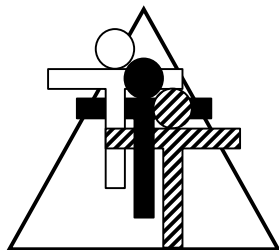


GRAPHIC ORGANIZERS FOR TEACHING ALGEBRA

By Brad Fulton
California League of Middle Schools
Educator of the Year, 2005
brad@tttpress.com



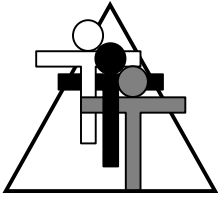
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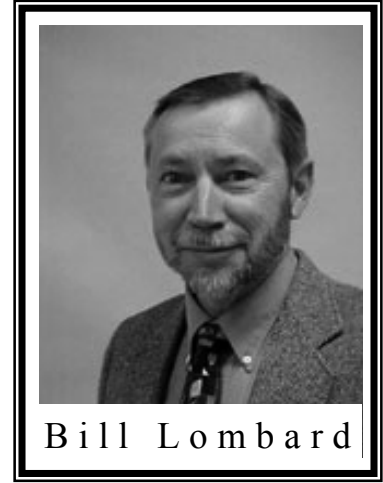
Brad Fulton and Bill Lombard Teacher to Teacher Press

"Building Mathematical Skill on a Foundation of Understanding"



Brad Fulton

- ◆ **Consultants**
- ◆ **Educators**
- ◆ **Authors**
- ◆ **Seminar leaders**
- ◆ **Teacher trainers**
- ◆ **Conference speakers**



Bill Lombard

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Known throughout the country for motivating and engaging teachers and students, Brad and Bill have authored over ten books that provide easy-to-teach yet mathematically-rich activities for busy teachers. In addition, they have co-authored six teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

Seminar leaders and trainers of mathematics teachers

- ◆ California Math Council and NCTM presenters
- ◆ Lead trainers for summer teacher training institutes
- ◆ Trainers/consultants for district, county, regional, and national workshops

Authors and co-authors of mathematics curriculum

- ◆ *Simply Great Math Activities* series: five books covering all major strands
- ◆ *Math Discoveries* series: bringing math alive for students in middle schools
- ◆ Teacher training seminar materials handbooks for elementary, middle, and secondary school

Available for workshops, keynote addresses, and conference sessions.

All workshops provide participants with complete and ready-to-use activities. These activities require minimal preparation, use materials commonly found in classrooms, and give clear and specific directions and format. Participants will also receive journal prompts, homework suggestions, and ideas for extensions and assessment.

Brad and Bill's math activities are the best I've seen in 30 years of teaching!

Wayne Dequer, 7th grade math teacher

"The high-energy, easy-to-follow handouts were clear. The instructors were great!"

DeLinda Van Dyke, middle school teacher

References available upon request

Give It Structure:

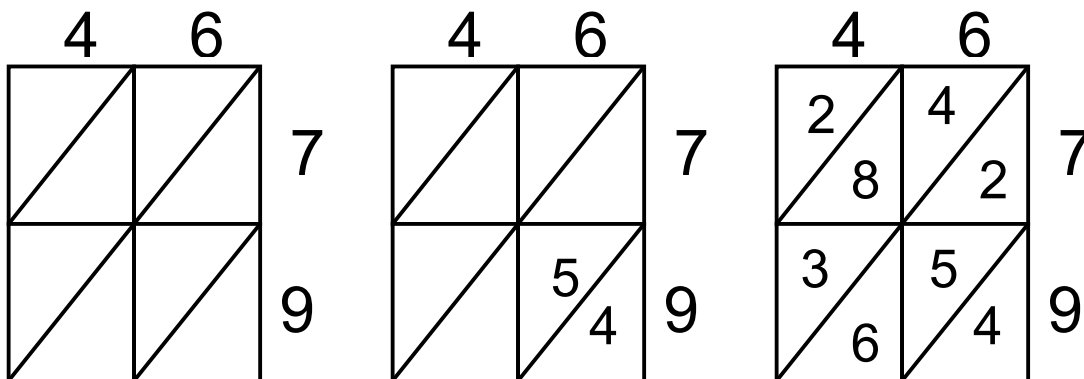
Students who struggle mathematically often fail for one of two reasons: either they struggle with the arithmetic that they encounter or they struggle with what to do with that arithmetic. For example, in solving a two-digit by two-digit multiplication problem like the one shown here, the student must begin by finding the product of the six and the nine. Given enough fingers, toes, and time, most students can find that $6 \times 9 = 54$. However, the student must also know that the four goes beneath the nine, and the five goes above the four. Then the nine and four must be multiplied while the five is added and both numbers go next to the four. Then for some inexplicable reason, a mystery zero appears. The student may approach mathematics with the thought that, "Ours is not to reason why—just invert and multiply."

$$\begin{array}{r} 46 \\ \times 79 \\ \hline \end{array}$$

Clearly we want our students to understand why an algorithm works, but realistically few of them do. Just as importantly, we want our students to be successful with math, and few of our intervention students are. For this reason, if we can provide students with a structural template or a graphic organizer, they need only concern themselves with the arithmetic involved in a problem. Following are a few graphic organizers that will help students with common mathematical tasks.

Lattice multiplication:

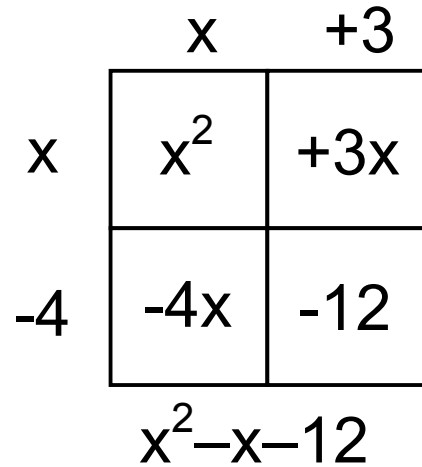
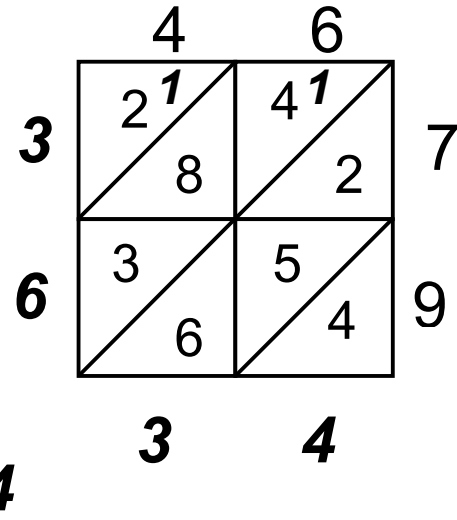
For centuries, multiplication was solved differently than it is now. A lattice was used to multiply 46 by 79 as shown in the left diagram. Each digit was multiplied and the product placed in the intersecting cells. The tens digit was placed above the diagonal and the ones was placed below. In the second diagram, the six and nine have been multiplied to get a product of 54. All cells have been filled out in the third diagram.



