentences, as the way we speak is a reflection of our thinking. To write or speak well is to think well. Here are some other similar questions that can be used to get the math conversation going:

- Will that strategy always work?
- Did you notice any patterns?
- Can you solve the problem a different way?
- What are the advantages of solving the problem that way?
- How close do you think your answer is?
- Can you paraphrase what that student said?

Asking students to explain or justify their reasoning and their answers is another valuable strategy for promoting deeper thinking. Instead of asking if two triangles are congruent, it is better to ask them to also explain their reasoning.
Though it is more difficult to manage, written language also offers many benefits in a math class. Recently my students had to design a tessellation. In addition to demonstrating it by tiling their pattern on a piece of paper, they had to illustrate how their tile could *translate, reflect, and rotate* on a coordinate plane. Additionally, they had to write a paragraph describing their tessellation similar to this example:

I began with a square. I cut a scalene triangle out of the bottom and translated it to the top and then reflected it horizontally. I cut an irregular curved shape out of the left side and performed a slide-flip to the right side. When I finished, my shape reminds me of a man wearing a hat.

The project was assessed not only on the mathematics modeled in the coordinate plane, but also on their artistic presentation and on their written paragraph. Spelling, grammar, punctuation, and mathematical vocabulary were all factors in their grade on the written portion.

Students can also be asked to write a summary of the day’s lesson elaborating on what skill was being learned, what they have grasped, and what they still need to learn.

Asking students to write a note to an absent student explaining how to do the day’s lesson is also a great way to help them take their thinking about the concept to a much deeper and more sophisticated level of understanding.

Taken collectively, these three strategies will go a long way in moving students toward the Common Core Mathematical Practices. Over time, we teachers will become more and more proficient in organizing our lessons to foster this type of thinking. And also over time, our students will gain more and more fluency with the eight mathematical practices.