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Take Your Places
Lessons 1–6

By Brad Fulton
Educator of the Year, 2005
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♦ California Math Council and NCTM national featured presenter
♦ Lead trainer for summer teacher training institutes
♦ Trainer/consultant for district, county, regional, and national workshops

Author and co-author of mathematics curriculum
♦ Simply Great Math Activities series: six books covering all major strands
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Brad’s math activities are the best I’ve seen in 38 years of teaching!
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Thanks and happy teaching,

Brad 😊
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Effective because they are classroom-tested and classroom-proven. These popular DVDs of Brad’s trainings have been utilized by teachers throughout the country for years.

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Great DVD presentations offer quality mathematics staff development at a fraction of the cost!
Overview:
This is a powerful lesson that enriches your students’ number sense and empowers their mathematical thinking. Each of the six lessons contains an activity master to use in class followed by two or three homework assignments. These will provide hours of classroom use, and of course you can easily design your own problem templates customized to the needs of your students. These six activities cover addition, subtraction, and multiplication of two-digit numbers, division by a one-digit divisor, and addition and subtraction of fractions.

Procedure:
This file contains lessons 1–6 from the DVD Take Your Places. The video is a site-licensed product that can save your school money over purchasing individually licensed copies of this handout. The DVD contains lessons 7–12 in addition to the six that are included here. The video also demonstrates many teaching tips similar to those that are described below.

1. Pass out one of the activity masters to each student and state the goal. For example, in activity 3, your goal might be to maximize the product of the two numbers. Explain that you will draw a digit from a set of ten numbered 0–9. When you draw the number, they are to write it in one of the empty cells. The students may not change the location of the digit once they have recorded it. I like to have my students seated in pairs so that they can see that the rules are being followed. You may also choose to have them write in ink.

2. Ask them where they chose to write the digit and why. For example, if the goal is to maximize the product, larger digits should go in the tens place and smaller numbers in the ones place, although I don’t tell them this. As they work, they will think about and discuss the number sense governing the problem and also consider the probability of drawing larger or smaller numbers.

3. Once everyone has recorded the digit, draw the second one. I like to keep the first digit removed so that it is not possible to draw it again. This forces students to reconsider the probability. For example, if the first draw was a 6, there is now only a \(\frac{3}{9}\) probability of drawing a larger digit (7, 8, or 9) since one of the ten original digits has been removed. The students should now record the second digit in one of the remaining empty cells.
4. Draw the third digit and have the students record it in one of the two remaining cells.
5. Then draw a fourth digit and have the students record it in the final empty cell.
6. Ask the students to calculate their problem and share responses. The number sense of this activity arises from three sources: the thinking and discussing the students do as they place their digits, the evolving probability that governs the process, and the questions you ask them to consider as they work. While the DVD goes into this final factor in great detail, here are some typical questions that will foster mathematical thinking:
   a. Where did you place your digit, and why did you place it there?
   b. What is the probability that the next digit is greater than 5?
   c. Does it matter where you place your digit? Why?
   d. Is it possible for two people to get the same final answer and not place their digits in the same locations?
   e. How many different arrangements (or problems) can be made using these digits?
   f. How many different answers can you get by rearranging your digits?
   g. If you could rearrange your digits now that you know all four of them, what changes would you make?
   h. What rules are helping you decide where you place your digits?
   i. What are you learning about how to best play this game?
7. This game can be played as a warm up or extended into a whole class period. The value comes as students play the game multiple times. Then they see how the numbers are functioning in the problem. For example, some students may not realize at first that larger digits should go in the tens place or that order doesn’t matter in a multiplication problem. Number sense is built as we play with and work with numbers over a period of time to see how they behave.
Homework 1-A
The six answers are:
   84  102
   111  120
   129  147
The greatest sum of 147 is
given by
   83 + 64
   64 + 83
   84 + 63
   63 + 84

Homework 1-B
A  93 + 75 = 168
B  82 + 64 = 146
C  91 + 65 = 156
D  57 + 39 = 96
E  78 + 69 = 147
F  45 + 07 = 52
G  47 + 20 = 67
H  49 + 21 = 70
I  36 + 19 = 55

Homework 2-A
The twelve positive solutions are:
   62 – 57 = 5
   72 – 65 = 7
   75 – 62 = 13
   67 – 52 = 15
   72 – 56 = 16
   76 – 52 = 24
   56 – 27 = 29
   57 – 26 = 31
   65 – 27 = 38
   67 – 25 = 42
   75 – 26 = 49
   76 – 25 = 51
The greatest and minimum differences are shown
in the list above.