The following diagram shows how the diagonals are added to produce the product. Notice that in two diagonals, a tens digit has been regrouped to the top of the next diagonal.

Although the lattice method does essentially the same steps as our more familiar process, I have observed that students who use this method tend to make fewer errors. I believe this is because they do not have to attend to the structure of the algorithm and simply focus on the arithmetic of the multiplication.

This method of multiplication also transitions very nicely into multiplication of binomials in algebra. Rarely do we teach students to multiply binomials the way we multiply multi-digit numbers. Instead more and more algebra teachers are using what is called the *generic rectangle* that is essentially the lattice. If we want to multiply two binomials such as $(x+3)(x-4)$ we can place them in an array. Again, we fill in each cell and *add diagonals* as we did in the lattice. Now however, we call it *combining like terms*.



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HANDOUT, MAXIMUM MATH FROM THE MULTIPLICATION TABLE

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<http://www.tttpress.com/download.html>

Proportion Boxes:

Solving proportions, percents, and the word problems involving them are easier with this graphic organizer. Let's look at a sample word problem that lends itself to a solution involving proportions.

Mr. Fulton sleeps 7 hours per day. How many of his 54 years has he spent sleeping?

Notice that we are comparing two ideas in two ways. We are comparing the part of his day he spends sleeping to his whole day. We are also comparing a single day to his whole life. Thus we could label our graphic organizer as shown.

This sets up the proportion in the proper arrangement. That is, we have $\frac{7}{24} = \frac{x}{54}$. We would now decide the best method of solving this proportion. In this case, cross products would be effective. Notice that if the labels are switched, the numbers switch with them and a valid proportion still results. This template will work for any proportion problem, even if it is not a word problem. In addition, it works for all types of percent problems.

	hours	years
part	7	x
whole	24	54

28 more pages!

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PROPORTIONAL REASONING

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Mixture Problems:

Combined mixture problems in algebra can baffle students. They tend to look like one of these two problems.

A chemist wants to mix 12 grams of a 10% salt solution with some 75% salt solution to get a 20% salt solution. How many grams of the 75% salt solution are needed?

A company wants to create a 20 mixture of peanuts and walnuts that sells for \$5 per pound. The peanuts sell for \$2 per pound and the walnuts sell for \$6 per pound. How many pounds of each must they use?

I have used the following approach with great success. First, I tell them these problems are “ca-ca”. They agree. Then I explain that they can be solved with an equation that spells “ca-ca”.

$$C_1A_1 + C_2A_2 = C_T A_T$$

This means the *concentration* of the first solution times the *amount* of the first solution plus the *concentration* of the second solution times the *amount* of the second solution equals *concentration* of the total solution times the *amount* of the total solution. In the second example the c stands for *cost* instead of *concentration*.

Before substituting values into the equation template, I introduce the graphic organizer shown here. The students must decide which numbers go in which cells. First I ask the students to think about the problem. Clearly, if we mix two solutions, the total concentration of the mixture will be stronger than one solution and weaker than the other. That is, the *total concentration* will be the middle percent. (This will also be true if a *cost* is involved.)

Thus the 20% is the total, and the 10% and the 75% are the concentrations of the two addends. Then the students merely match the amounts with the respective percents and use an x for the missing number. The completed graphic is shown. Notice that the two *amounts* in the top cells must add up to the *total amount*. The template for the second problem is also shown.

		C	A
1	10%	12	
+2	75%	x	
T	20%	x+12	

		C	A
1			
+2			
T			

		C	A
1	\$2	x	
+2	\$6	20-x	
T	\$5	20	

It is also a good idea to have the students estimate the answer ahead of time. For example, in the second problem, the total mixture is one dollar less than the \$6 price of the walnuts but three dollars more than the \$2 price of the peanuts. Thus we will need more of the walnuts than we will of the peanuts. Often students can use this information to correctly estimate the answer.

to divide first, then add.

- Placing numbers in sections A and D or C and D will require students to subtract first, then multiply.
- Placing numbers in sections B and D will require students to study the various combinations of sums and products that satisfy the given answers.

$$12 - 8 = 4$$

$$4 \times 8 = 32$$

5. Eventually, you may wish to increase the difficulty through examples like these.

$$1 \times 20 = 20 \quad 1 + 20 = 21$$

$$2 \times 10 = 20 \quad 2 + 10 = 12$$

$$4 \times 5 = 20 \quad 4 + 5 = 20$$

6. You can also use the guess and check method to solve complex puzzles. Research has shown that the guess and check method is not only a valuable skill, it helps children transition to solving equations in algebra. Here is how to solve problems like the one on the right using this method. Pick a pair of numbers that add up to 100 such as 50 and 50. Write them in columns a and b.

Then multiply them to find the product. In this case, it is 2500, which is too high. We mark our check with an "H" to signify that this is too high. This tells us that the number in column a is too high.

a	b	check
50	50	2500 H
40	60	2400 L
45	55	2475 L
48	52	2496 H
47	53	2491 ☺

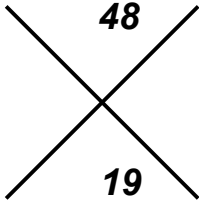
Let's adjust our guess by trying 40 and 60. Remember that our guesses must add to 100. It is also very important to note that the smaller of the two numbers must go in column a.

The product of these two numbers is 2400, which is too low. This is marked with an "L".

Our next guess for column a must be greater than 40 but less than 50. Let's try 45. This makes $b = 55$. Our new product, 2475 is too low also.

Our fourth guess will be 48. Now $b = 52$, and our product is 2496. Although this is too high, it is very close.

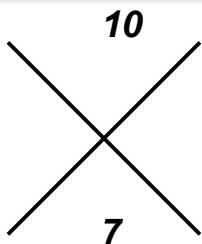
For our next guess, we try 47 for a , and 53 for b . This gives us the product we wanted.



Good Tip!



These drills are a great way to practice number concepts throughout the year. Worksheets can be created on the spot to be used as homework or warm-ups. If you are studying fraction multiplication, simply have the students copy a set of these problems as you write them on the board.

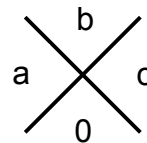


Journal Prompts:



Explain to a student how you would find the solution to the problem on the right.

What could you tell about the value of a and c in the example on the below? What can you tell about the value of b ? Explain.



Homework:

Assign one of the accompanying activity masters.

You can make a homework worksheet by placing numbers in a copy of the blank activity master. Alternately, the students can copy down problems as you write them on the board.

Taking a Closer Look:



The difficulty of these drills can be varied by the numbers chosen and their placement. Using decimals, fractions, or negative numbers can also increase the complexity.

Algebra students can practice factoring polynomials this way too.

