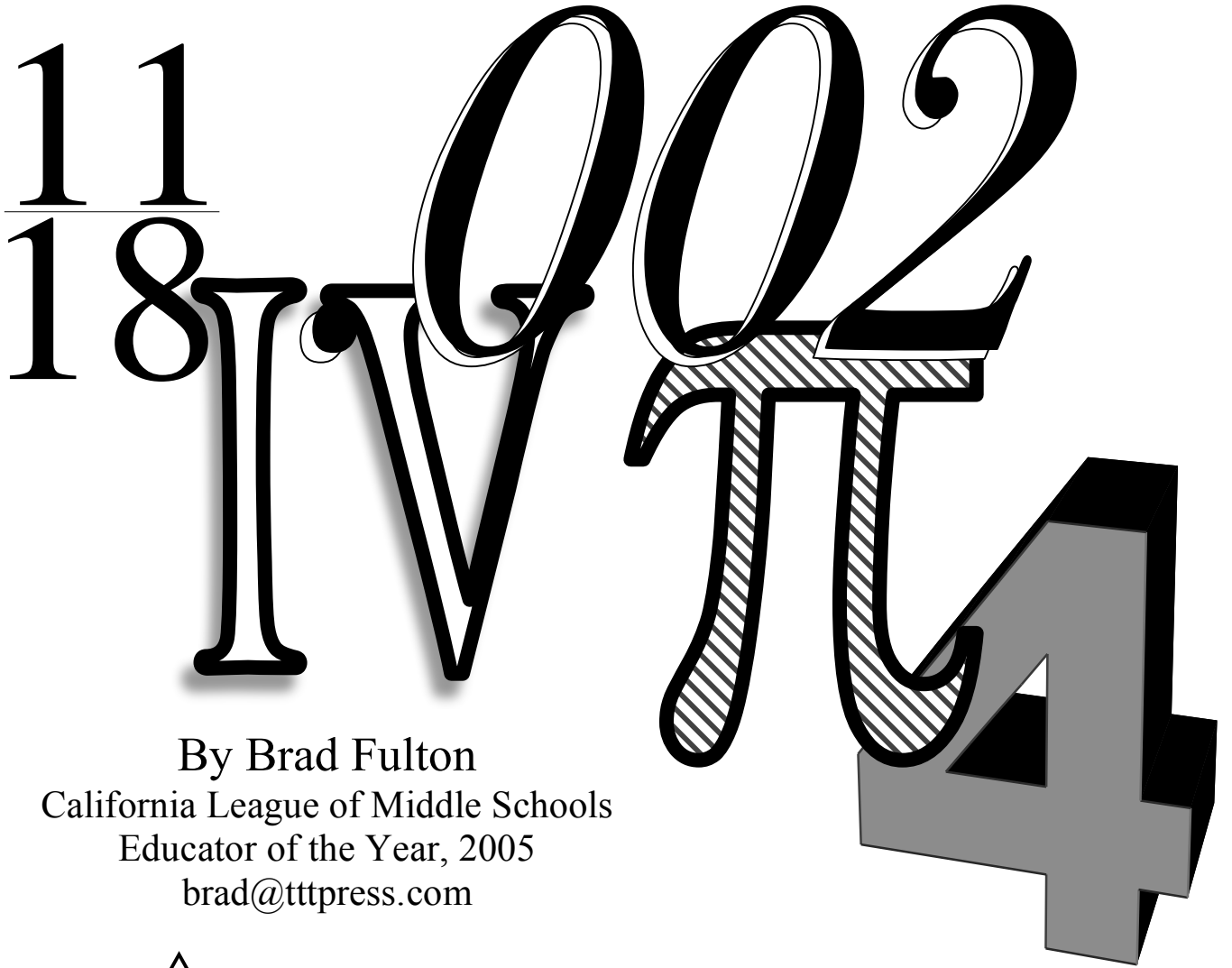


BUILDING NUMBER SENSE

THROUGH ENGAGING ACTIVITIES

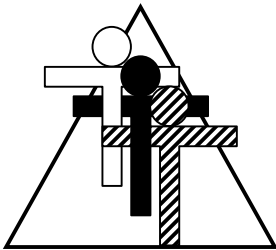


By Brad Fulton

California League of Middle Schools

Educator of the Year, 2005

brad@tttpress.com



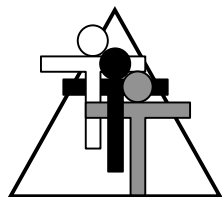
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Brad Fulton and Bill Lombard
Teacher to Teacher Press
“Building Mathematical Skill on a Foundation of Understanding”



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- ◆ **Educators**
- ◆ **Authors**
- ◆ **Seminar leaders**
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Known throughout the country for motivating and engaging teachers and students, Brad and Bill have authored over ten books that provide easy-to-teach yet mathematically-rich activities for busy teachers. In addition, they have co-authored six teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

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- ◆ Lead trainers for summer teacher training institutes
- ◆ Trainers/consultants for district, county, regional, and national workshops

Authors and co-authors of mathematics curriculum

- ◆ *Simply Great Math Activities* series: five books covering all major strands
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Brad and Bill's math activities are the best I've seen in 30 years of teaching!

