Graphic Oranizers

For Teaching Algebra

Solving Proportions

Multiplying Binomials

Factoring Polynomials

Combined Work Prolems

Working with Integers

By Brad Fulton
Educator of the Year, 2005
brad@tttpress.com www.tttpress.com
530-547-4687
P.O. Box 233, Millville, CA 96062

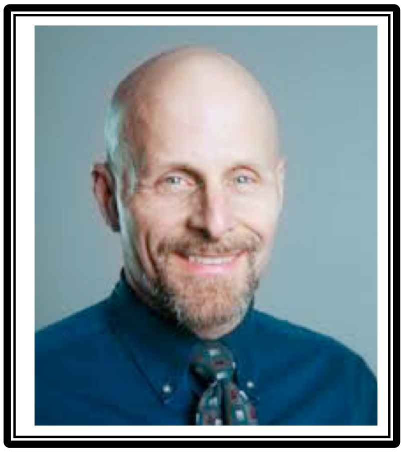
Teacher to Teacher Press

Join us!

Facebook: TeacherToTeacherPress
Twitter: @tttpress
YouTube /watchtttpress

Brad Fulton

Educator of the Year



PO Box 233, Millville, CA 96062
(530) 547-4687
brad@tttpress.com

- ♦ **Consultant**
- ♦ **Educator**
- ♦ **Author**
- ♦ **Keynote presenter**
- ♦ **Teacher trainer**
- ♦ **Conference speaker**

Known throughout the country for motivating and engaging teachers and students, Brad has co-authored over a dozen books that provide easy-to-teach yet mathematically rich activities for busy teachers while teaching full time for over 30 years. In addition, he has co-authored over 40 teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

Seminar leader and trainer of mathematics teachers

- ♦ 2005 California League of Middle Schools Educator of the Year
- ♦ California Math Council and NCTM national featured presenter
- ♦ Lead trainer for summer teacher training institutes
- ♦ Trainer/consultant for district, county, regional, and national workshops

Author and co-author of mathematics curriculum

- ♦ Simply Great Math Activities series: six books covering all major strands
- ♦ Angle On Geometry Program: over 400 pages of research-based geometry instruction
- ♦ Math Discoveries series: bringing math alive for students in middle schools
- ♦ Teacher training seminar materials handbooks for elementary, middle, and secondary school

Available for workshops, keynote addresses, and conferences

All workshops provide participants with complete, ready-to-use activities that require minimal preparation and give clear and specific directions. Participants also receive journal prompts, homework suggestions, and ideas for extensions and assessment.

Brad's math activities are the best I've seen in 38 years of teaching!
Wayne Dequer, 7th grade math teacher, Arcadia, CA

“I can't begin to tell you how much you have inspired me!”
Sue Bonesteel, Math Dept. Chair, Phoenix, AZ

“Your entire audience was fully involved in math!! When they chatted, they chatted math. Real thinking!”
Brenda McGaffigan, principal, Santa Ana, CA

“Absolutely engaging. I can teach algebra to second graders!”
Lisa Fellers, teacher

References available upon request

Like my activities? How about giving me a favorable rating on the Teachers Pay Teachers website? Four stars would be much appreciated and would help me sleep better at night.

Like me even more? Then please don't make copies for your colleagues. I know it's tempting when they say, "Wow! Groovy activity! Can I have a copy?" But this is how I make my money, and why are they still saying "groovy" anyway?

If we make copies for our friends, can we honestly tell our students not to copy or take things that don't belong to them? (Ouch!)

Half priced site licensed copies are available on the TPT website. Please encourage them to take advantage of this affordable option. Okay?

Thanks and happy teaching,

Brad ☺

I want...

○ a) Effective staff development

○ b) Affordable staff development

○ c) Ongoing staff development

● d) *ALL OF THE ABOVE!*

www.tttpress.com
brad@tttpress.com

Great DVD presentations offer quality mathematics staff development at a fraction of the cost!

- **Effective because** they are classroom-tested and classroom-proven. These popular DVDs of Brad's trainings have been utilized by teachers throughout the country for years.
- **Affordable because** they are site-licensed. Buy only one copy for your whole school, print as many copies of the handouts as you need.
- **Ongoing because** when you hire new staff, simply hit "play" and the training begins. There's no need to bring back the consultant.

Give It Structure:

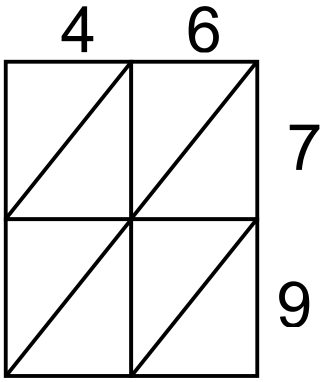
Students who struggle mathematically often fail for one of two reasons: either they struggle with the arithmetic that they encounter or they struggle with what to do with that arithmetic. For example, in solving a two-digit by two-digit multiplication problem like the one shown here, the student must begin by finding the product of the six and the nine. Given enough fingers, toes, and time, most students can find that 6x9=54. However, the student must also know that the four goes beneath the nine, and the five goes above the four. Then the nine and four must be multiplied while the five is added and both numbers go next to the four. Then for some inexplicable reason, a mystery zero appears. The student may approach mathematics with the thought that, “Ours is not to reason why–just invert and multiply.”

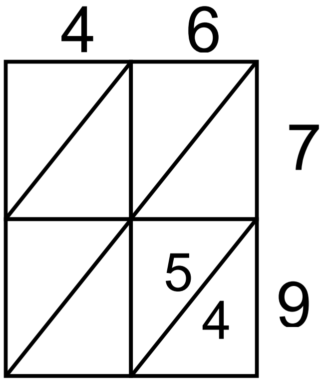
$$\begin{array}{r} 46 \\ \underline{x79} \end{array}$$

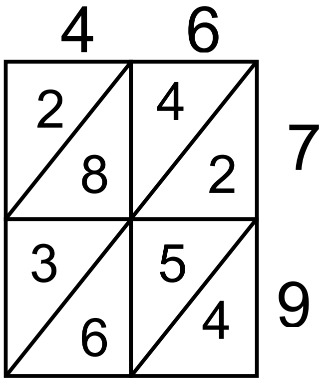
Clearly we want our students to understand why an algorithm works, but realistically few of them do. Just as importantly, we want our students to be successful with math, and few of our intervention students are. For this reason, if we can provide students with a structural template or a graphic organizer, they need only concern themselves with the arithmetic involved in a problem. Following are a few graphic organizers that will help students with common mathematical tasks.

Lattice multiplication:

For centuries, multiplication was solved differently than it is now. A lattice was be used to multiply 46 by 79 as shown in the left diagram. Each digit was multiplied and the product placed in the intersecting cells. The tens digit was placed above the diagonal and the ones was placed below. In the second diagram, the six and nine have been multiplied to get a product of 54. All cells have been filled out in the third diagram.

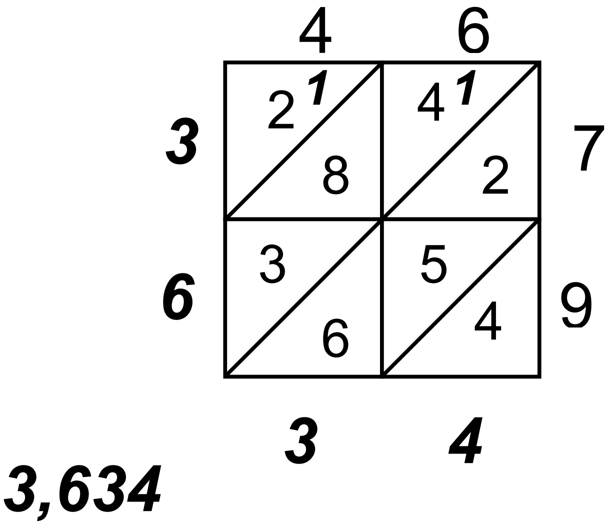




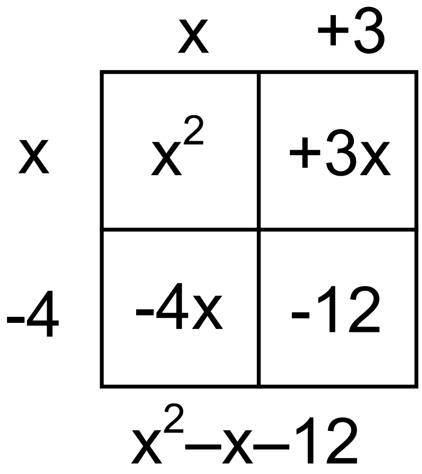


The following diagram shows how the diagonals are added to produce the product. Notice that in two diagonals, a tens digit has been regrouped to the top of the next diagonal.