For the polynomial $x^2 + 7x + 10 = 0$, students would construct the problem shown to find the solutions 2 and 5. The expression factors into the following binomials:

$$x^2 + 7x + 10 = (x + 2)(x + 5)$$

The solution to the equation then is $x = -2$ and $x = -5$.

Assessment:



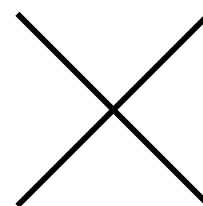
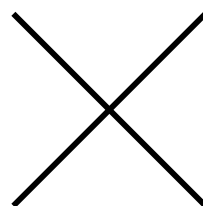
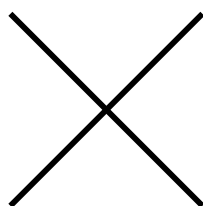
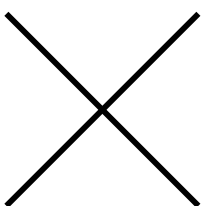
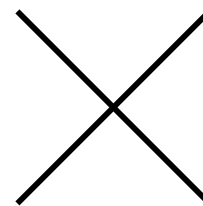
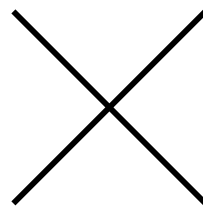
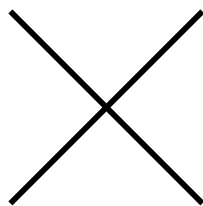
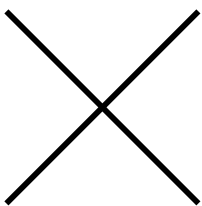
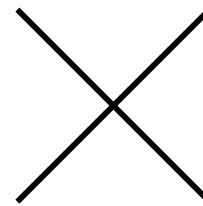
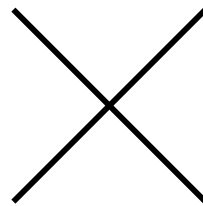
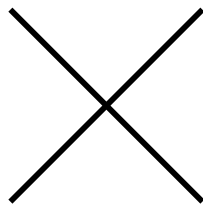
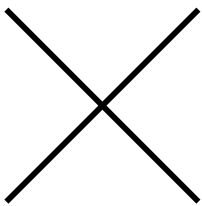
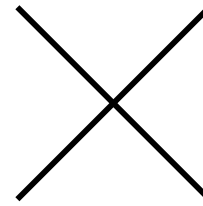
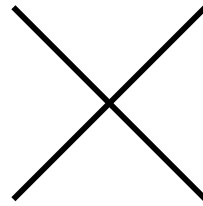
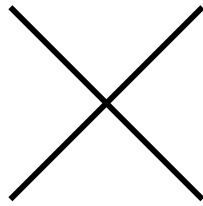
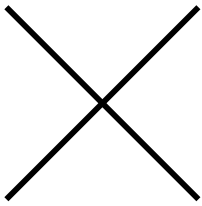
These drills can be spot checked for accuracy or students can exchange papers to check them.

You may also use the answer keys for the accompanying activity masters.

Answer Key:

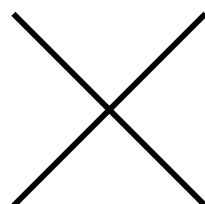
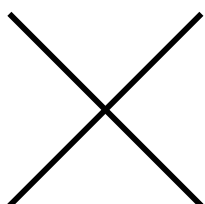
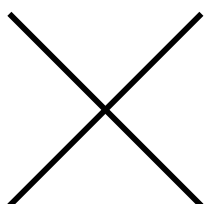
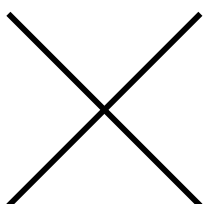
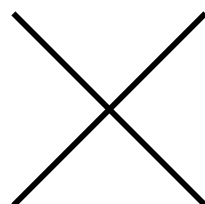
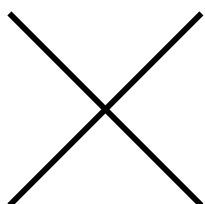
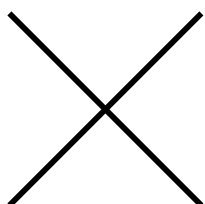
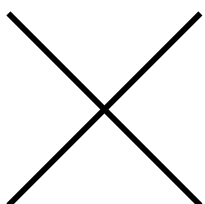
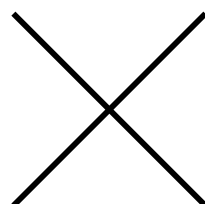
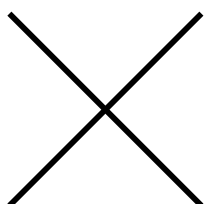
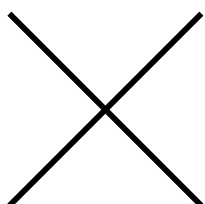
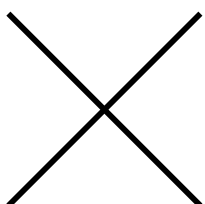
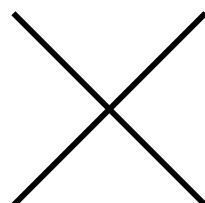
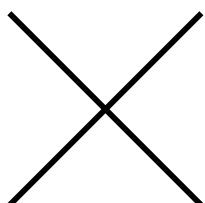
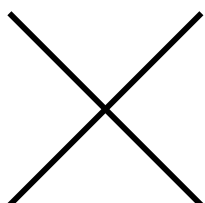
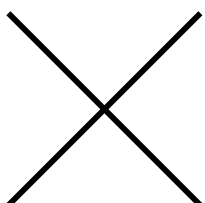
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a	14	9	a	5	35	a	8	10	a	4	7
b	45	14	b	6	24	b	7	14	b	4	8
c	12	7	c	0	0	c	6	14	c	3	12
d	8	9	d	7	49	d	7	13	d	6	8
e	36	12	e	12	60	e	9	13	e	6	6
f	36	13	f	9	54	f	12	15	f	0	12
g	48	14	g	11	22	g	6	12	g	4	9
h	12	8	h	9	108	h	4	16	h	7	9
i	0	7	i	12	96	i	6	10	i	8	8
j	9	6	j	10	10	j	8	15	j	6	10
k	20	9	k	12	48	k	7	11	k	5	12
l	25	10	l	12	120	l	0	9	l	8	11
m	33	14	m	2	12	m	3	8	m	10	12
n	60	16	n	12	132	n	2	11	n	11	12
o	24	14	o	0	0	o	7	10	o	7	12
p	77	18	p	6	42	p	12	21	p	12	12
q	72	20	q	9	72	q	11	21	q	8	9
r	60	16	r	12	0	r	12	23	r	9	11
s	60	17	s	12	144	s	1	9	s	9	9
Set 5	top	bottom	Set 6	top	bottom	Set 7	side	side	Set 8	side	side
a	1	7.2	a	-14	-5	a	-36	-1	a	47	53
b	0.45	1.4	b	45	-14	b	-1	36	b	23	63
c	1.2	4.3	c	-12	1	c	-6	6	c	50	60
d	0.008	0.18	d	8	-9	d	2	14	d	211	289
e	3.6	6.6	e	-36	0	e	-14	-2			
f	0.36	1.3	f	36	-13	f	-7	4			
g	0.048	0.68	g	-48	2	g	-4	7			
h	0.12	0.8	h	-12	4	h	-9	-2			
i	0	0.7	i	0	-7	i	-6	3			
j	0.9	3.3	j	9	-6	j	-6	-3			
k	0.002	0.09	k	-20	1	k	-18	-1			
l	0.025	0.55	l	-25	0	l	-9	-5			
m	0.33	1.4	m	33	-14	m	-15	-3			
n	6	10.6	n	-60	-4	n	-15	3			
o	0.24	2.12	o	24	-14	o	-1	45			
p	0.077	0.81	p	77	-18	p	-8	-6			
q	0.96	1.28	q	-96	-4	q	-6	8			
r	0.6	10.06	r	60	-16	r	-12	4			
s	6	12.5	s	-60	-7	s	-2	24			