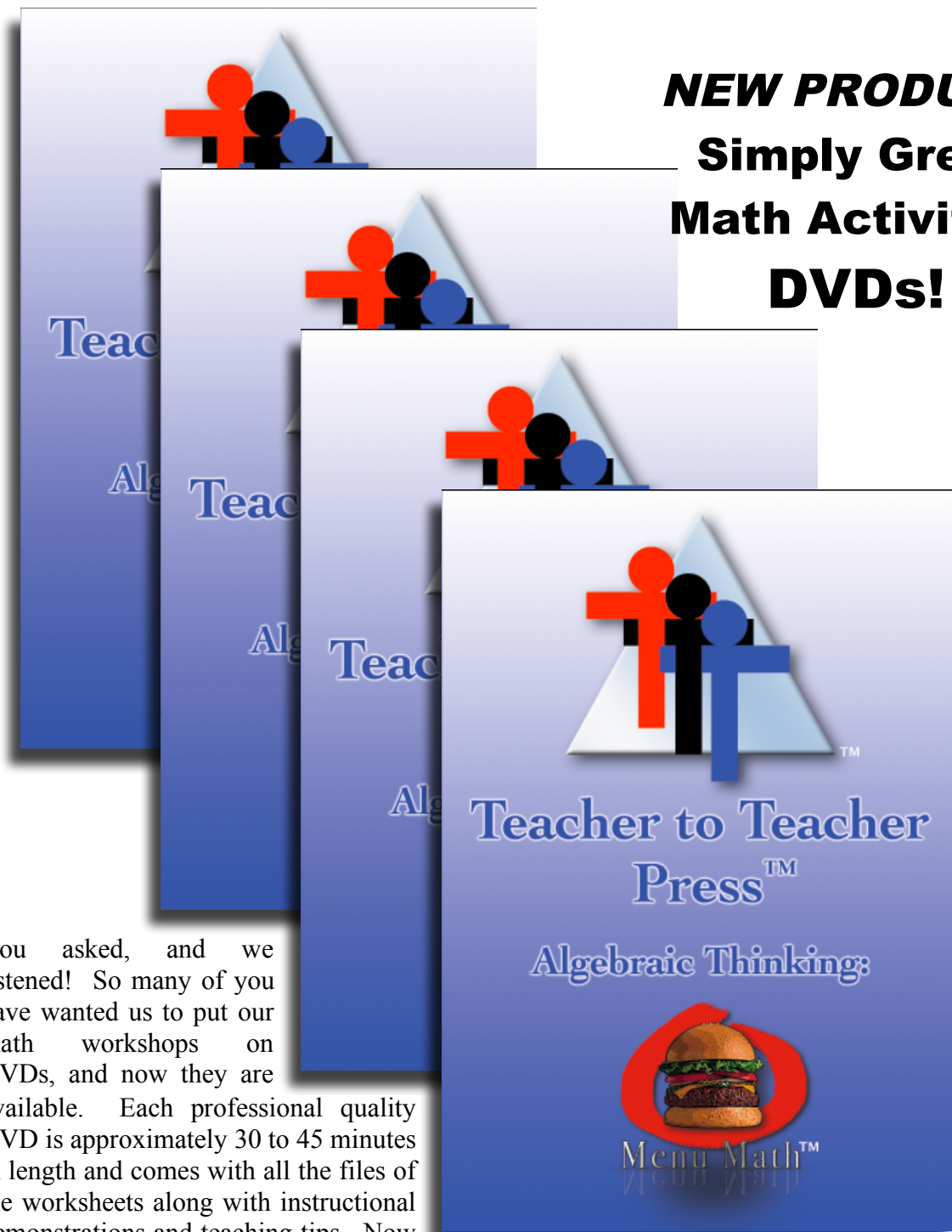
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"The high-energy, easy-to-follow handouts were clear. The instructors were great!"

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References available upon request

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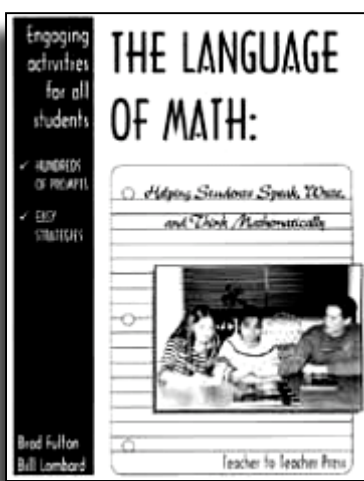
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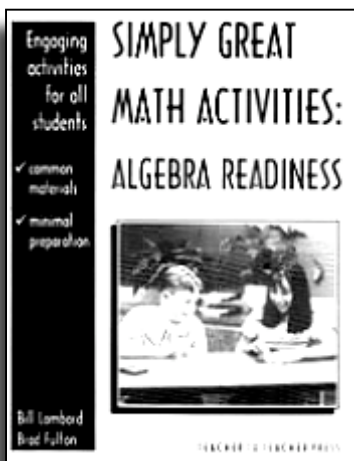
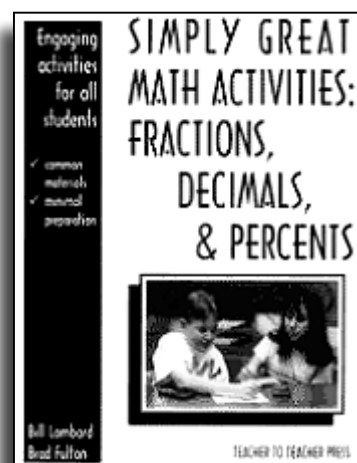
Happy surfing!

Books by Brad and Bill



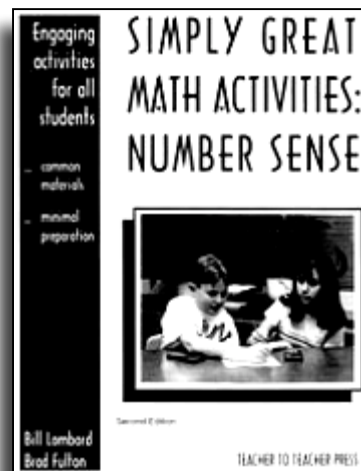
The Language of Math helps teachers create a classroom environment rich in mathematical thinking by showing them how to easily incorporate oral and written language into their math classes. Over 100 journal and discussion starters are included along with extensive instructions for making the most of your math time.

Here are a dozen unique and conceptual activities that will help your students add, subtract, multiply and divide fractions as well as connect them to decimal and percent representations. Both you and your students will love the novel and creative approach.



Teachers are raving about how effective these activities have been in their classrooms. Children as young as fourth grade and college students alike say that algebra is easy and makes sense because of this incredible approach.

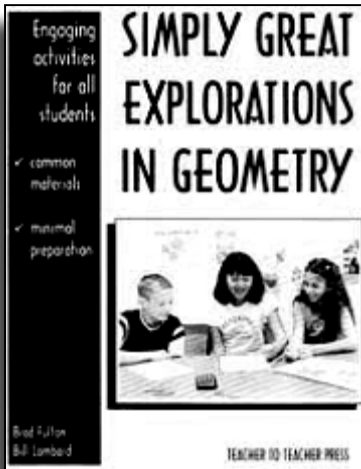
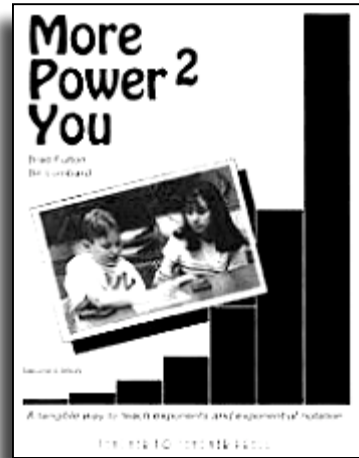
Students don't even think they are doing math sometimes because these activities are so fun and engaging, but they are developing rich and valuable number sense as they explore these eleven creative activities.