Although the lattice method does essentially the same steps as our more familiar process, I have observed that students who use this method tend to make fewer errors. I believe this is because they do not have to attend to the structure of the algorithm and simply focus on the arithmetic of the multiplication.



This method of multiplication also transitions very nicely into multiplication of binomials in algebra. Rarely do we teach students to multiply binomials the way we multiply multi-digit numbers. Instead more and more algebra teachers are using what is called the *generic rectangle* that is essentially the lattice. If we want to multiply two binomials such as (x+3)(x-4) we can place them in an array. Again, we fill in each cell and *add diagonals* as we did in the lattice. Now however, we call it *combining like terms*.



Proportion Boxes:

Solving proportions, percents, and the word problems involving them are easier with this graphic organizer. Let's look at a sample word problem that lends itself to a solution involving proportions.

> Mr. Fulton sleeps 7 hours per day. How many of his 54 years has he spent sleeping?

Notice that we are comparing two ideas in two ways. We are comparing the part of his day he spends sleeping to his whole day. We are also comparing a single day to his whole life. Thus we could label our graphic organizer as shown.

This sets up the proportion in the proper arrangement. That is, we have $^{7}/_{24}={}^{x}/_{54}$. We would now decide the best method of solving this proportion. In this case, cross products would be effective. Notice that if the labels are switched, the numbers switch with them and a valid proportion still results. This template will work for any proportion problem, even if it is not a word problem. In addition, it works for all types of percent problems.

	hours	years
part	7	x
whole	24	54

Problems about percent can be solved using the same strategy. Simply change the labels as shown. To solve the problem, we simply put the numbers into their correct cells. What percent of 56 is 21? This gives us the proportion:

$$^{21}/_{56} = {}^{x}/_{100}$$

	fraction	percent
part	21	x
whole	56	100

Combined Mixture Problems:

Combined mixture problems in algebra can baffle students. They tend to look like one of these two problems.

A chemist wants to mix 12 grams of a 10% salt solution with some 75% salt solution to get a 20% salt solution. How many grams of the 75% salt solution are needed?

A company wants to create a 20 mixture of peanuts and walnuts that sells for $5 per pound. The peanuts sell for $2 per pound and the walnuts sell for $6 per pound. How many pounds of each must they use?

I have used the following approach with great success. First, I tell them these problems are “ca-ca”. They agree. Then I explain that they can be solved with an equation that contains a lot of “ca-ca”.

$$C_1A_1+C_2A_2=C_TA_T$$

This means the *concentration* of the first solution times the *amount* of the first solution plus the *concentration* of the second solution times the *amount* of the second solution equals *concentration* of the total solution times the *amount* of the total solution. In the second example the *c* stands for *cost* instead of *concentration*.

Before substituting values into the equation template, I introduce the graphic organizer shown here. The students must decide which numbers go in which cells. First I ask the students to think about the problem. Clearly, if we mix two solutions, the total concentration of the mixture will be stronger than one solution and weaker than the other. That is, the *total concentration* will be the middle percent. (This will also be true if a *cost* is involved.) Thus the 20% is the total, and the 10% and the 75% are the concentrations of the two addends. Then the students merely match the amounts with the respective percents and use an *x* for the missing number. The completed graphic is shown. Notice that the two *amounts* in the top cells must add up to the *total amount*. The template for the second problem is also shown.

	C	A
1		
+2		
T		

	C	A
1	10%	12
+2	75%	x
T	20%	$x+12$

	C	A
1	$2	x
+2	$6	$20-x$
T	$5	20

It is also a good idea to have the students estimate the answer ahead of time. For example, in the second problem, the total mixture is one dollar less than the $6 price of the walnuts but three dollars more than the $2 price of the peanuts. Thus we will need more of the walnuts than we will of the peanuts. Often students can use this information to correctly estimate the answer.

ACTIVITY

Materials:

☒ paper

☐ transparency master

☐ activity master

"X" Marks the Spot

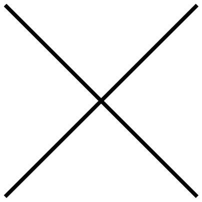
Overview: The rules are so simple you can teach them without saying a word! Yet the math is rich and abundant. You can use these simple drills to reinforce basic addition, subtraction, multiplication, and division facts and develop number sense without boring your students. Fractions, decimals, and negative numbers can also be used. You can even factor polynomials using this simple method!

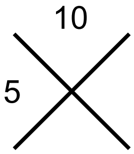
Vocabulary: product, difference, quotient, polynomial

PROCEDURE

Skills:

- Adding, subtracting, multiplying, and dividing integers, fractions, and decimals
- Problem solving
- Factoring polynomials



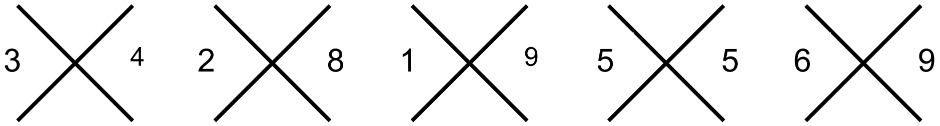


$^{10}/_5 = 2$

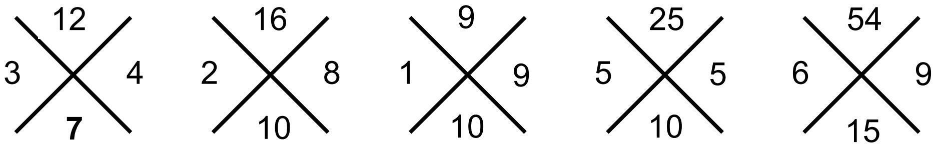
$5 + 2 = 7$

1. Tell the class that this game has only two simple rules...but you won't tell them what they are. They will have to figure out the rules by themselves. As soon as a student knows how to play, he or she can come up to the board and write down the answer.

2. Have them copy these five problems onto a piece of paper as you write them on the board.



Then begin writing in the answers by adding the numbers on the left and right to get the bottom number and multiplying them to get the top number.

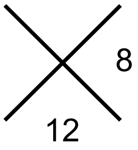


3. If you work slowly, pausing as if to ponder before writing each answer, some of the students will soon catch on. After a majority of the class has discovered the rules of the game, allow a student to explain them.

4. Then you can continue to play the game by varying the format.
   - Placing numbers in sections A and B will require students

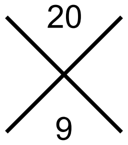
to divide first, then add.

- Placing numbers in sections A and D or C and D will require students to subtract first, then multiply.
- Placing numbers in sections B and D will require students to study the various combinations of sums and products that satisfy the given answers.



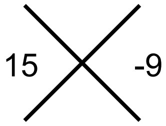
12 – 8 = 4

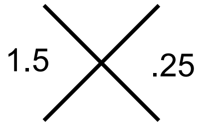
4 x 8 = 32

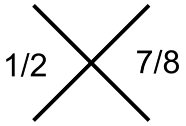


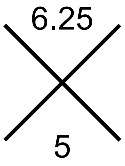
1 x 20 = 20	1 + 20 = 21
2 x 10 = 20	2 + 10 = 12
4 x 5 = 20	**4 + 5 = 20**

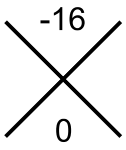
5. Eventually, you may wish to increase the difficulty through examples like these.