“X” Marks the Spot



“X” Marks the Spot

Multiply the two side numbers and put the product on the top. Add the two side numbers and put the sum on the bottom



"X" Marks the Spot 1

Name _____

Multiply the two side numbers and put the product on the top. Add the two side numbers and put the sum on the bottom as shown.

$$\begin{array}{ccc} & 56 & \\ 7 & \times & 8 \\ & 4 & \end{array}$$

$$\begin{array}{ccc} a & & \\ 2 & \times & 7 \\ & & \end{array}$$

$$\begin{array}{ccc} b & & \\ 5 & \times & 9 \\ & & \end{array}$$

$$\begin{array}{ccc} c & & \\ 3 & \times & 4 \\ & & \end{array}$$

$$\begin{array}{ccc} d & & \\ 8 & \times & 1 \\ & & \end{array}$$

$$\begin{array}{ccc} e & & \\ 6 & \times & 6 \\ & & \end{array}$$

$$\begin{array}{ccc} f & & \\ 9 & \times & 4 \\ & & \end{array}$$

$$\begin{array}{ccc} g & & \\ 6 & \times & 8 \\ & & \end{array}$$

$$\begin{array}{ccc} h & & \\ 2 & \times & 6 \\ & & \end{array}$$

$$\begin{array}{ccc} i & & \\ 7 & \times & 0 \\ & & \end{array}$$

$$\begin{array}{ccc} j & & \\ 3 & \times & 3 \\ & & \end{array}$$

$$\begin{array}{ccc} k & & \\ 5 & \times & 4 \\ & & \end{array}$$

$$\begin{array}{ccc} l & & \\ 5 & \times & 5 \\ & & \end{array}$$

$$\begin{array}{ccc} m & & \\ 11 & \times & 3 \\ & & \end{array}$$

$$\begin{array}{ccc} n & & \\ 6 & \times & 10 \\ & & \end{array}$$

$$\begin{array}{ccc} o & & \\ 12 & \times & 2 \\ & & \end{array}$$

$$\begin{array}{ccc} p & & \\ 7 & \times & 11 \\ & & \end{array}$$

$$\begin{array}{ccc} q & & \\ 12 & \times & 8 \\ & & \end{array}$$

$$\begin{array}{ccc} r & & \\ 10 & \times & 6 \\ & & \end{array}$$

$$\begin{array}{ccc} s & & \\ 5 & \times & 12 \\ & & \end{array}$$

"X" Marks the Spot 2

Name _____

The number on the bottom is the sum of the two numbers on the sides. Find the missing side number. Then multiply the two side numbers and write the product on the top.

$$\begin{array}{ccc} & 56 & \\ & \diagdown \quad \diagup & \\ 7 & & 8 \\ & \diagup \quad \diagdown & \\ & 15 & \end{array}$$

$$\begin{array}{ccc} & a & \\ & \diagdown \quad \diagup & \\ 7 & & \\ & \diagup \quad \diagdown & \\ & 12 & \end{array}$$

$$\begin{array}{ccc} & b & \\ & \diagdown \quad \diagup & \\ 4 & & \\ & \diagup \quad \diagdown & \\ & 10 & \end{array}$$

$$\begin{array}{ccc} & c & \\ & \diagdown \quad \diagup & \\ 9 & & \\ & \diagup \quad \diagdown & \\ & 9 & \end{array}$$

$$\begin{array}{ccc} & d & \\ & \diagdown \quad \diagup & \\ & & 7 \\ & \diagup \quad \diagdown & \\ & 14 & \end{array}$$

$$\begin{array}{ccc} & e & \\ & \diagdown \quad \diagup & \\ 5 & & \\ & \diagup \quad \diagdown & \\ & 17 & \end{array}$$

$$\begin{array}{ccc} & f & \\ & \diagdown \quad \diagup & \\ & & 6 \\ & \diagup \quad \diagdown & \\ & 15 & \end{array}$$

$$\begin{array}{ccc} & g & \\ & \diagdown \quad \diagup & \\ 2 & & \\ & \diagup \quad \diagdown & \\ & 13 & \end{array}$$

$$\begin{array}{ccc} & h & \\ & \diagdown \quad \diagup & \\ & & 12 \\ & \diagup \quad \diagdown & \\ & 21 & \end{array}$$

$$\begin{array}{ccc} & i & \\ & \diagdown \quad \diagup & \\ & & 8 \\ & \diagup \quad \diagdown & \\ & 20 & \end{array}$$

$$\begin{array}{ccc} & j & \\ & \diagdown \quad \diagup & \\ 1 & & \\ & \diagup \quad \diagdown & \\ & 11 & \end{array}$$

$$\begin{array}{ccc} & k & \\ & \diagdown \quad \diagup & \\ & & 4 \\ & \diagup \quad \diagdown & \\ & 16 & \end{array}$$

$$\begin{array}{ccc} & l & \\ & \diagdown \quad \diagup & \\ 10 & & \\ & \diagup \quad \diagdown & \\ & 22 & \end{array}$$

$$\begin{array}{ccc} & m & \\ & \diagdown \quad \diagup & \\ & & 6 \\ & \diagup \quad \diagdown & \\ & 8 & \end{array}$$

$$\begin{array}{ccc} & n & \\ & \diagdown \quad \diagup & \\ & & 11 \\ & \diagup \quad \diagdown & \\ & 23 & \end{array}$$

$$\begin{array}{ccc} & o & \\ & \diagdown \quad \diagup & \\ 8 & & \\ & \diagup \quad \diagdown & \\ & 8 & \end{array}$$

$$\begin{array}{ccc} & p & \\ & \diagdown \quad \diagup & \\ & & 7 \\ & \diagup \quad \diagdown & \\ & 13 & \end{array}$$

$$\begin{array}{ccc} & q & \\ & \diagdown \quad \diagup & \\ 8 & & \\ & \diagup \quad \diagdown & \\ & 17 & \end{array}$$

$$\begin{array}{ccc} & r & \\ & \diagdown \quad \diagup & \\ 0 & & \\ & \diagup \quad \diagdown & \\ & 12 & \end{array}$$

$$\begin{array}{ccc} & s & \\ & \diagdown \quad \diagup & \\ & & 12 \\ & \diagup \quad \diagdown & \\ & 24 & \end{array}$$