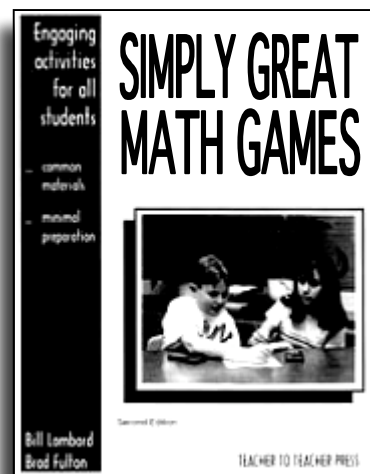
Our first book is still one of our most popular. Every teacher we talk to who has tried this approach to functions has been amazed at what their students have learned and accomplished. Over 150 pages of multiple representations of functions cover such concepts as slope, intercept, and function notation. Even elementary students have developed an understanding of functions with this book.

Exponents will finally make sense to your students after they participate in the unique activities found in this book. Both positive and negative exponents are demonstrated conceptually. Your students will even be able to explain *why* $n^0 = 1$.



Over one dozen geometry activities will excite your students as they discover the connections between geometry and fractions, decimals, percents, and even algebra. Area formulas, angle measurement, polygon attributes, vocabulary, and construction are covered.

A dozen engaging and educational games await you and your students in this creative and highly adaptable book. You'll find games that reinforce basic operations with whole numbers, fractions, decimals, and integers as well as algebraic skills. Game masters will serve a spectrum of grade levels and skill levels. Your students will beg for more!



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The Number Sense Problem

As we improve our performance on high-stakes tests, number sense skills seem to be declining. This may be due to more procedural instruction that favors better performance on state tests.

Number sense is a top priority in lower grades, but it is often replaced with more abstract instruction as children get older. Like physical health, our “number health” atrophies with a lack of exercise.

Research shows that number sense is developed only through frequent instruction over a period of years. We cannot cover number sense as a separate unit of instruction as we can with other areas of mathematics. It cannot be learned from a chapter in a textbook nor assessed with standardized testing. Yet we know when a student has it and when he does not. Like great writing, number sense is hard to define yet immediately recognizable.

Though it eludes definition, we can recognize it by its characteristics. Students with number sense have an ease and fluidity with numbers and their operations. They demonstrate coordination with mathematics much like a gymnast shows coordination in physical movement. Students with number sense can take numbers apart and recompose them in more expedient ways. This is what allows a student to see that $9+7=10+6$ or that $24 \times 3 = 8 \times 3 \times 3 = 8 \times 9$.

A sense about numbers also allows a student to know when an answer is reasonable. Such students have not only a sense for the magnitude of numbers, but also of the meaning of a mathematical problem. This helps them know if their answer fits the situation or if an error is present.

At its core, number sense is built upon a solid foundation in place value. It has been said that if you have place value, you have number sense, and if you have number sense, you have place value. Yet this is only the foundation and not the entire structure. Much of higher mathematics shows a separation from place value. Exponents are one example of this. Though 9^9 is composed of single digits, that is those in the ones place, it is an extremely large number. Algebra, geometry, and other branches of higher mathematics include many areas that are far beyond the scope of simple place value.

As students get older, we must be sure we are preparing them with a solid foundation in “algebra sense”, “geometry sense”, “trigonometry sense” and so on.

Fortunately, not only can we identify the characteristics of number sense but also the strategies that foster its development. As shown in the following pages of this handout, the inclusion of simple components of classroom instruction will help your students to show great growth in their number sense skills.

The Five Components Of Number Sense

- Estimation
- Mental Computation
- Mathematical properties
- Effects of operations
- Number magnitude

Estimation

Estimate these problems:

1. Which is greater, 86×38 or 88×36 ?
Why?

2. 0.52×789

3. 40×26.7

4. 43×52

5. $4953 \div 68$

6. $0.4 \times 58.6 \times 5 \times 3$

7. Write $\frac{715}{1866}$ as a percent.

Answers

1. $86 \times 38 = 3,268$ while $88 \times 36 = 3,168$. Two sets of numbers have the same sum, the pair with the lesser difference will produce a greater product.
2. 410.28
3. 1,068
4. 2,236
5. Approximately 72.8
6. 351.6
7. Approximately 38.3%

Estimation relies on two fundamental skills: approximation and mental computation. According to Case, the typical child is not capable of handling both of these tasks until around age 11.

R. Case
Intellectual Development
1985

Research has shown that...“For students to become highly skilled at estimation, it had to be incorporated into their regular instruction over several years.”

*From Research Ideas for the Classroom:
Middle Grades Mathematics
Douglas T. Owens*

Solve these problems mentally if possible.

1. 16×15

2. 45% of 250

3. $3467 - 1588$

4. Find a fraction between $\frac{11}{18}$ and $\frac{5}{7}$.

5. Find a fraction between $\frac{5}{7}$ and $\frac{13}{18}$.

6. $5 \times 2,379$

Answers:

1. 240
2. 112.5
3. 1,879
4. $\frac{2}{3}$ is one example
5. $\frac{18}{25}$ is one example
6. 11,895

Mathematical properties

Use mathematical properties to solve each problem. Explain how the properties work.

1. 16×52

2. $37 + 256 + 19 + 50$

3. $9(4,555)$

4. $10,002 - 4,566$

5. $18 \div 1.5$

6. $1.5 \div \frac{2}{3}$

Answers:

1. 832. The distributive property can be applied. You could also cut 16 in half while doubling 52. Eventually you will get 1×832 . This is an application of the associative property of multiplication.

2. 362. The associative property allows us to rearrange the addends this way:

$$\begin{aligned} &37+19+(44+6)+256= \\ &(37+19+44)+(6+256)= \\ &100+262= \\ &362 \end{aligned}$$

3. 40,995. The distributive property can be used two ways:

$$\begin{aligned} &9(4,555)= \\ &9(4,000+500+50+5)= \\ &36,000+4,500+450+45 \end{aligned}$$

or

$$\begin{aligned} &9(4,555)= \\ &(10-1)(4,555)= \\ &45,000-4,555= \\ &45,000-4,000-500-5--5 \end{aligned}$$

4. 5,436. $10,002-4,566=9,999-4,563$. This is called the compensation method of subtraction.

5. 12. $18 \div 1.5 = 36 \div 3$. If we think of the division problem as a fraction: $\frac{18}{1.5}$, then

we have multiplied it by one in the form of $\frac{2}{2}$.