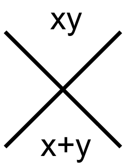


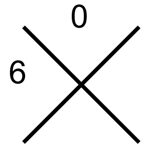


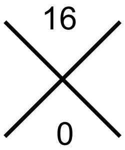




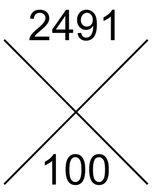








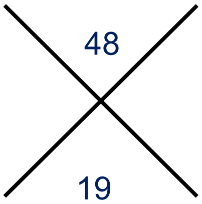
6. You can also use the guess and check method to solve complex puzzles. Research has shown that the guess and check method is not only a valuable skill, it helps children transition to solving equations in algebra. Here is how to solve problems like the one on the right using this method. Pick a pair of numbers that add up to 100 such as 50 and 50. Write them in columns a and b.
Then multiply them to find the product. In this case, it is 2500, which is too high. We mark our check with an "H" to signify that this is too high. This tells us that the number in column a is too high.
Let's adjust our guess by trying 40 and 60. Remember that our guesses must add to 100. It is also very important to note that the smaller of the two numbers must go in column a.
The product of these two numbers is 2400, which is too low. This is marked with an "L".
Our next guess for column a must be greater than 40 but less than 50. Let's try 45. This makes *b* = 55. Our new product, 2475 is too low also.



a	*b*	check
50	50	2500 H
40	60	2400 L
45	55	2475 L
48	52	2496 H
47	53	2491 ☺

Our fourth guess will be 48. Now *b* = 52, and our product is 2496. Although this is too high, it is very close.

For our next guess, we try 47 for *a*, and 53 for *b*. This gives us the product we wanted.

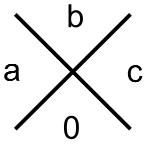


* * *

Journal Prompts:

Explain to a student how you would find the solution to the problem on the left.

What could you tell about the value of a and c in the example on the below? What can you tell about the value of b? Explain.



Homework:

Assign one of the accompanying activity masters.

You can make a homework worksheet by placing numbers in a copy of the blank activity master. Alternately, the students can copy down problems as you write them on the board.

Good Tip!

These drills are a great way to practice number concepts throughout the year. Worksheets can be created on the spot to be used as homework or warm-ups. If you are studying fraction multiplication, simply have the students copy a set of these problems as you write them on the board.

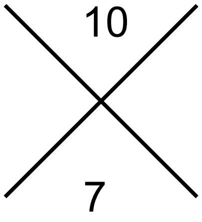
Taking a Closer Look:

The difficulty of these drills can be varied by the numbers chosen and their placement. Using decimals, fractions, or negative numbers can also increase the complexity.

Algebra students can practice factoring polynomials this way too. For the polynomial x^2 + **7**x + **10** = 0, students would construct the problem shown. I tell them to put the *b* term in the *basement* and the *c* term in the *ceiling*. The solutions are 2 and 5. The expression factors into the following binomials:

$$x^2 + 7x + 10 = (x + 2)(x + 5)$$

The solution to the equation then is x = –2 and x = –5.



Assessment:

These drills can be spot checked for accuracy or students can exchange papers to check them.

You may also use the accompanying answer keys for the accompanying activity masters.

Answer Key:

Set 1	top	bottom
a	14	9
b	45	14
c	12	7
d	8	9
e	36	12
f	36	13
g	48	14
h	12	8
i	0	7
j	9	6
k	20	9
l	25	10
m	33	14
n	60	16
o	24	14
p	77	18
q	72	20
r	60	16
s	60	17

Set 2	side	top
a	5	35
b	6	24
c	0	0
d	7	49
e	12	60
f	9	54
g	11	22
h	9	108
i	12	96
j	10	10
k	12	48
l	12	120
m	2	12
n	12	132
o	0	0
p	6	42
q	9	72
r	12	0
s	12	144

Set 3	side	bottom
a	8	10
b	7	14
c	6	14
d	7	13
e	9	13
f	12	15
g	6	12
h	4	16
i	6	10
j	8	15
k	7	11
l	0	9
m	3	8
n	2	11
o	7	10
p	12	21
q	11	21
r	12	23
s	1	9

Set 4	side	side
a	4	7
b	4	8
c	3	12
d	6	8
e	6	6
f	0	12
g	4	9
h	7	9
i	8	8
j	6	10
k	5	12
l	8	11
m	10	12
n	11	12
o	7	12
p	12	12
q	8	9
r	9	11
s	9	9

Set 5	top	bottom
a	1	7.2
b	0.45	1.4
c	1.2	4.3
d	0.008	0.18
e	3.6	6.6
f	0.36	1.3
g	0.048	0.68
h	0.12	0.8
i	0	0.7
j	0.9	3.3
k	0.002	0.09
l	0.025	0.55
m	0.33	1.4
n	6	10.6
o	0.24	2.12
p	0.077	0.81
q	0.96	1.28
r	0.6	10.06
s	6	12.5

Set 6	top	bottom
a	-14	-5
b	45	-14
c	-12	1
d	8	-9
e	-36	0
f	36	-13
g	-48	2
h	-12	4
i	0	-7
j	9	-6
k	-20	1
l	-25	0
m	33	-14
n	-60	-4
o	24	-14
p	77	-18
q	-96	-4
r	60	-16
s	-60	-7

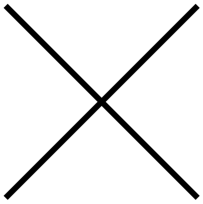
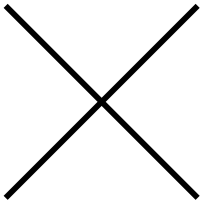
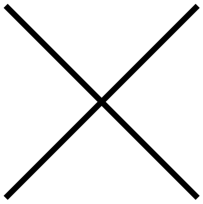
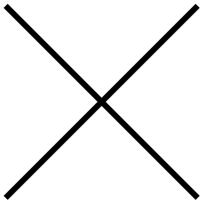
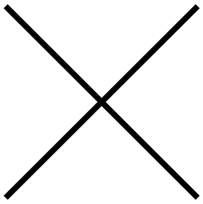
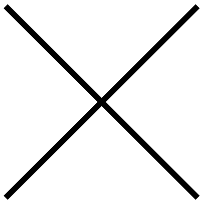
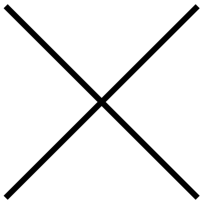
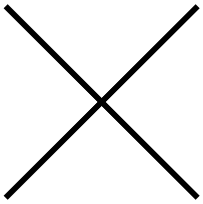
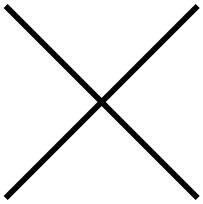
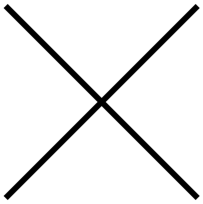
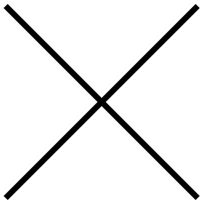
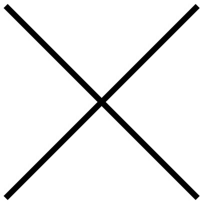
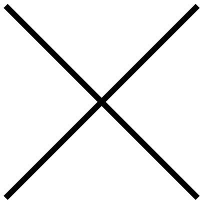
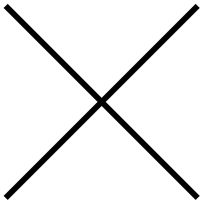
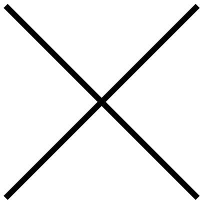
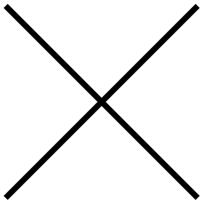
Set 7	side	side
a	-36	-1
b	-1	36
c	-6	6
d	2	14
e	-14	-2
f	-7	4
g	-4	7
h	-9	-2
i	-6	3
j	-6	-3
k	-18	-1
l	-9	-5
m	-15	-3
n	-15	3
o	-1	45
p	-8	-6
q	-6	8
r	-12	4
s	-2	24

Set 8	side	side
a	47	53
b	23	63
c	50	60
d	211	289

Name_________________________

“X” Marks the Spot

Multiply the two side numbers and put the product on the top. Add the two side numbers and put the sum on the bottom.