"X" Marks the Spot 3

Name _____

The top number is the product of the two numbers on the sides. Find the missing side number. Then add the two side numbers and write the sum on the bottom.

$$\begin{array}{ccc} & 56 & \\ & \diagdown \quad \diagup & \\ 7 & & 8 \\ & \diagup \quad \diagdown & \\ & 15 & \end{array}$$

$$\begin{array}{ccc} a & 16 & \\ & \diagdown \quad \diagup & \\ 2 & & \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} b & 49 & \\ & \diagdown \quad \diagup & \\ & & 7 \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} c & 48 & \\ & \diagdown \quad \diagup & \\ & & 8 \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} d & 42 & \\ & \diagdown \quad \diagup & \\ 6 & & \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} e & 36 & \\ & \diagdown \quad \diagup & \\ 4 & & \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} f & 36 & \\ & \diagdown \quad \diagup & \\ 3 & & \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} g & 36 & \\ & \diagdown \quad \diagup & \\ & & 6 \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} h & 48 & \\ & \diagdown \quad \diagup & \\ & & 12 \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} i & 24 & \\ & \diagdown \quad \diagup & \\ 4 & & \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} j & 56 & \\ & \diagdown \quad \diagup & \\ 7 & & \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} k & 28 & \\ & \diagdown \quad \diagup & \\ & & 4 \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} l & 0 & \\ & \diagdown \quad \diagup & \\ & & 9 \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} m & 15 & \\ & \diagdown \quad \diagup & \\ 5 & & \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} n & 18 & \\ & \diagdown \quad \diagup & \\ 9 & & \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} o & 21 & \\ & \diagdown \quad \diagup & \\ & & 3 \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} p & 108 & \\ & \diagdown \quad \diagup & \\ 9 & & \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} q & 110 & \\ & \diagdown \quad \diagup & \\ & & 10 \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} r & 132 & \\ & \diagdown \quad \diagup & \\ 11 & & \\ & \diagup \quad \diagdown & \end{array}$$

$$\begin{array}{ccc} s & 8 & \\ & \diagdown \quad \diagup & \\ & & 8 \\ & \diagup \quad \diagdown & \end{array}$$

"X" Marks the Spot 4

Name _____

The top number is the product of the two missing side numbers. The bottom number is the sum of the two missing side numbers. Find the missing side numbers.

$$\begin{array}{ccc} & 56 & \\ 7 & \times & 8 \\ & 15 & \end{array}$$

$$\begin{array}{ccc} a & 28 & \\ & \times & \\ & 11 & \end{array}$$

$$\begin{array}{ccc} b & 32 & \\ & \times & \\ & 12 & \end{array}$$

$$\begin{array}{ccc} c & 36 & \\ & \times & \\ & 15 & \end{array}$$

$$\begin{array}{ccc} d & 48 & \\ & \times & \\ & 14 & \end{array}$$

$$\begin{array}{ccc} e & 36 & \\ & \times & \\ & 12 & \end{array}$$

$$\begin{array}{ccc} f & 0 & \\ & \times & \\ & 12 & \end{array}$$

$$\begin{array}{ccc} g & 36 & \\ & \times & \\ & 13 & \end{array}$$

$$\begin{array}{ccc} h & 63 & \\ & \times & \\ & 16 & \end{array}$$

$$\begin{array}{ccc} i & 64 & \\ & \times & \\ & 16 & \end{array}$$

$$\begin{array}{ccc} j & 60 & \\ & \times & \\ & 16 & \end{array}$$

$$\begin{array}{ccc} k & 60 & \\ & \times & \\ & 17 & \end{array}$$

$$\begin{array}{ccc} l & 88 & \\ & \times & \\ & 19 & \end{array}$$

$$\begin{array}{ccc} m & 120 & \\ & \times & \\ & 22 & \end{array}$$

$$\begin{array}{ccc} n & 132 & \\ & \times & \\ & 23 & \end{array}$$

$$\begin{array}{ccc} o & 84 & \\ & \times & \\ & 19 & \end{array}$$

$$\begin{array}{ccc} p & 144 & \\ & \times & \\ & 24 & \end{array}$$

$$\begin{array}{ccc} q & 72 & \\ & \times & \\ & 17 & \end{array}$$

$$\begin{array}{ccc} r & 99 & \\ & \times & \\ & 20 & \end{array}$$

$$\begin{array}{ccc} s & 81 & \\ & \times & \\ & 18 & \end{array}$$

"X" Marks the Spot 5

Name _____

Multiply the two side numbers and put the product on the top. Add the two side numbers and put the sum on the bottom as shown.

$$\begin{array}{ccc} & 5.6 & \\ 7 & \times & .8 \\ & 7.8 & \end{array}$$

$$\begin{array}{ccc} a & & \\ .2 & \times & 7 \\ & & \end{array}$$

$$\begin{array}{ccc} b & & \\ .5 & \times & .9 \\ & & \end{array}$$

$$\begin{array}{ccc} c & & \\ .3 & \times & 4 \\ & & \end{array}$$

$$\begin{array}{ccc} d & & \\ .08 & \times & .1 \\ & & \end{array}$$

$$\begin{array}{ccc} e & & \\ 6 & \times & .6 \\ & & \end{array}$$

$$\begin{array}{ccc} f & & \\ .9 & \times & .4 \\ & & \end{array}$$

$$\begin{array}{ccc} g & & \\ .6 & \times & .08 \\ & & \end{array}$$

$$\begin{array}{ccc} h & & \\ .2 & \times & .6 \\ & & \end{array}$$

$$\begin{array}{ccc} i & & \\ .7 & \times & 0 \\ & & \end{array}$$

$$\begin{array}{ccc} j & & \\ 3 & \times & .3 \\ & & \end{array}$$

$$\begin{array}{ccc} k & & \\ .05 & \times & .04 \\ & & \end{array}$$

$$\begin{array}{ccc} l & & \\ .05 & \times & .5 \\ & & \end{array}$$

$$\begin{array}{ccc} m & & \\ 1.1 & \times & .3 \\ & & \end{array}$$

$$\begin{array}{ccc} n & & \\ .6 & \times & 10 \\ & & \end{array}$$

$$\begin{array}{ccc} o & & \\ .12 & \times & 2 \\ & & \end{array}$$

$$\begin{array}{ccc} p & & \\ .7 & \times & .11 \\ & & \end{array}$$

$$\begin{array}{ccc} q & & \\ 1.2 & \times & .08 \\ & & \end{array}$$

$$\begin{array}{ccc} r & & \\ 10 & \times & .06 \\ & & \end{array}$$

$$\begin{array}{ccc} s & & \\ .5 & \times & 12 \\ & & \end{array}$$

"X" Marks the Spot 6

Name _____

Multiply the two side numbers and put the product on the top. Add the two side numbers and put the sum on the bottom as shown.