Homework 2-B
A  43 – 32 = 31
B  98 – 67 = 31
C  65 – 02 = 63
D  31 – 24 = 7
E  86 – 79 = 7
F  72 – 39 = 33
G  57 – 08 = 49
H  46 – 05 = 41
I  87 – 56 = 31

Homework 3-A
The twelve products are:
   48 x 37 = 1,776
   47 x 38 = 1,786
   78 x 34 = 2,652
   74 x 38 = 2,812
   87 x 34 = 2,958
   84 x 37 = 3,108
   78 x 43 = 3,354
   73 x 48 = 3,504
   83 x 47 = 3,901
   87 x 43 = 3,741
   84 x 73 = 6,132
   83 x 74 = 6,142
The greatest product can
also be obtained by
commuting the factors:
   74 x 83 = 6,142

Homework 3-B
A  74 x 65 = 4,810
B  51 x 43 = 2,193
C  80 x 76 = 6,080
D  57 x 46 = 2,622
E  35 x 14 = 490
F  78 x 06 = 468*
G  53 x 46 = 2,438
H  54 x 36 = 1,944
I  46 x 35 = 1,610
* Notice that if one of the
digits is zero, the two
greatest digits do not
go in the ones column.
Homework 3-C
1  da x cb
2  ab x dc
Arranged from least to greatest the numbers are:
   dc, db, da, cd, cb, ca, 
   bd, bc, ba, ad, ac, ab

Homework 4-A
The six possible problems and quotients are:
   63 ÷ 7 = 9
   36 ÷ 7 = 5 R1
   73 ÷ 6 = 9 R1
   37 ÷ 6 = 6 R1
   76 ÷ 3 = 25 R1
   67 ÷ 3 = 22 R1

Homework 4-B
A  876 ÷ 5 = 175 R1
B  960 ÷ 4 = 240
C  567 ÷ 8 = 70 R 7
D  046 ÷ 9 = 5 R1

Homework 4-C
1  abc ÷ d  This assumes that d ≠ 0
2  dcb ÷ a
The highest possible value of d is 6, because then you would have: c = 7, b = 8, and a = 9.

Homework 5-A
1  \( \frac{9}{2} + \frac{6}{3} = \frac{13}{2} = 6\frac{1}{2} \)
2  \( \frac{2}{9} + \frac{3}{6} = \frac{35}{54} \)
3  There are two solutions:
   \( \frac{2}{6} + \frac{3}{9} = \frac{2}{3} \)
   \( \frac{2}{3} + \frac{6}{9} = \frac{4}{3} = 1\frac{1}{3} \)

Homework 5-B
1  \( \frac{a}{d} + \frac{b}{c} \)
   and \( \frac{b}{c} + \frac{a}{d} \)
2  \( \frac{d}{a} + \frac{c}{b} \)
   and \( \frac{c}{b} + \frac{d}{a} \)
3  \( \frac{c}{b} \)
4  \( \frac{a}{b} \)
5  \( \frac{a}{d} \)

Homework 6-A
1  \( \frac{8}{1} - \frac{3}{5} = \frac{37}{5} = \frac{7^2}{5} \)
2  \( \frac{3}{8} - \frac{1}{5} = \frac{7}{40} \)
3  \( \frac{8}{5} - \frac{1}{3} = \frac{19}{15} = \frac{1^4}{15} \)

Homework 6-B
1  \( \frac{a}{d} - \frac{c}{b} \)
2  \( \frac{c}{a} - \frac{d}{b} \)
3  \( 3, 4, 5, 6, 7, 8, 9 \)
4  \( 2, 3, 4, 5, 6, 7, 8 \)
5  \( 1, 2, 3, 4, 5, 6, 7 \)
6  \( 0, 1, 2, 3, 4, 5, 6 \)
As your teacher calls out digits, place them in the empty boxes to achieve the goal. Once you write in a digit, it cannot be moved.

Goal: ____________________________

A  +  B  +  C  +
   +  +  +

D  +  E  +  F  +
   +  +  +

G  +  H  +  I  +
   +  +  +
Place the digits 4, 6, 8, and 3 in the four cells of each problem to get the six different answers.

A  +  

B  +  

C  +  

D  +  

E  +  

F  +  

Which problem gave you the greatest sum?  __________

Show three more ways to arrange the digits that will give the same sum.

G  +  

H  +  

I  +  

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Place the digits in each problem to get the **maximum** sum.

3, 5, 7, 9

A

2, 4, 6, 8

B

1, 9, 6, 5

C

Place the digits in each problem to get the **minimum** sum.

3, 5, 7, 9

D

6, 7, 8, 9

E

7, 4, 0, 5

F

Place the digits in each problem to make the sum as **close to 68** as possible.