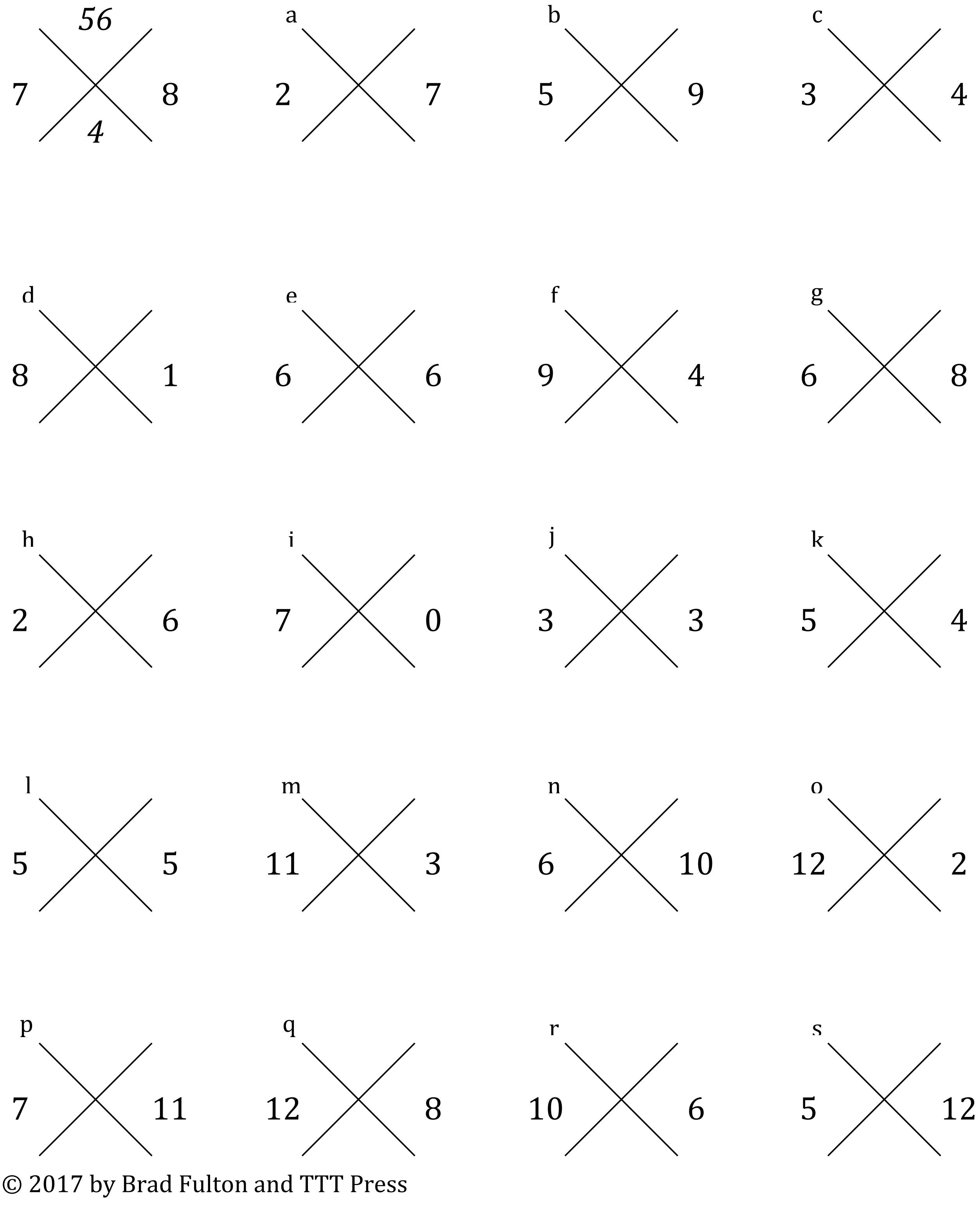

















Activity Master

"X" Marks the Spot 1

Name________________

Multiply the two side numbers and put the product on the top. Add the two side numbers and put the sum on the bottom as shown.

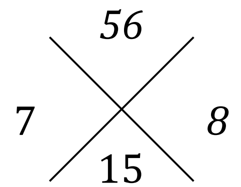


Activity Master

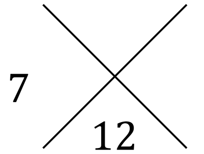
"X" Marks the Spot 2

Name_______________

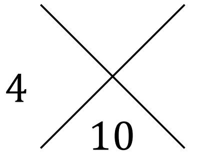
The number on the bottom is the sum of the two numbers on the sides. Find the missing side number. Then multiply the two side numbers and write the product on the top.



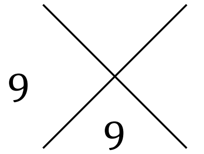
a



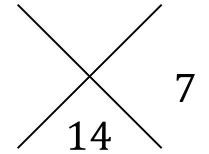
b



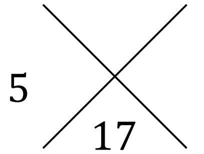
c



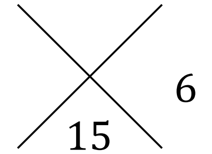
d



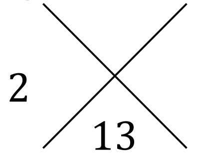
e



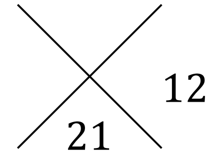
f



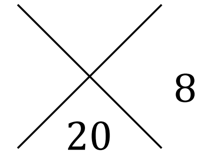
g



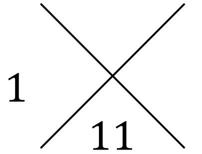
h



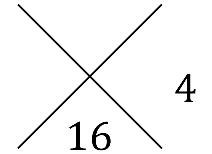
i



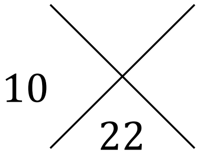
j



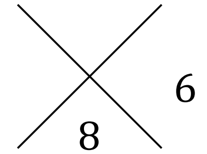
k



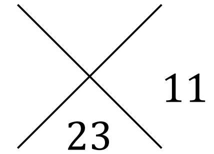
l



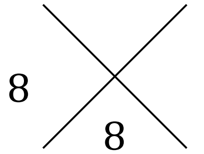
m



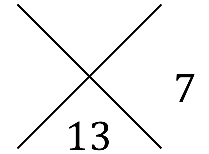
n



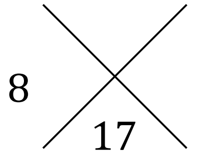
o



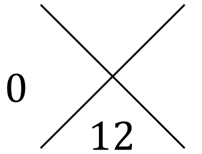
p



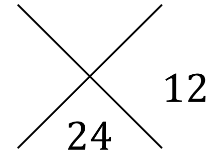
q



r



s



Activity Master

"X" Marks the Spot 3

Name________________

The top number is the product of the two numbers on the sides. Find the missing side number. Then add the two side numbers and write the sum on the bottom.

56
7 *8*
15

a 16
2

b 49
7

c 48
8

d 42
6

e 36
4

f 36
3

g 36
6

h 48
12

i 24
4

j 56
7

k 28
4

l 0
9

m 15
5

n 18
9

o 21
3

p 108
9

q 110
10

r 132
11

s 8
8

Activity Master

"X" Marks the Spot 4

Name________________

The top number is the product of the two missing side numbers. The bottom number is the sum of the two missing side numbers. Find the missing side numbers.

56
7 *8*
15

a 28
11

b 32
12

c 36
15

d 48
14

e 36
12

f 0
12

g 36
13

h 63
16

i 64
16

j 60
16

k 60
17

l 88
19

m 120
22

n 132
23

o 84
19

p 144
24

q 72
17

r 99
20

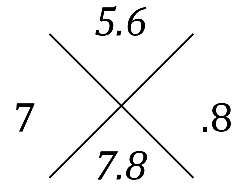
s 81
18

Activity Master

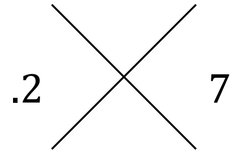
"X" Marks the Spot 5

Name_______________

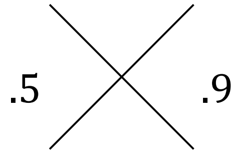
Multiply the two side numbers and put the product on the top. Add the two side numbers and put the sum on the bottom as shown.