$$\begin{array}{ccc} & -56 & \\ 7 & \times & -8 \\ & -1 & \end{array}$$

$$\begin{array}{ccc} a & & \\ 2 & \times & -7 \\ & & \end{array}$$

$$\begin{array}{ccc} b & & \\ -5 & \times & -9 \\ & & \end{array}$$

$$\begin{array}{ccc} c & & \\ -3 & \times & 4 \\ & & \end{array}$$

$$\begin{array}{ccc} d & & \\ -8 & \times & -1 \\ & & \end{array}$$

$$\begin{array}{ccc} e & & \\ -6 & \times & 6 \\ & & \end{array}$$

$$\begin{array}{ccc} f & & \\ -9 & \times & -4 \\ & & \end{array}$$

$$\begin{array}{ccc} g & & \\ -6 & \times & 8 \\ & & \end{array}$$

$$\begin{array}{ccc} h & & \\ -2 & \times & 6 \\ & & \end{array}$$

$$\begin{array}{ccc} i & & \\ -7 & \times & 0 \\ & & \end{array}$$

$$\begin{array}{ccc} j & & \\ -3 & \times & -3 \\ & & \end{array}$$

$$\begin{array}{ccc} k & & \\ 5 & \times & -4 \\ & & \end{array}$$

$$\begin{array}{ccc} l & & \\ -5 & \times & 5 \\ & & \end{array}$$

$$\begin{array}{ccc} m & & \\ -11 & \times & -3 \\ & & \end{array}$$

$$\begin{array}{ccc} n & & \\ 6 & \times & -10 \\ & & \end{array}$$

$$\begin{array}{ccc} o & & \\ -12 & \times & -2 \\ & & \end{array}$$

$$\begin{array}{ccc} p & & \\ -7 & \times & -11 \\ & & \end{array}$$

$$\begin{array}{ccc} q & & \\ -12 & \times & 8 \\ & & \end{array}$$

$$\begin{array}{ccc} r & & \\ -10 & \times & -6 \\ & & \end{array}$$

$$\begin{array}{ccc} s & & \\ 5 & \times & -12 \\ & & \end{array}$$

"X" Marks the Spot 7

Name _____

The top number is the product of the two missing side numbers. The bottom number is the sum of the two missing side numbers. Find the missing side numbers.

$$\begin{array}{c} 36 \\ \diagdown \quad \diagup \\ 36 \quad \quad \quad 1 \\ \diagup \quad \diagdown \\ 37 \end{array}$$

$$\begin{array}{c} a \quad 36 \\ \diagdown \quad \diagup \\ \quad \quad \quad -37 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} b \quad -36 \\ \diagdown \quad \diagup \\ \quad \quad \quad 35 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} c \quad -36 \\ \diagdown \quad \diagup \\ \quad \quad \quad 0 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} d \quad 28 \\ \diagdown \quad \diagup \\ \quad \quad \quad 16 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} e \quad 28 \\ \diagdown \quad \diagup \\ \quad \quad \quad -16 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} f \quad -28 \\ \diagdown \quad \diagup \\ \quad \quad \quad -3 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} g \quad -28 \\ \diagdown \quad \diagup \\ \quad \quad \quad 3 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} h \quad 18 \\ \diagdown \quad \diagup \\ \quad \quad \quad -11 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} i \quad -18 \\ \diagdown \quad \diagup \\ \quad \quad \quad -3 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} j \quad 18 \\ \diagdown \quad \diagup \\ \quad \quad \quad -9 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} k \quad 18 \\ \diagdown \quad \diagup \\ \quad \quad \quad -19 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} l \quad 45 \\ \diagdown \quad \diagup \\ \quad \quad \quad -14 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} m \quad 45 \\ \diagdown \quad \diagup \\ \quad \quad \quad -18 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} n \quad -45 \\ \diagdown \quad \diagup \\ \quad \quad \quad -12 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} o \quad -45 \\ \diagdown \quad \diagup \\ \quad \quad \quad 44 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} p \quad 48 \\ \diagdown \quad \diagup \\ \quad \quad \quad -14 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} q \quad -48 \\ \diagdown \quad \diagup \\ \quad \quad \quad 2 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} r \quad -48 \\ \diagdown \quad \diagup \\ \quad \quad \quad -8 \\ \diagup \quad \diagdown \end{array}$$

$$\begin{array}{c} s \quad -48 \\ \diagdown \quad \diagup \\ \quad \quad \quad 22 \\ \diagup \quad \diagdown \end{array}$$

"X" Marks the Spot 8

Name _____

Use a guess and check table to find the missing side numbers. Always put your lower number in column a.

a

$$\begin{array}{r} 2491 \\ \times 100 \\ \hline \end{array}$$

a	b	check

b

$$\begin{array}{r} 1449 \\ \times 86 \\ \hline \end{array}$$

a	b	check

c

$$\begin{array}{r} 3000 \\ \times 110 \\ \hline \end{array}$$

a	b	check

d

$$\begin{array}{r} 60979 \\ \times 500 \\ \hline \end{array}$$

a	b	check

Using “X” Marks the Spot to Solve Combined Work Problems.

One of the more challenging types of problems for algebra students is the combined work problems. These often look like these two examples:

Andy can paint a room in three hours. Zoe can paint the same room in four hours.

How long would it take them to paint the room working together?

One pipe can fill a tub in three hours. A second pipe can fill the same tub in four hours. How long will it take to fill the tub if both pipes are used?

Typically students either add the two numbers or average them, failing to realize that the job must get done more quickly than either given time. Thus the answer must be *less* than either given number.

The most common way to solve the problem is by using the following formula:

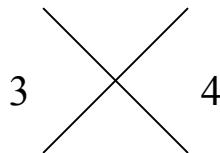
$$\frac{x}{3} + \frac{x}{4} = 1$$

The variable, x , represents the total time for the combined work. The first fraction means that Andy paints one third of a room per hour, and the second fraction shows that Zoe paints one fourth of a room per hour. The two fractions add up to one room being painted.