2, 0, 7, 4

G

1, 9, 4, 2

H

3, 6, 1, 9

I

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As your teacher calls out digits, place them in the empty boxes to achieve the goal. Once you write in a digit, it cannot be moved.

Goal: ____________________________________

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Place the digits 2, 7, 5, and 6 in the four cells of each problem to get twelve different positive differences.

Which problem gave you the greatest difference?  __________

Which problem gave you the least difference  __________
Place the digits in each problem to get the **maximum** difference.

1, 2, 3, 4

2, 3, 4, 1

6, 7, 8, 9

8, 7, 9, 6

0, 2, 6, 5

5, 6, 0, 2

Place the digits in each problem to get the **minimum** difference.

1, 2, 3, 4

2, 3, 4, 1

6, 7, 8, 9

8, 7, 9, 6

3, 7, 2, 9

2, 3, 9, 7

Place the digits in each problem to make the difference as **close to 42** as possible.

0, 5, 7, 8

0, 8, 7, 5

5, 0, 6, 4

5, 4, 0, 6

8, 7, 6, 5

8, 5, 6, 7

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As your teacher calls out digits, place them in the empty boxes to achieve the goal. Once you write in a digit, it cannot be moved.

Goal:________________________________________
Place the digits 3, 8, 4, and 7 in the four cells of each problem to get the twelve different products.

Which problem gave you the greatest product? ________

Arrange the digits a different way to get the same product.

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Place the digits in the four cells of each problem to get the **maximum** product.

```
4, 5, 6, 7  4, 5, 3, 1  6, 8, 0, 7
A B C
X X X
___ ___ ___
```

Place the digits in the four cells of each problem to get the **minimum** product.

```
4, 5, 6, 7  4, 5, 3, 1  6, 8, 0, 7
D E F
X X X
___ ___ ___
```

Place the digits 4, 6, 3, and 5 in the four cells of each problem to get the **as close as possible** to the target product.

```
Target: 2,500  Target: 1,950  Target: 1,583
G H I
X X X
___ ___ ___
```

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The letters \( a, b, c, \) and \( d \) are variables representing single digit numbers such that \( a > b > c > d \). That is \( a \) represents the digit of greatest value and \( d \) represents the digit of least value.

1. Place the variables in the four cells to get the **maximum** product.

2. Now place the variables in the four cells to get the **minimum** product.

Use the variables to form two-digit numbers such as \( c,b \) where \( c \) represents the digit in the tens’ place and \( b \) represents the digit in the ones place.

3. How would you write the greatest two-digit number without repeating a variable?

4. How would you write the least two-digit number without repeating a variable?

It is possible to make twelve two-digit numbers such as \( c, b \) using the four variables. Write them on the number line below from least to greatest.
As your teacher calls out digits, place them in the empty boxes to achieve the goal. Once you write in a digit, it cannot be moved.

Goal: ____________________________________

A

B

C

D
Place the digits 6, 7, and 3 in the cells of each problem to show all six quotients.

A

B

C

D

E

F
Place the digits in the four cells of each problem to get the **maximum** quotient.

5, 6, 7, 8

A

6, 4, 9, 0

B

Place the digits in the four cells of each problem to get the **minimum** quotient.

5, 6, 7, 8

C

6, 4, 9, 0

D
The letters $a$, $b$, $c$, and $d$ are variables representing single digit nonzero numbers such that $a > b > c > d$. That is $a$ represents the digit of greatest value and $d$ represents the digit of least value.

1. Place the variables in the four cells to get the **maximum** quotient.

2. Now place the variables in the four cells to get the **minimum** quotient.

3. What is the highest value that $d$ could represent? ______ Why?
As your teacher calls out digits, place them in the empty boxes to achieve the goal. Once you write in a digit, it cannot be moved.

Goal: __________________________________________

A

\[
\begin{array}{ccc}
\phantom{0} & \phantom{0} & + \\
\phantom{0} & \phantom{0} & \\
\phantom{0} & \phantom{0} & \\
\phantom{0} & \phantom{0} & \\
\end{array}
\]

B

\[
\begin{array}{ccc}
\phantom{0} & \phantom{0} & + \\
\phantom{0} & \phantom{0} & \\
\phantom{0} & \phantom{0} & \\
\end{array}
\]

C

\[
\begin{array}{ccc}
\phantom{0} & \phantom{0} & + \\
\phantom{0} & \phantom{0} & \\
\phantom{0} & \phantom{0} & \\
\end{array}
\]

D

\[
\begin{array}{ccc}
\phantom{0} & \phantom{0} & + \\
\phantom{0} & \phantom{0} & \\
\phantom{0} & \phantom{0} & \\
\end{array}
\]