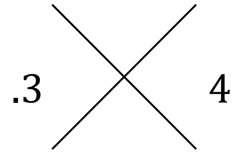
a



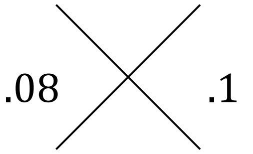
b



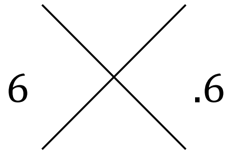
c



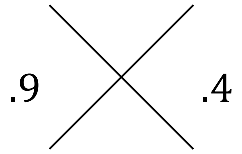
d



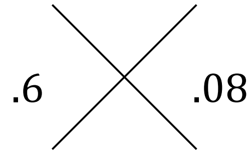
e



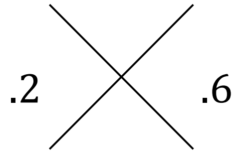
f



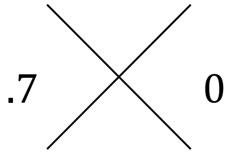
g



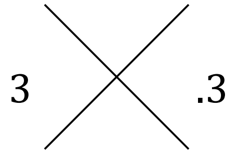
h



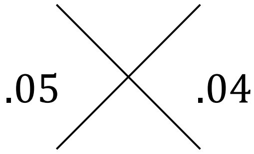
i



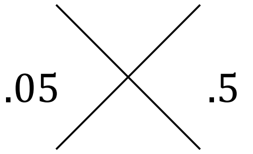
j



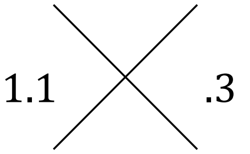
k



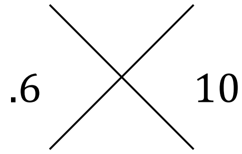
l



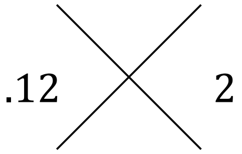
m



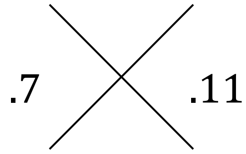
n



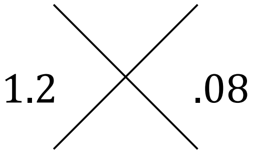
o



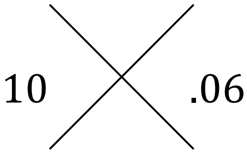
p



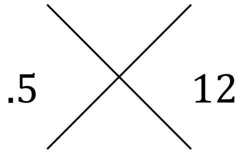
q



r



s

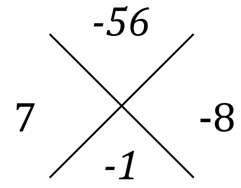


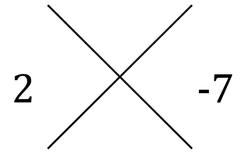
Activity Master

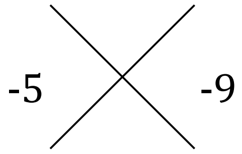
"X" Marks the Spot 6

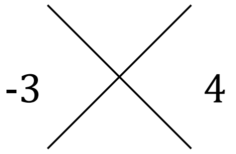
Name________________

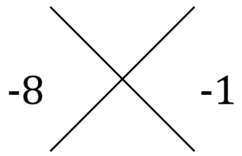
Multiply the two side numbers and put the product on the top. Add the two side numbers and put the sum on the bottom as shown.

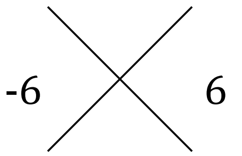


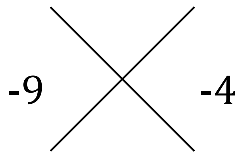
a


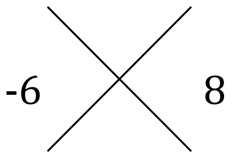
b


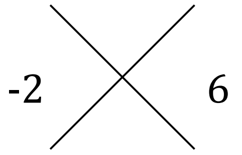
c


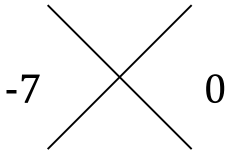
d


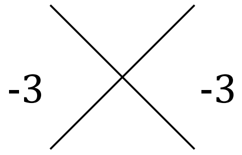
e


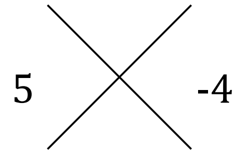
f


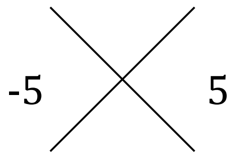
g


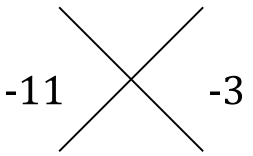
h


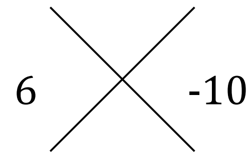
i


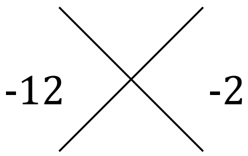
j


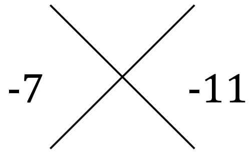
k


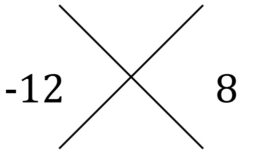
l


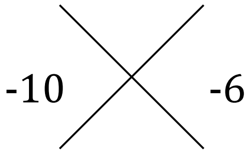
m


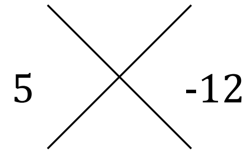
n


o


p


q


r


s


Activity Master

"X" Marks the Spot 7

Name________________

The top number is the product of the two missing side numbers. The bottom number is the sum of the two missing side numbers. Find the missing side numbers.

36
36 *1*
37

a 36
-37

b -36
35

c -36
0

d 28
16

e 28
-16

f -28
-3

g -28
3

h 18
-11

i -18
-3

j 18
-9

k 18
-19

l 45
-14

m 45
-18

n -45
-12

o -45
44

p 48
-14

q -48
2

r -48
-8

s -48
22

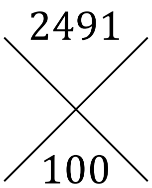
Activity Master

"X" Marks the Spot 8

Name________________

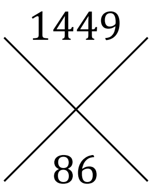
Use a guess and check table to find the missing side numbers. Always put your lower number in column a.

a



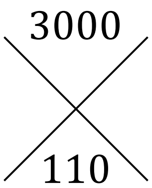
a	b	check

b



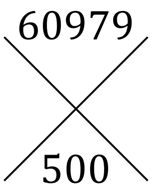
a	b	check

c



a	b	check

d



a	b	check

Using “X” Marks the Spot to Solve Combined Work Problems.

One of the more challenging types of problems for algebra students is the combined work problems. These often look like these two examples:

> Andy can paint a room in three hours. Zoe can paint the same room in four hours. How long would it take them to paint the room working together?
>
> One pipe can fill a tub in three hours. A second pipe can fill the same tub in four hours. How long will it take to fill the tub if both pipes are used?

Typically students either add the two numbers or average them, failing to realize that the job must get done more quickly than either given time. Thus the answer must be *less* than either given number.

The most common way to solve the problem is by using the following formula:

$$\frac{x}{3}+\frac{x}{4}=1$$

The variable, x, represents the total time for the combined work. The first fraction means that Andy paints one third of a room per hour, and the second fraction shows that Zoe paints one fourth of a room per hour. The two fractions add up to one room being painted.