While this procedure makes sense to the teacher, its origin and development is beyond the ability of most students. With enough practice, students may become proficient with this, but it remains questionable if the equation holds meaning for them. Let’s look at the solution to this problem using the equation.

$$\begin{aligned} \frac{x}{3} + \frac{x}{4} &= 1 \\ \frac{4x}{12} + \frac{3x}{12} &= 1 \\ \frac{7x}{12} &= 1 \\ \left(\frac{12}{7}\right)\frac{7x}{12} &= 1\left(\frac{12}{7}\right) \\ x &= \frac{12}{7} \end{aligned}$$

Now examine the solution to the following problem from “X” Marks the Spot:



Notice that the solution to this problem is also $12/7$! Certainly this is a much simpler way to achieve the correct answer, but will it always work? In fact, it does. Let’s demonstrate this by generalizing the problem:

Andy paints a room in a hours, and Zoe paints the same room in b hours. How

long will it take them to paint the room working together?

$$\begin{aligned} \frac{x}{a} + \frac{x}{b} &= 1 \\ \frac{bx}{ab} + \frac{ax}{ab} &= 1 \\ \frac{(a+b)x}{ab} &= 1 \\ \left(\frac{ab}{(a+b)}\right) \frac{(a+b)x}{ab} &= 1 \left(\frac{ab}{(a+b)}\right) \\ x &= \frac{ab}{(a+b)} \end{aligned}$$

This is also the solution to the simpler problem when we use a and b as our side numbers:

$$\begin{array}{c} \diagdown \quad ab \quad \diagup \\ a \quad \quad b \\ \diagup \quad a+b \quad \diagdown \end{array}$$

This method will also work when only one person's time is given along with the combined work time:

One pipe can fill a tub in 3 hours. With a second pipe running, the tub can be filled in only two hours. How long would the second pipe take if it was used alone?

The traditional algorithm would look like this:

$$\frac{2}{3} + \frac{2}{x} = 1$$

Using the "X" Marks the Spot strategy, we have:

$$\begin{array}{c} \diagdown \quad 3x \quad \diagup \\ 3 \quad \quad x \\ \diagup \quad x+3 \quad \diagdown \end{array}$$

We also know that the answer is two. Thus the top number divided by the bottom number must equal two:

$$\begin{aligned} \frac{3x}{x+3} &= 2 \\ \frac{3x}{x+3} &= \frac{2}{1} \\ 3x &= 2(x+3) \\ 3x &= 2x+6 \\ x &= 6 \end{aligned}$$

Be the Teacher

We underestimate our importance, value, and effect as a teacher. It is too easy to assume that students will learn math from a math book. We don't take this approach with other content areas. Though they get information from a textbook, chemistry students learn, *really learn*, the bulk of their knowledge from doing experiments. History students learn more about culture from hosting a Renaissance fair than from reading about it. Yet we sometimes fall into the clutches of the false premise that *anyone* can teach math given a good textbook. In fact, a math textbook represents the most difficult of all content area reading materials.

Thus our role as instructors is even greater in the math classroom than in other domains. How we present a concept, how we sequence the problems our students encounter, how we bring them incrementally toward our final desired level of mathematical rigor is an orchestration of the most demanding detail.

For this reason, it is important that we design each math lesson with great attention to detail. We should begin by teaching the concept at hand. Initially this should be done with positive whole numbers. Once the students have begun to grasp the concept, we slowly and incrementally introduce integers or rational numbers as needed. Because it develops mathematical understanding incrementally from basic concepts to full rigor, this approach is called *Conceptual Layering*. It has proven to be a very effective technique not only for students in an intervention setting but for regular education students as well.

It is much like building a house. No walls are erected until a firm foundation is established. Then and only then can we build a solid and sound structure. Due to limits on their length, textbooks cannot take time to present material this way any more than a dictionary can develop language. It merely defines it. Textbooks are a reference work too. New material is often introduced at an abstract and rigorous level. Students cannot start there anymore than a passenger can climb aboard a train moving at full speed. The teacher's task is to slow down the train to get the students aboard and then take them up to speed.

This process will look different in each classroom. Your expertise as the teacher is the best determinant for where to begin, how big of steps are taken in moving between each conceptual layer, and what level of rigor is the finishing point.

The textbook can still play a crucial role in this process. It is a vast resource of practice problems and often provides effective examples for students who get home and need more instruction to work successfully. The best perspective is one in which we view textbooks as reference materials, which they are by definition, and not as sacred works. Students don't learn math from math books. They don't even learn math from math teachers. They learn math best from people they value and respect. Thus the teacher who is optimistic, positive, helpful, and kind will do more for his or her students than any textbook can ever accomplish. Remember that you, not the textbook, are the teacher.

As you browse our website, you might find the following handouts of use with your intervention students:

Developing Number Sense (31 pages)

Math Projects (34 pages)

Algebra for All 8th Graders (41 pages)

Answering Your Students Why Questions in Mathematics (8 pages)

Fractions 2006 (48 pages)

Algebra for Everyone: Hundreds Magic (22 pages)

Managing the Math Class (33 pages)

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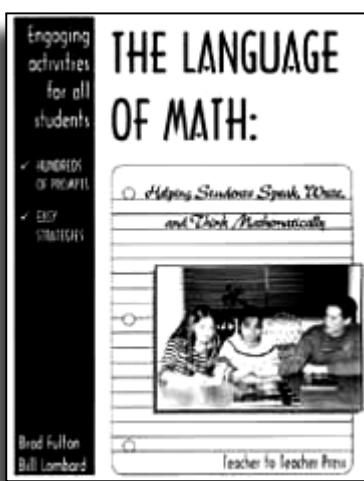
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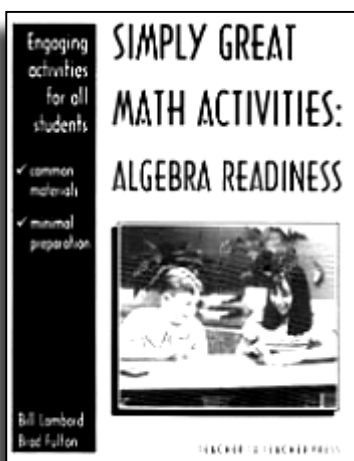
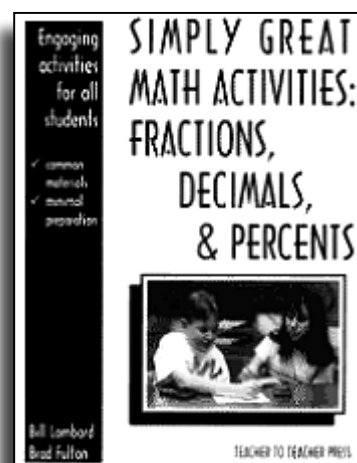
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Books by Brad and Bill



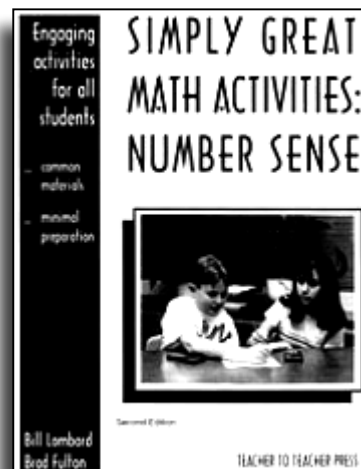
The Language of Math helps teachers create a classroom environment rich in mathematical thinking by showing them how to easily incorporate oral and written language into their math classes. Over 100 journal and discussion starters are included along with extensive instructions for making the most of your math time.