6. 2.25. Using the method above, we can multiply $\frac{1.5}{2}$ by $\frac{3}{3}$ to get $\frac{4.5}{2}$. Half

of that would be 2.25.

Explain your answer to each question and give examples when necessary.

1. If two numbers are added, will the sum always be greater?
2. If you subtract one number from another will the result be smaller than the first number?
3. If numbers are multiplied is the product always greater?
4. When you square a number, is the answer always greater?
5. If divide one number by another, will the quotient always be smaller?
6. If you keep adding one to both the numerator and denominator of $\frac{1}{2}$, how big will the number eventually be?

Answers:

1. No, if you add zero or a negative number, the sum is not greater.
2. No, if you subtract zero or a negative number, the difference is greater.
3. If you multiply by one or less, the product is equal or less.
4. The square will be greater unless the value of the number is between zero and one.
5. If you divide by a number x such that $0 < x \leq 1$, the quotient is not greater.
6. Eventually the fraction approaches 1.

Number magnitude

1. How long will it take to live a million seconds?
2. How long will it take to live a billion seconds?
3. How long will it take to live a trillion seconds?
4. How tall are a million dollar bills?
5. If you cut your paper in half twenty times, how big would it be?
6. How many times will you blink in your life?

Answers:

1. About 11.6 days
2. 31.7 years
3. 31,700 years
4. Over 650 feet tall. Each dollar is $\frac{1}{125}$ " thick.
5. Less than one ten-thousandth of a square inch, or about $\frac{1}{100}$ " by $\frac{1}{100}$ "
6. Between 300 billion and 500 billion times. Estimates vary.

Strategies that Develop Number Sense

- Playing with numbers
- Solving problems in multiple ways
- Creative practice
- Thinking, talking, and writing about numbers
- Exploring patterns



Pyramid Math

Building numbers sense is of critical importance if students are to be successful now and in later mathematics courses. But how do we develop number sense and provide skills practice at the same time? Fortunately we are providing a simple format for accomplishing both of these goals. This activity can be tailored to suit any grade level, and it can even be made self-correcting. Students will prefer this practice method to the standard textbook assignment, and they will be able to measure their own success. We even provide five student activity masters and answer keys!

Procedure:

- 1 Display the Activity Master 4 and enter four numbers in the cells of the bottom row as shown. (Use single-digit whole numbers at first so students can focus on the structure of the problem instead of stumbling on the computation at this point.) To solve the pyramid, an adjacent number pair is added. The sum is written in the box above the number pair. This is repeated for the other number pairs in the bottom row. Then this process is repeated for the second row to fill the third row. Finally the number pair in the third row is added to get the final top number as shown.

Required Materials:

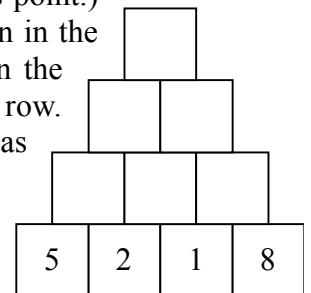
Paper

Activity Master

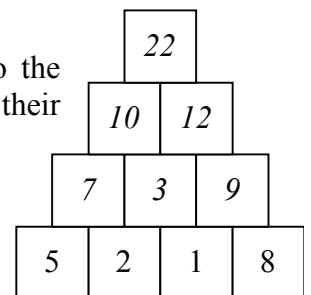
Optional Materials:

Transparency of Activity Master

- 2 Since each sum is based on the sums below, all students should get the same answer in the top cell. Thus they only need to check the top answer. If that is correct, all other cells are likely correct too.



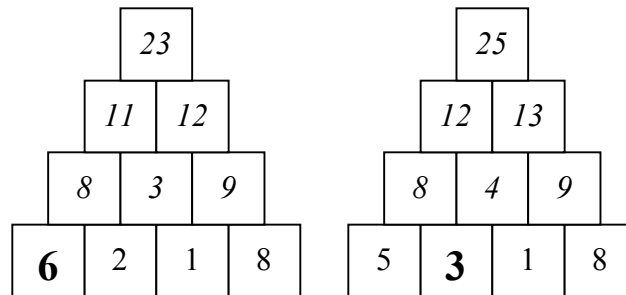
- 3 Now try another pyramid using new numbers. Students will catch on to the process quickly and will be eager to check their answer with those of their classmates. (No more correcting papers!)



- 4 As students understand how the problems work, introduce appropriate numbers. If you are studying decimals, throw in a few decimal points. If you have covered integers, use some negative numbers. Fractions make these problems much more difficult. Try one yourself before asking the students to do so. We suggest beginning with like denominators. Or you could use fractions that have a fairly small common denominator. For example, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{2}$, and $\frac{3}{4}$ can all use fourths for a common denominator.

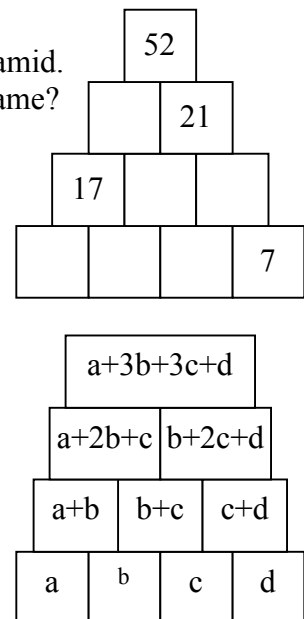
- 5 Try to make slight variations in the arrangement and values of the numbers to help children focus on the number sense involved. For example, in the first problem if we

increase the five by one, making it a six, the top number also increases by one, but if



we change the two to a three, the top number increases by three. Why does this occur? What would happen if we added two to the first or second cell of the bottom row? What happens when we do this to a five-row pyramid? As students answer these questions they will develop number sense.

- 6 Ask students to change the order of the numbers in the bottom row of a pyramid. How does this affect the top cell? Is the result in the top row always the same? How does the commutative property affect this result?
- 7 Introduce subtraction as shown in this sample. You can create one of your own easily, or have students create them for their classmates to solve.
- 8 Explore what happens when all odd numbers or all even numbers are used. What if all four cells in the bottom row contain the same number? What patterns occur?
- 9 The pyramid at the right shows the algebra involved in these problems. Notice that the numbers in the two middle cells get added three times on the way to the top while the numbers in the two outer cells do not. This explains why increasing the first cell by one causes the top row to increase by one, but increasing either of the two middle cells by one increases the top row by three.



We always love to hear from our readers. Email us with your comments on this activity at either

brad@tttpress.com or **bill@tttpress.com**

Good Tip!