While this procedure makes sense to the teacher, its origin and development is beyond the ability of most students. With enough practice, students may become proficient with this, but it remains questionable if the equation holds meaning for them. Let’s look at the solution to this problem using the equation.

$$\frac{x}{3}+\frac{x}{4}=1$$

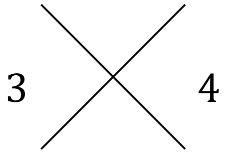
$$\frac{4x}{12}+\frac{3x}{12}=1$$

$$\frac{7x}{12}=1$$

$$\left(\frac{12}{7}\right)\frac{7x}{12}=1\left(\frac{12}{7}\right)$$

$$x=\frac{12}{7}$$

Now examine the solution to the following problem from “X” Marks the Spot:



Notice that the solution to this problem is also $^{12}/_{7}$! Certainly this is a much simpler way to achieve the correct answer, but will it always work? In fact, it does. Let’s demonstrate this by generalizing the problem:

> Andy paints a room in a hours, and Zoe paints the same room in b hours.

> How long will it take them to paint the room working together?

$$\frac{x}{a}+\frac{x}{b}=1$$

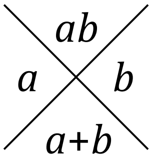
$$\frac{bx}{ab}+\frac{ax}{ab}=1$$

$$\frac{(a+b)x}{ab}=1$$

$$\left(\frac{ab}{(a+b)}\right)\frac{(a+b)x}{ab}=1\left(\frac{ab}{(a+b)}\right)$$

$$x=\frac{ab}{(a+b)}$$

This is also the solution to the simpler problem when we use *a* and *b* as our side numbers:



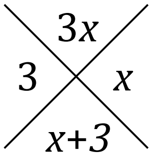
This method will also work when only one person's time is given along with the combined work time:

> One pipe can fill a tub in 3 hours. With a second pipe running, the tub can be filled in only two hours. How long would the second pipe take if it was used alone?

The traditional algorithm would look like this:

$$\frac{2}{3}+\frac{2}{x}=1$$

Using the "X" Marks the Spot strategy, we have:



We also know that the answer is two. Thus the top number divided by the bottom number must equal two:

$$
\begin{aligned}
\frac{3x}{x+3} &= 2\\
\frac{3x}{x+3} &= \frac{2}{1}\\
3x &= 2(x+3)\\
3x &= 2x+6\\
x &= 6
\end{aligned}
$$

Multiplying binomials using the box

In this activity, students use a 2x2 box to multiply binomials. It is much like filling in a multiplication table. Each cell of the box contains the product of the factors above and to the left of it.

Multiply (2x+1)(3x–4)

	$2x$	$+1$
$3x$	$6x^2$	$3x$
-4	$-8x$	-4

$$6x^2 - 5x - 4$$

Two worksheets are provided. In the first, the *a* term is 1, and in the second it is greater than 1.

Multiplying Binomials

Name______________________________

Date___________________ Class_______

Use the area model to multiply these binomial pairs.

A

	x	$+4$
x		
$+2$		

$(x + 4)(x + 2) =$ ____________

B

	x	$+7$
x		
$+9$		

$(x + 7)(x + 9) =$ ____________

C

	x	$+3$
x		
$+8$		

$(x + 3)(x + 8) =$ ____________

D

	x	-4
x		
$+2$		

$(x - 4)(x + 2) =$ ____________

E

	x	$+5$
x		
-9		

$(x + 5)(x - 9) =$ ____________

F

	x	-6
x		
$+1$		

$(x - 6)(x + 1) =$ ____________

G

	x	-8
x		
-4		

$(x - 8)(x - 4) =$ ____________

H

	x	-2
x		
-7		

$(x - 2)(x - 7) =$ ____________

I

	x	-9
x		
-7		

$(x - 9)(x - 7) =$ ____________

Multiplying Binomials

Name________________________________

Date____________________ Class________

Use the area model to multiply these binomial pairs.

A

	$2x$	$+1$
x		
$+1$		

$(2x + 1)(x + 1) =$ ______________

B

	x	$+5$
$4x$		
$+3$		

$(x + 5)(4x + 3) =$ ______________

C

	$3x$	$+1$
$2x$		
$+1$		

$(3x + 1)(2x + 1) =$ ______________

D

	$3x$	-2
$3x$		
$+4$		

$(3x - 2)(3x + 4) =$ ______________

E

	$4x$	-5
$2x$		
-9		

$(4x - 5)(2x - 9) =$ ______________

F

	$6x$	-5
$5x$		
$+1$		

$(6x - 5)(5x + 1) =$ ______________

G

	$7x$	-8
$8x$		
$+7$		

$(7x - 8)(8x + 7) =$ ______________

H

	$6x$	-1
$6x$		
-5		

$(6x - 1)(6x - 5) =$ ______________

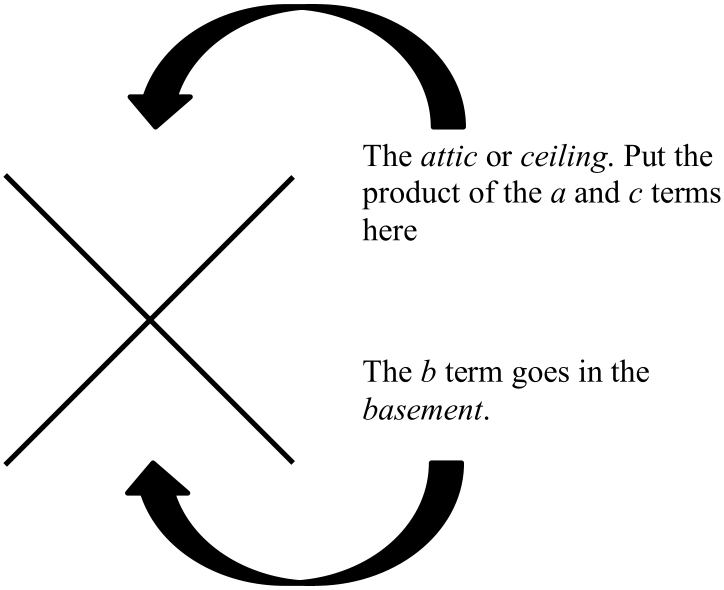
I

	$2x$	-11
$4x$		
-3		

$(2x - 11)(4x - 3) =$ ______________

Factoring polynomials where the *a* term is 1 using the X

Now students will use what they learned about using “X Marks the Spot” to factor trinomials in which the *a* term is 1. The *b* term goes in the *basement*. We put the product of the *a* and *c* terms in the *attic* and *ceiling*.

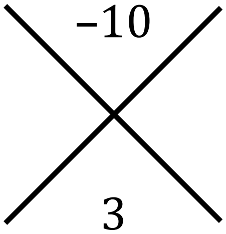


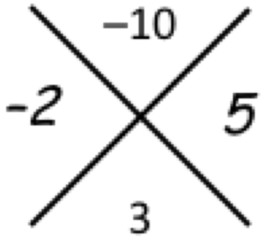
The *attic* or *ceiling*. Put the product of the *a* and *c* terms here

The *b* term goes in the *basement*.

To factor, simply solve as we did with “X Marks the Spot”.

Factor $x^2 + 3x - 10$
$a = 1$, $b = 3$, $c = -10$
$ac = -10$





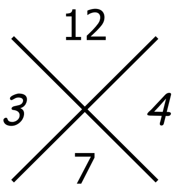
$$x^2 + 3x - 10 = (x - 2)(x + 5)$$

Factoring Polynomials

Name______________________________

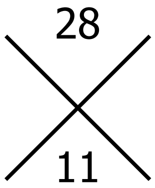
Date___________________ Class_______

Use the X to factor each polynomial. Write the answer in factored form as in the example.