Here are a dozen unique and conceptual activities that will help your students add, subtract, multiply and divide fractions as well as connect them to decimal and percent representations. Both you and your students will love the novel and creative approach.



Teachers are raving about how effective these activities have been in their classrooms. Children as young as fourth grade and college students alike say that algebra is easy and makes sense because of this incredible approach.

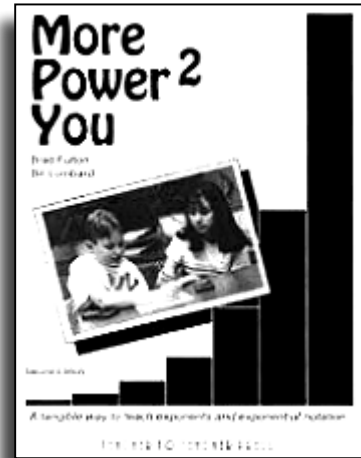
Students don't even think they are doing math sometimes because these activities are so fun and engaging, but they are developing rich and valuable number sense as they explore these eleven creative activities.



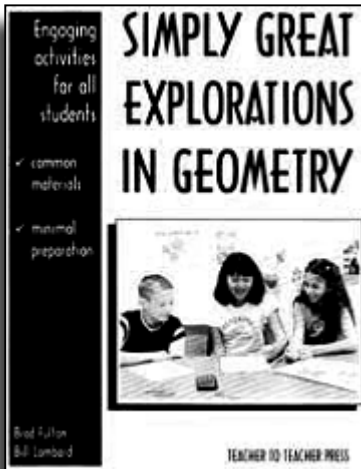


Our first book is still one of our most popular. Every teacher we talk to who has tried this approach to functions has been amazed at what their students have learned and accomplished. Over 150 pages of multiple representations of functions cover such concepts as slope, intercept, and function notation. Even elementary students have developed an understanding of functions with this book.

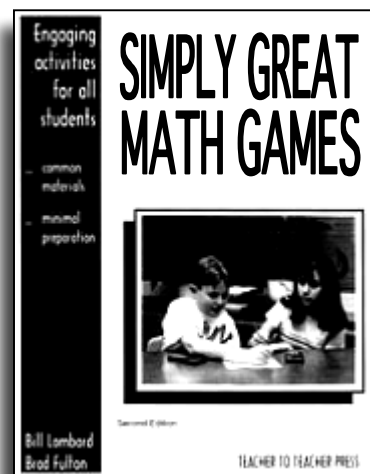
Exponents will finally make sense to your students after they participate in the unique activities found in this book. Both positive and negative exponents are demonstrated conceptually. Your students will even be able to explain $why n^0 = 1$.



Over one dozen geometry activities will excite your students as they discover the connections between geometry and fractions, decimals, percents, and even algebra. Area formulas, angle measurement, polygon attributes, vocabulary, and construction are covered.

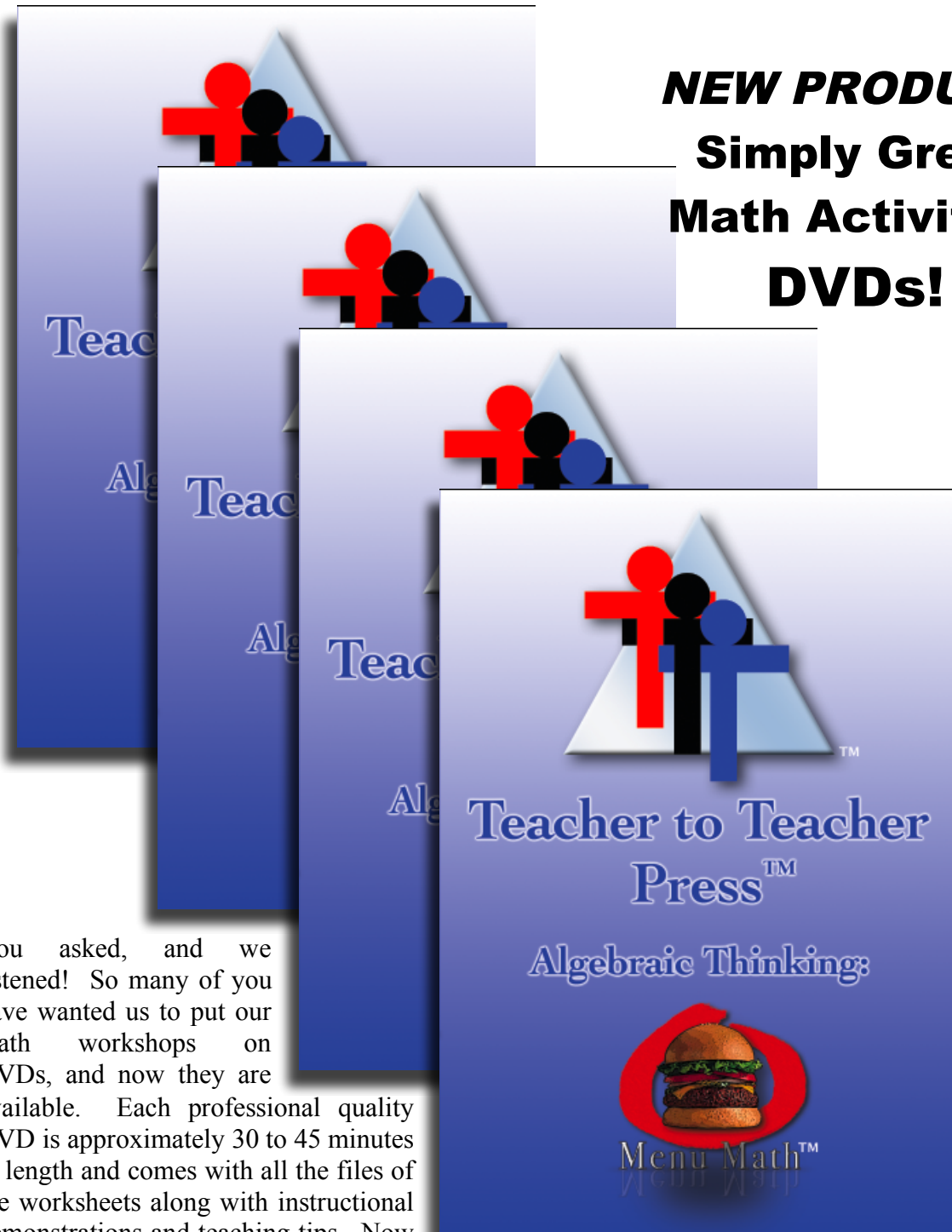


A dozen engaging and educational games await you and your students in this creative and highly adaptable book. You'll find games that reinforce basic operations with whole numbers, fractions, decimals, and integers as well as algebraic skills. Game masters will serve a spectrum of grade levels and skill levels. Your students will beg for more!



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