E

\[
\begin{array}{ccc}
\phantom{0} & \phantom{0} & + \\
\phantom{0} & \phantom{0} & \\
\phantom{0} & \phantom{0} & \\
\end{array}
\]

F

\[
\begin{array}{ccc}
\phantom{0} & \phantom{0} & + \\
\phantom{0} & \phantom{0} & \\
\phantom{0} & \phantom{0} & \\
\end{array}
\]
1. Place the digits 2, 6, 9, and 3 in the four cells to maximize the sum.

   \[
   \[
   \[
   \[
\]
\]

2. Place the digits 2, 6, 9, and 3 in the four cells to minimize the sum.

   \[
   \[
   \[
   \[
\]
\]

3. Place the digits 2, 6, 9, and 3 in the four cells to get a sum as close as possible to one.

   \[
   \[
   \[
   \[
\]
\]
The letters \( a, b, c, \) and \( d \) are variables representing single digit numbers such that \( a > b > c > d \). That is \( a \) represents the digit of greatest value and \( d \) represents the digit of least value.

1. Place the four variables in the cells in order to maximize the sum. There are two ways to do this.

\[
\begin{array}{c}
\phantom{a} \\
\phantom{b} \\
\phantom{c} \\
\phantom{d}
\end{array}
\begin{array}{c}
\phantom{a} \\
\phantom{b} \\
\phantom{c} \\
\phantom{d}
\end{array} +
\begin{array}{c}
\phantom{a} \\
\phantom{b} \\
\phantom{c} \\
\phantom{d}
\end{array}
= 
\begin{array}{c}
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\end{array}
\begin{array}{c}
\phantom{a} \\
\phantom{b} \\
\phantom{c} \\
\phantom{d}
\end{array}
\]

2. Place the four variables in the cells in order to minimize the sum. There are two ways to do this.

\[
\begin{array}{c}
\phantom{a} \\
\phantom{b} \\
\phantom{c} \\
\phantom{d}
\end{array}
\begin{array}{c}
\phantom{a} \\
\phantom{b} \\
\phantom{c} \\
\phantom{d}
\end{array} +
\begin{array}{c}
\phantom{a} \\
\phantom{b} \\
\phantom{c} \\
\phantom{d}
\end{array}
= 
\begin{array}{c}
\phantom{a} \\
\phantom{b} \\
\phantom{c} \\
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\end{array}
\begin{array}{c}
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\end{array}
= 
\begin{array}{c}
\phantom{a} \\
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\phantom{c} \\
\phantom{d}
\end{array}
\begin{array}{c}
\phantom{a} \\
\phantom{b} \\
\phantom{c} \\
\phantom{d}
\end{array}
\]

Circle the greater fraction in each pair.

3. \( \frac{a}{d} + \frac{b}{c} \)

4. \( \frac{b}{a} + \frac{a}{b} \)

5. \( \frac{a}{d} + \frac{b}{c} \)
As your teacher calls out digits, place them in the empty boxes to achieve the goal. Once you write in a digit, it cannot be moved.

Goal: ________________________________

A

B

C

D

E

F
1. Place the digits 8, 3, 5, and 1 in the four cells to **maximize** the difference.

```
    __   __
   __  __
```

2. Place the digits 8, 3, 5, and 1 in the four cells to **minimize** the difference while keeping the answer positive.

```
    __   __
   __  __
```

3. Place the digits 8, 3, 5, and 1 in the four cells to get a difference **as close as possible** to one.

```
    __   __
   __  __
```
The letters $a$, $b$, $c$, and $d$ are variables representing single digit numbers such that $a > b > c > d$. That is $a$ represents the digit of greatest value and $d$ represents the digit of least value.

1. Place the four variables in the cells in order to maximize the difference. There are two ways to do this.

2. Place the four variables in the cells in order to minimize the difference. There are two ways to do this.

3. What possible values could the variable $a$ represent? __________________

4. What possible values could the variable $b$ represent? __________________

5. What possible values could the variable $c$ represent? __________________

6. What possible values could the variable $d$ represent? __________________

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To use the spinners, set a paper clip on the center of a spinner. Place a pencil point on the center and spin the paper clip as shown.
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Similar activities include:

• **Take Your Places: Activities 7-12** (Covering multiplication and division of fractions, order of operations, parentheses, and the compound interest formula.)
• **Math Maps: Developing the Mathematical Practices**
• **Sum Thing Interesting: Finding Amazing Patterns in Addition**
• **Array We Go: Building: An Engaging and Visual Representation of Factors, Multiples, Primes, and Composites, and More**
• **Developing Number Sense**

Feel free to contact me if you have questions or comments or would like to discuss a staff development training or keynote address at your site.

Happy teaching,

*Brad*

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