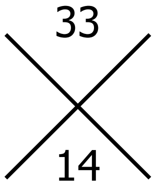
$x^2 + 7x + 12 =$ <u>$(x + 3)(x + 4)$</u>

A



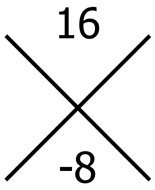
$x^2 + 11x + 28 =$ ________________________

B



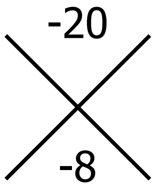
$x^2 + 14x + 33 =$ ________________________

C



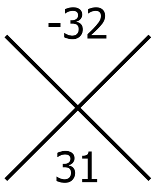
$x^2 - 8x + 16 =$ ________________________

D



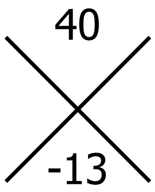
$x^2 - 8x - 20 =$ ________________________

E



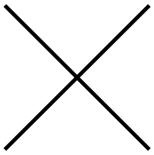
$x^2 + 31x - 32 =$ ________________________

F



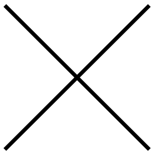
$x^2 - 13x + 40 =$ ________________________

G



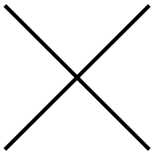
$x^2 + 3x - 28 =$ ______________________

H



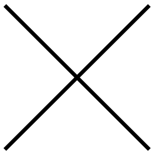
$x^2 + 16x + 48 =$ ______________________

I



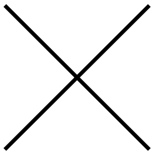
$x^2 - 8x - 48 =$ ______________________

J



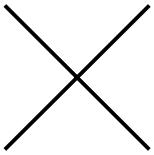
$x^2 - 19x + 48 =$ ______________________

K



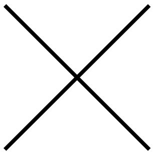
$x^2 - 3x - 54 =$ ______________________

L



$x^2 + 25x - 54 =$ ______________________

M



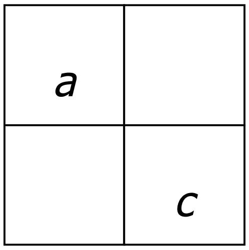
$x^2 - 19x - 42 =$ ______________________

Factoring polynomials when a >1 using the box or the X and box

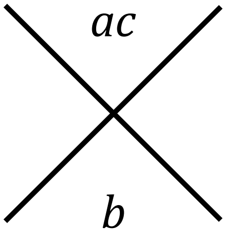
In the first of the two worksheets, students factor trinomials when $a > 1$ using the box. This requires some trial and error.

In the second worksheet, students combine the power of the X and box to solve these challenging polynomials with no guesswork.

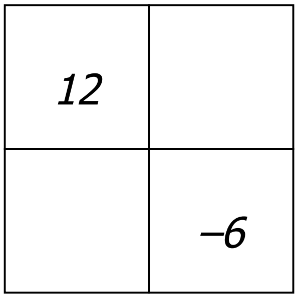
First we recall that in the box method, the *a* term goes in the upper left cell, and the *c* term goes in the lower right. The be term is the sum of the two remaining cells.

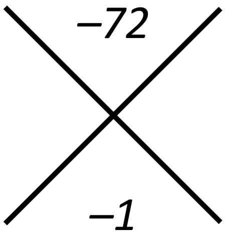


Remember also that when using the X, the product of the *a* and *c* terms goes in the *attic-ceiling* and the *b* term goes in the *basement*.

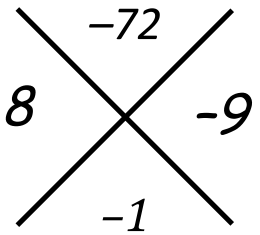


Let's factor $12x^2 – x – 6$. We fill in the X and box with what we know.
$a = 12$, $b = –1$, $c = –6$
$ac = –72$





Solving the X gives us 8 and –9. These are placed into the remaining cells of the box. It doesn't matter in which order they are placed.



$12x^2$	$8x$
$-9x$	-6

Now we simply factor the columns and rows.

	$3x$	$+2$
$4x$	$12x^2$	$8x$
-3	$-9x$	-6

Notice that the bottom row has a common factor of 3. However, since the lead term ($-9x$) is negative, the factor is also negative.

The factorization of $12x^2 - x - 6$ is $(3x + 2)(4x - 3)$.

Challenging Polynomials $a \geq 1$

Name________________________________

Date____________________ Class________

Use a box to factor each polynomial. Write the answer in factored form as in the example.

	$2x$	$+5$
x	$2x^2$	$5x$
$+2$	$4x$	$+10$

$2x^2 + 9x + 10 = \underline{(2x + 5)(x + 2)}$

A

x^2	
	$+15$

$x^2 + 8x + 15 =$ _________________

B

x^2	
	-14

$x^2 - 5x - 14 =$ _________________

C

x^2	
	-32

$x^2 + 4x - 32 =$ _________________

D

$2x^2$	
	$+1$

$2x^2 + 3x + 1 =$ _________________

E

x	$3x^2$	
$+2$		$+2$

$3x^2 + 7x + 2 =$ _________________

F

		+1
x	$2x^2$	
		+3

$2x^2 + 7x + 3 =$ ________________

G

	$3x$	
+2		

$3x^2 + 5x - 2 =$ ________________

H

$9x^2 - 6x + 1 =$ ________________

I

$6x^2 + 7x + 2 =$ ________________

J

$4x^2 - 9 =$ ________________

K

$4x^2 - x - 3 =$ ________________

L

$8x^2 + 2x - 1 =$ ________________

M

$8x^2 + 6x - 5 =$ ________________

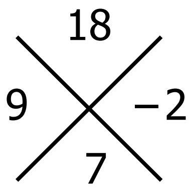
Using the "X" and Box $a \geq 1$

Name______________________________

Date____________________ Class________

Use the X and box to factor each polynomial as in the example. Remember to write in the correct *attic*, *ceiling*, and *basement* numbers. Write your answer in factored form.

$6x^2 + 7x - 3 = (\underline{2x + 3})(\underline{3x - 1})$



	$2x$	$+3$
$3x$	$6x^2$	$9x$
-1	$-2x$	-3

A. __________________=(_______)(_______)

B. __________________=(_______)(_______)

C. ____________________=(________)(________)

D. ____________________=(________)(________)

E. ____________________=(________)(________)

F. ____________________=(________)(________)

Using the

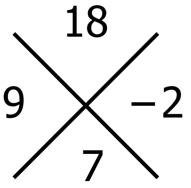
“X” andBlx $a \geq 1$

Name______________________________

Date____________________ Class________

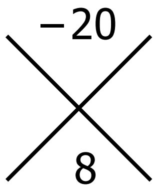
Use the X and box to factor each polynomial as in the example. Remember to write in the correct *attic*, *ceiling*, and *basement* numbers. Write your answer in factored form.

$6x^2 + 7x - 3 = (\underline{2x + 3})(\underline{3x - 1})$



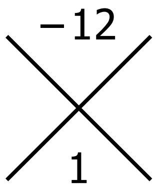
	$2x$	$+3$
$3x$	$6x^2$	$9x$
-1	$-2x$	-3

A. $4x^2 + 8x - 5 =$ (_______)(_______)



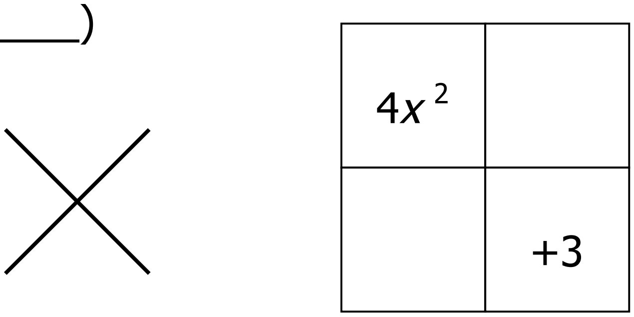
$4x^2$	
	-5

B. $4x^2 + x - 3 =$ (_______)(_______)

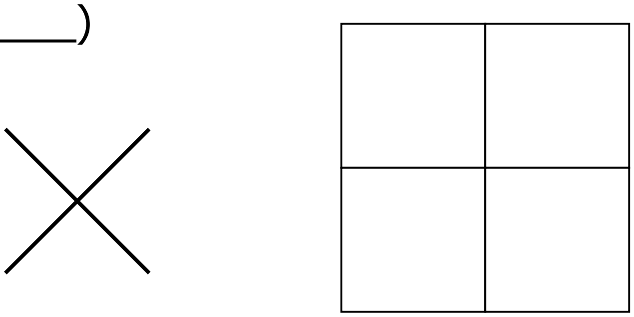


$4x^2$	
	-3

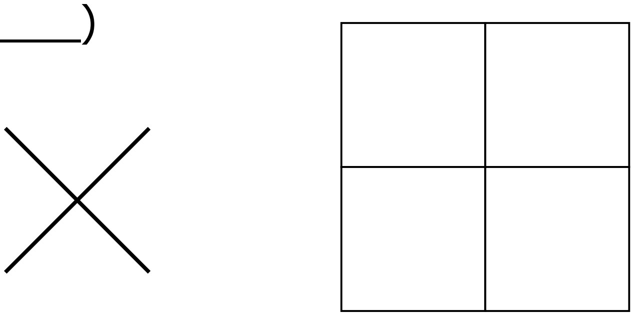
C. $4x^2 - 13x + 3 = (\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_)$