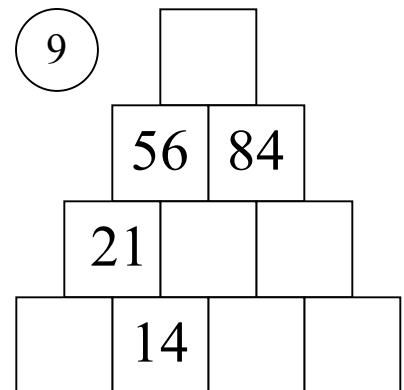
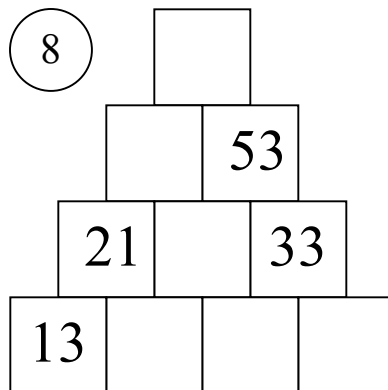
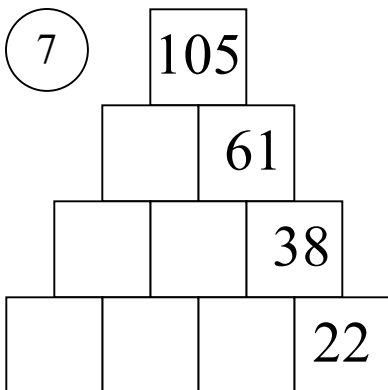
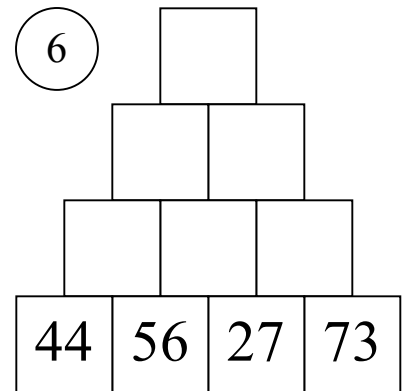
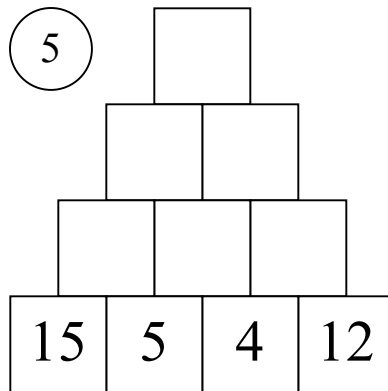
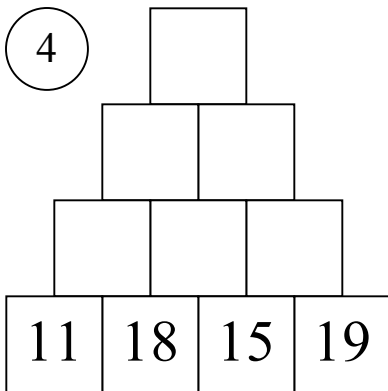
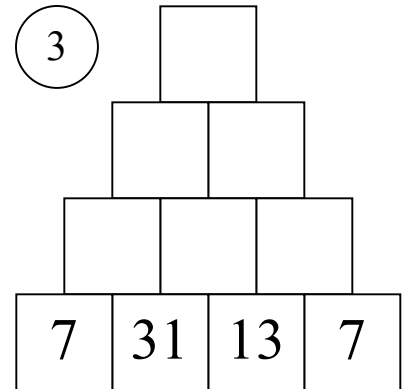
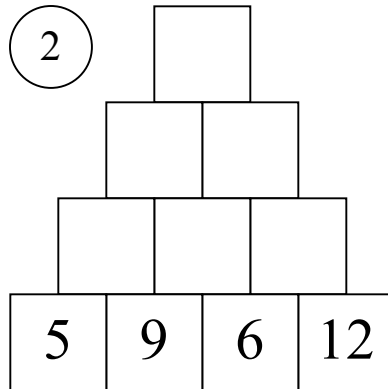
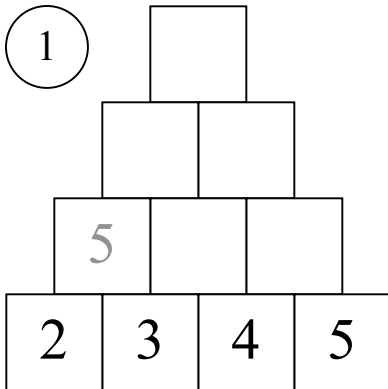


To make the activity self-assessing, write the answers to the problems at the bottom of the student's worksheet. As a student solves a problem, the answer can be crossed off from the list. If the answer is not found, the problem can be solved again.

Activity Master 1
Pyramid Math

Name _____

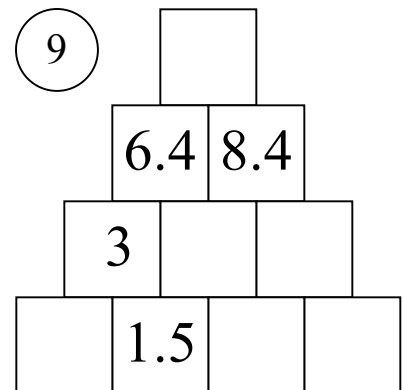
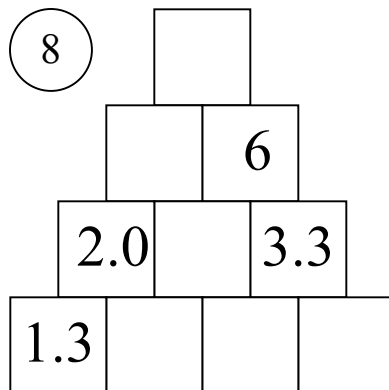
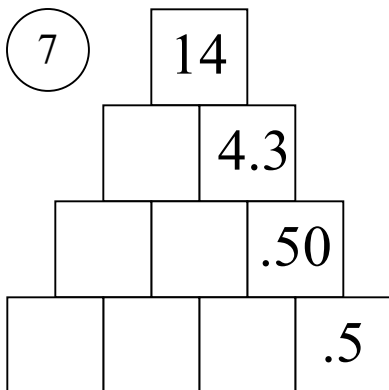
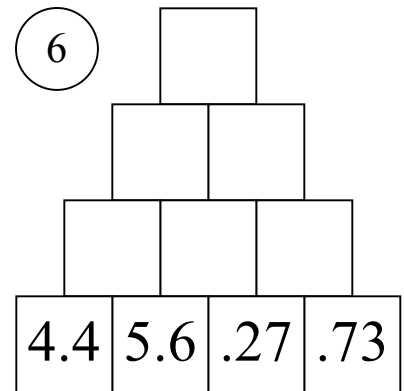
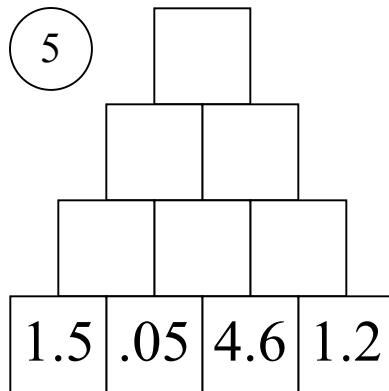
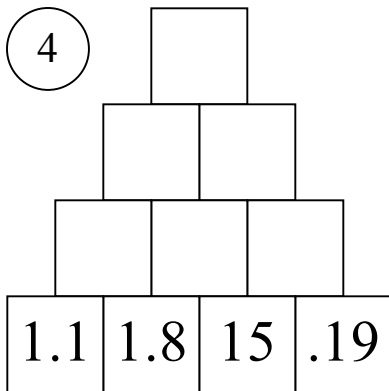
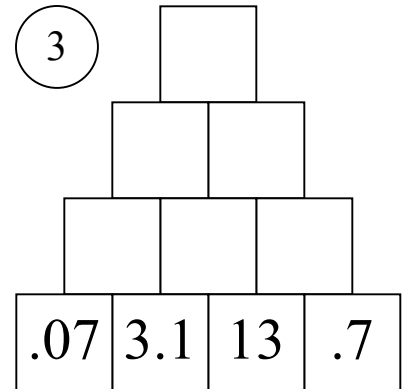
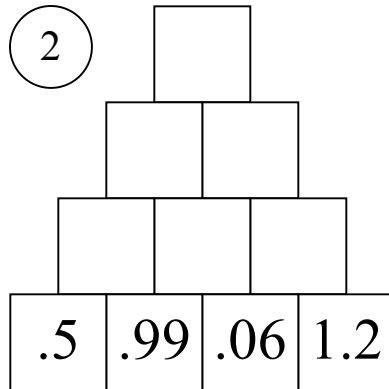
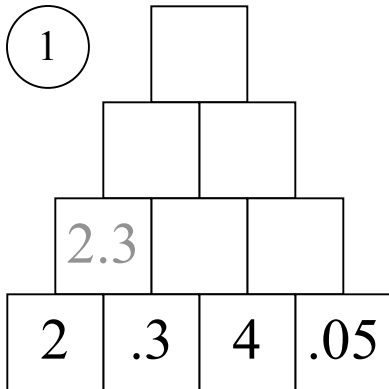
Add adjacent numbers and write their sum in the box above them as shown in the example. Continue until you reach the top of the pyramid. On some problems, you may need to work backwards.



Activity Master 2
Pyramid Math

Name _____

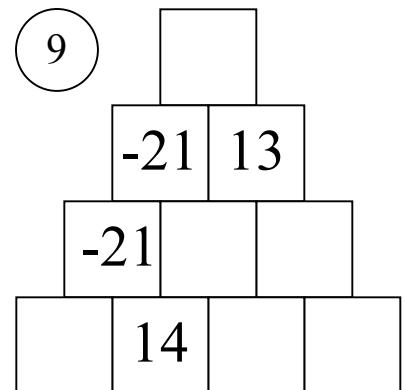
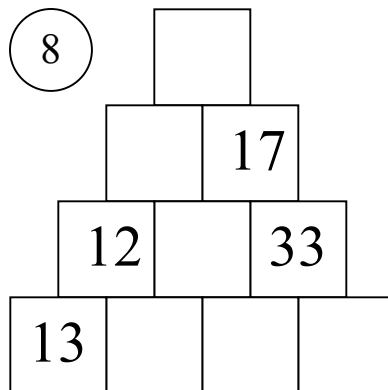
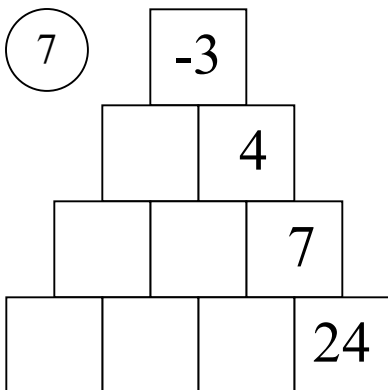
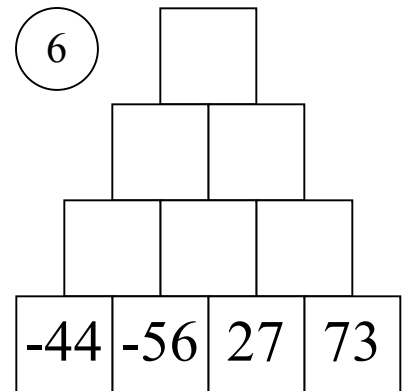
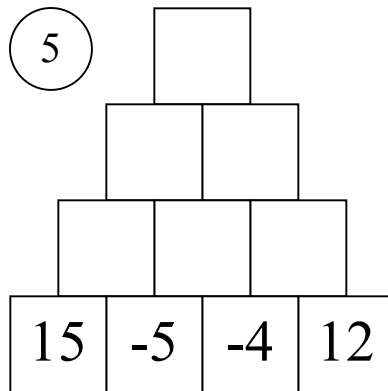
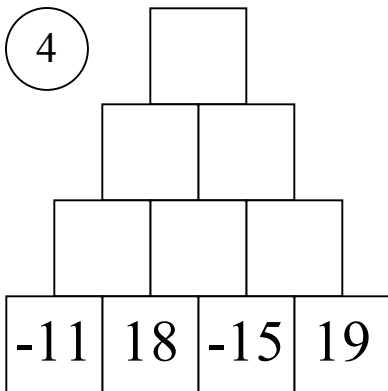
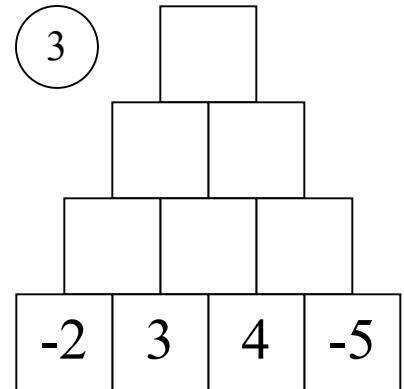
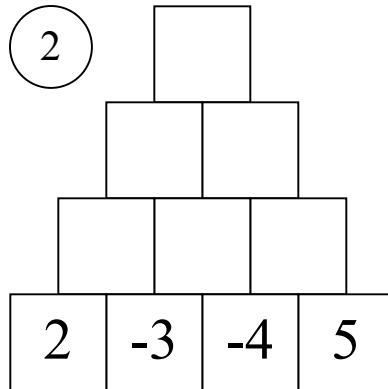
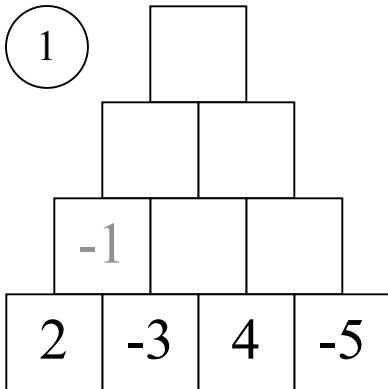
Add adjacent numbers and write their sum in the box above them as shown in the example. Continue until you reach the top of the pyramid. On some problems, you may need to work backwards.