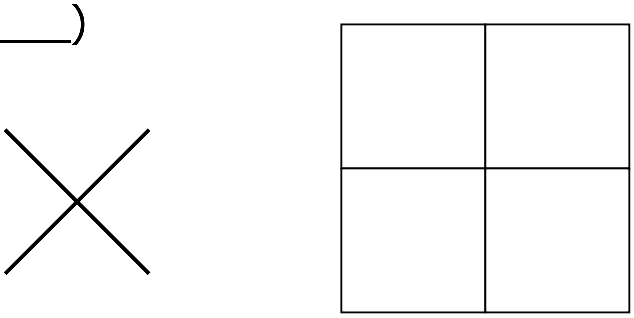
D. $8x^2 - 2x - 15 = (\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_)$



E. $6x^2 + 35x - 6 = (\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_)$



F. $20x^2 - 8x - 1 = (\_\_\_\_\_\_\_)(\_\_\_\_\_\_\_)$



Answer Keys

Multiplying Binomials 1

A) $x^2 + 6x + 8$
B) $x^2 + 16x + 63$
C) $x^2 + 11x + 24$
D) $x^2 - 2x - 8$
E) $x^2 - 4x - 45$
F) $x^2 - 5x - 6$
G) $x^2 - 12x + 32$
H) $x^2 - 9x + 14$
I) $x^2 - 16x + 63$

Factoring Polynomials 1

A) $(x + 4)(x + 7)$
B) $(x + 3)(x + 11)$
C) $(x - 4)(x - 4)$
D) $(x - 10)(x + 2)$
E) $(x - 1)(x + 32)$
F) $(x - 8)(x - 5)$
G) $(x - 4)(x + 7)$
H) $(x + 4)(x + 12)$
I) $(x - 12)(x + 4)$
J) $(x - 16)(x - 3)$
K) $(x - 9)(x + 6)$
L) $(x - 2)(x + 27)$
M) $(x - 21)(x + 2)$

Challenging Polynomials 1

A) $(x + 3)(x + 5)$
B) $(x - 7)(x + 2)$
C) $(x - 4)(x + 8)$
D) $(2x + 1)(x + 1)$
E) $(3x + 1)(x + 2)$
F) $(2x + 1)(x + 3)$
G) $(3x - 1)(x + 2)$
H) $(3x - 1)^2$
I) $(3x + 2)(2x + 1)$
J) $(2x + 3)(2x - 3)$
K) $(4x + 3)(x - 1)$
L) $(4x - 1)(2x + 1)$
M) $(4x + 5)(2x - 1)$

Using the X and Box 1

A) $(2x + 5)(2x - 1)$
B) $(4x - 3)(x + 1)$
C) $(4x - 1)(x - 3)$
D) $(4x + 5)(2x - 3)$
E) $(x + 6)(6x - 1)$
F) $(10x + 1)(2x - 1)$

Standards alignment:

GRADE 4

CCSS.MATH.CONTENT.4.NBT.B.5
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

GRADE 6

CCSS.MATH.CONTENT.6.RP.A.3
Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

CCSS.MATH.CONTENT.6.RP.A.3.B
Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

CCSS.MATH.CONTENT.6.RP.A.3.C
Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

CCSS.MATH.CONTENT.6.RP.A.3.D
Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

GRADE 7

CCSS.MATH.CONTENT.7.RP.A.2
Recognize and represent proportional relationships between quantities.

CCSS.MATH.CONTENT.7.RP.A.2.B
Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

CCSS.MATH.CONTENT.7.RP.A.2.C
Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.*

CCSS.MATH.CONTENT.7.RP.A.3
Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

HIGH SCHOOL

CCSS.MATH.CONTENT.HSA.SSE.B.3
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*

CCSS.MATH.CONTENT.HSA.SSE.B.3.A
Factor a quadratic expression to reveal the zeros of the function it defines.

CCSS.MATH.CONTENT.HSA.APR.B.3
Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

CCSS.MATH.CONTENT.HSA.APR.C.5
Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[1]

CCSS.MATH.CONTENT.HSA.APR.D.7
Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Remember that you are the teacher

We underestimate our importance, value, and effect as a teacher. It is too easy to assume that students will learn math from a math book. We don't take this approach with other content areas. Though they get information from a textbook, chemistry students learn, *really learn*, the bulk of their knowledge from doing experiments. History students learn more about culture from hosting a Renaissance fair than from reading about it. Yet we sometimes fall into the clutches of the false premise that *anyone* can teach math given a good textbook. In fact, a math textbook represents the most difficult of all content area reading materials.

Thus our role as instructors is even greater in the math classroom than in other domains. How we present a concept, how we sequence the problems our students encounter, how we bring them incrementally toward our final desired level of mathematical rigor is an orchestration of the most demanding detail.

For this reason, it is important that we design each math lesson with great attention to detail. We should begin by teaching the concept at hand. Initially this should be done with positive whole numbers. Once the students have begun to grasp the concept, we slowly and incrementally introduce integers or rational numbers as needed. Because it develops mathematical understanding incrementally from basic concepts to full rigor, this approach is called *Conceptual Layering*. It has proven to be a very effective technique not only for students in an intervention setting but for regular education students as well.

It is much like building a house. No walls are erected until a firm foundation is established. Then and only then can we build a solid and sound structure. Due to limits on their length, textbooks cannot take time to present material this way any more than a dictionary can develop language. It merely defines it. Textbooks are a reference work too. New material is often introduced at an abstract and rigorous level. Students cannot start there anymore than a passenger can climb aboard a train moving at full speed. The teacher's task is to slow down the train to get the students aboard and then take them up to speed.

This process will look different in each classroom. Your expertise as the teacher is the best determinant for where to begin, how big of steps are taken in moving between each conceptual layer, and what level of rigor is the finishing point.

The textbook can still play a crucial role in this process. It is a vast resource of practice problems and often provides effective examples for students who get home and need more instruction to work successfully. The best perspective is one in which we view textbooks as reference materials, which they are by definition, and not as sacred works. Students don't learn math from math books. They don't even learn math from math teachers. They learn math best from people they value and respect. Thus the teacher who is optimistic, positive, helpful, and kind will do more for his or her students than any textbook can ever accomplish. Remember that you, not the textbook, are the teacher.

If you liked this activity, you might also like some of the other lessons available in my TeachersPayTeachers store. Simply search for "Brad Fulton".

You can also find many free and inexpensive resources on my personal website, www.tttpress.com. Be sure to subscribe to receive monthly newsletters, blogs, and activities.

Similar activities include:

- *Function Fun, Parts 1–5:* Students as young as 4th grade have made sense of functions using the multi-representational approach!
- *Solving Linear Equations:* dozens of worksheets are sequenced from easy one-step equations to multistep problems. Choose from positive, negative, or fractional solutions to suit your students' needs.
- *Take Your Places: Activities 1–6* (Covering addition, subtraction, multiplication, and division of whole numbers, and addition and subtraction of fractions

Feel free to contact me if you have questions or comments or would like to discuss a staff development training or keynote address at your site.

Happy teaching,

Brad