Activity Master 3
Pyramid Math

Name _____

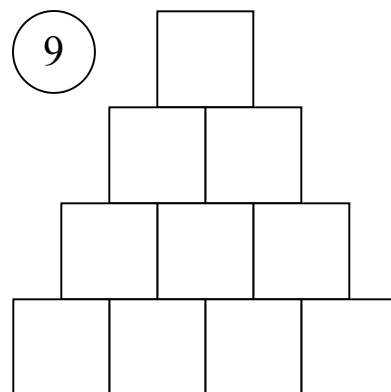
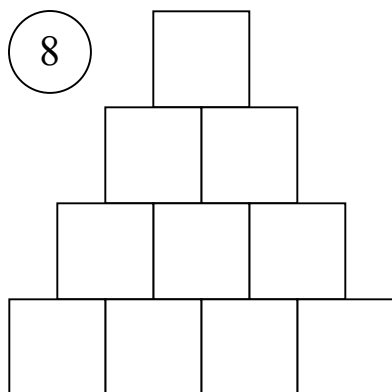
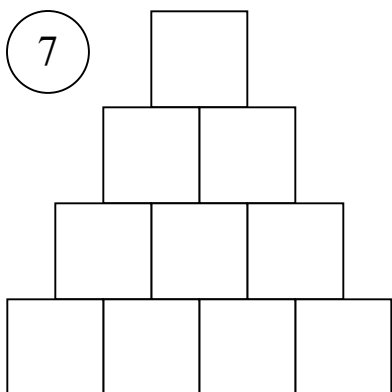
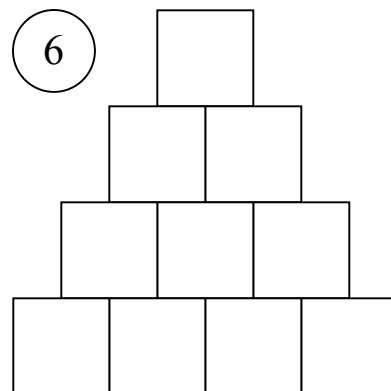
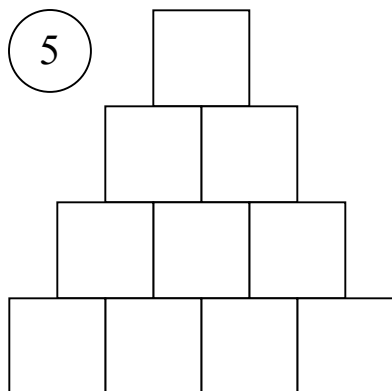
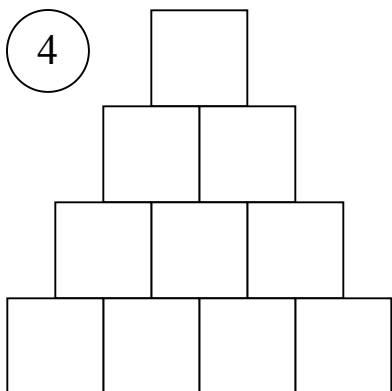
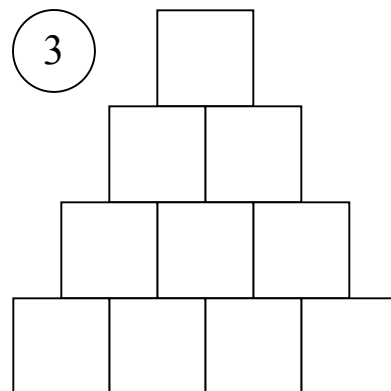
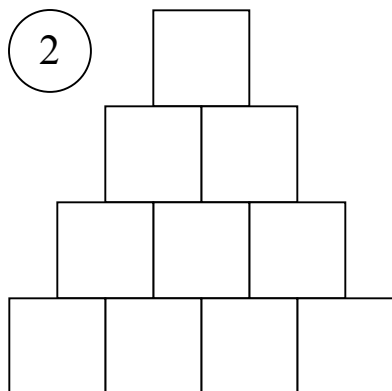
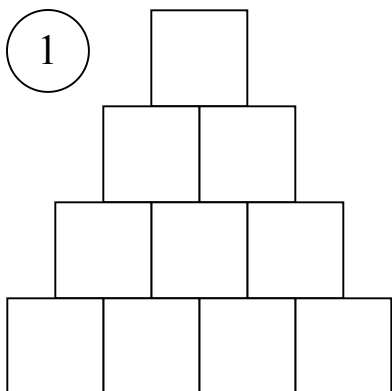
Add adjacent numbers and write their sum in the box above them as shown in the example. Continue until you reach the top of the pyramid. On some problems, you may need to work backwards.



Activity Master 4
Pyramid Math

Name _____

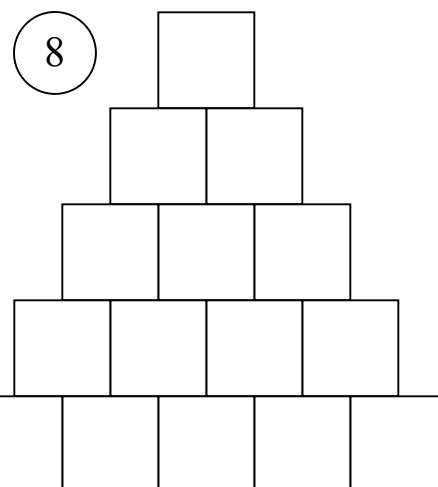
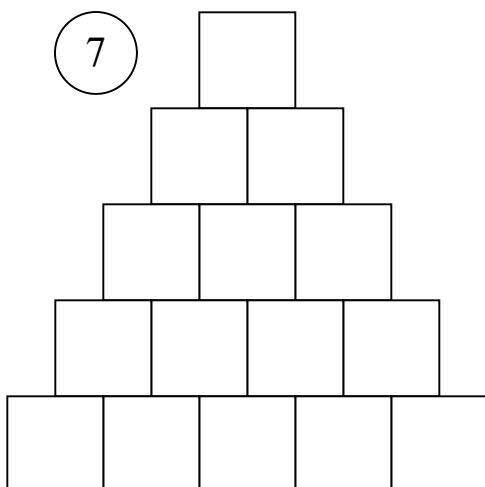
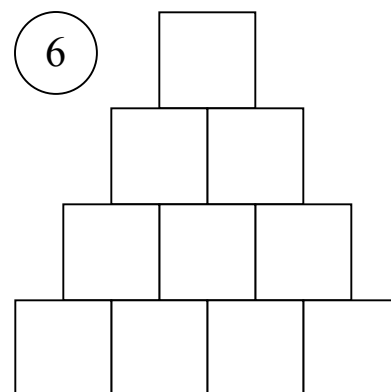
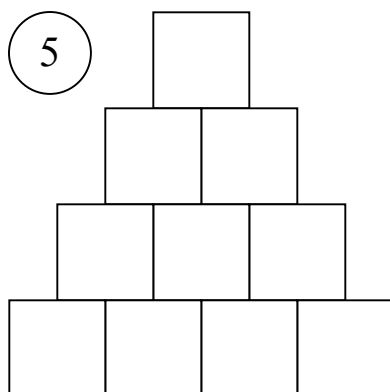
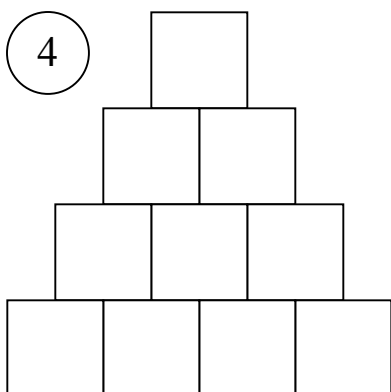
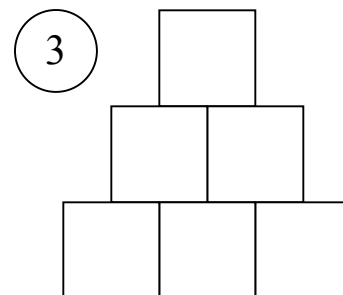
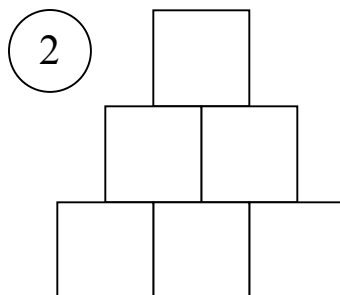
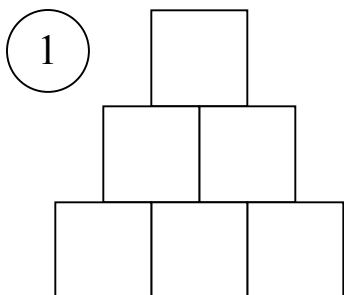
Add adjacent numbers and write their sum in the box above them. Continue until you reach the top of the pyramid. On some problems, you may need to work backwards.



Activity Master 5
Pyramid Math

Name _____

Add adjacent numbers and write their sum in the box above them. Continue until you reach the top of the pyramid. On some problems, you may need to work backwards.



Answer Key

Answers are provided for the top box of the pyramid except for problem 7 of each set. The answer for the left box of the bottom row is provided for problem 7.

Activity

Master:	Problem:								
	1	2	3	4	5	6	7	8	9
1	28	62	146	119	54	366	14	94	140
2	14.05	4.85	49.07	51.69	16.65	62.34	2.1	10.7	14.8
3	0	-14	14	18	0	-58	-18	13	-8

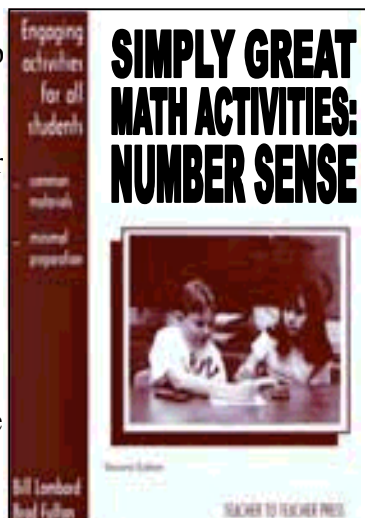
Research Says:

Though many researchers have trouble explaining exactly what number sense is or how to develop it, most of us as teachers can recognize when a student does or does not have number sense. By looking at a wide variety of research samples however, a framework of number sense components starts to take shape. Five of the most important components are 1) Estimation; 2) Mental computation; 3) Mathematical properties; 4) Mathematical operations; and 5) Number magnitude. In addition, numerous strategies begin to emerge for fostering number sense in students. They are 1) Toying with numbers; 2) Multiple computational methods; 3) Creative practice; 4) Exploring patterns; and 5) Use of written and oral language. Providing students with opportunities to use these strategies will help them build the components of number sense.

Keep in mind though that textbooks don't tend to utilize these strategies. That is because the purpose of a text is to increase math skill, not number sense. However, both are necessary. This means that as teachers we will need to work beyond the book to build number sense. The development of number sense is an ongoing endeavor that results more from experiences by the student than from direct instruction. Activities like Pyramid Math can be used to provide these experiences.

Further Help in Developing Number Sense!

For more activities that provide students with opportunities to develop number sense, take a look at our recently revised book on number sense. The second edition includes even more pages and a more teacher-friendly format that makes your job much easier. Students will enjoy making sense of numbers while building their computational skills.



www.tttpress.com

Characteristics of Number Sense

- Nonalgorithmic
- Tends to be complex
- Involves meaning
- Relies on judgment
- Is “effortful”
- Relies on self-regulation of the thinking process
- May involve uncertainty
- May yield multiple solutions or multiple solution paths

Number sense is something that
“unfolds” rather than something that
is “taught” directly.

P. R. Trafton
Establishing foundations for research
on number sense and related topics
1989