The Language of Math: Helping Students Speak, Write, and Think Mathematically

California Math Council – SS
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by
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Here is a proven method, including sample activities and lessons, for providing success for all students. While providing for struggling students, these activities will make even your brightest students think about math in new ways.

This 55 page presentation is ready to use in your classroom. Complete with overhead transparencies and activities, it is available in its entirety ready to download at the author's website, www.tttpress.com

Here are duplicable activities and concepts for all teachers for all their students.
All the transparencies, research, and activities that model effective teaching strategies as well as more "stuff" are on the author's website.

Five Concepts & Key Ideas:

- ✓ Rationale for incorporating oral and written language
- ✓ Easy tips for leading rich mathematical discussions
- ✓ Quotations about speaking, writing, and thinking mathematically
- ✓ Quick questions that promote good discussions
- ✓ Overhead transparencies of ready-to-use journal prompts

Teachers will become empowered with skills and activities so their students will become powerful problem solvers and creative mathematical thinkers.

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The Language of Math: Helping Students Speak, Write, and Think Mathematically

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Abstract

We want teachers to be models of mathematical reasoning and problem solving. By providing teachers with language skills, they can build the confidence of their students. They can check student progress for understanding, encourage conjectures, provide collective and collaborative thinking, problem solving, and reflections.

Using simple activities that can be modified for use in multiple grades, straightforward questions anyone can use will develop deep concepts of mathematics in number sense, algebraic thinking, and geometry. The activities lead naturally into extensions allowing use of these activities over several months, allowing students to become competent and confident in their skills.

The comprehensive handout will enable all participants to become experts at communicating this essential skill and will enable math

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leaders/trainers to inservice their staffs immediately.

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Problem solving is the reason we teach mathematics.

How can we design problem solving activities so students see the value in mathematics?

* * * * * *

Teachers affect eternity.

We are ordinary people
doing extraordinary things.

Hours pass into days
And time slips away,
But the words we speak
And the things we do,
Touch the lives of others
And live on forever.

Emily Matthews

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“We can think of ourselves not
as teachers but as gardeners.
A gardener does not “grow flowers”;
he tries to give them what he thinks
will help them grow, and they grow by
themselves.
A child’s mind, like a flower, is a living
thing. We can’t make it grow any more
than we can make a flower grow by
gluing on leaves and petals. All we can
do is surround the growing mind with
what it needs for growing and have faith
that it will take what it needs and grow.”

– John Holt

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"The terms and circumstances of human existence can be expected to change radically during the next life span. Science, mathematics, and technology will be at the center of that change – causing it, shaping it, responding to it. Therefore, they will be essential to the education of today's children for tomorrow's world."

Benchmarks for Science Literacy

* * * * * *

Innumeracy

In today's world, "innumeracy" is an even greater danger than illiteracy, and is perhaps even more common.

– Arthur C. Clarke

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Here is a sample of mathematical quotations for classroom use from the author's website. These can be used for a class 'Quote of the Day', good discussion topics, extra credit assignments, and many other mathematically rich themes.

"The eye sees only what the mind is prepared to comprehend."
– Henri L. Bergson

"Each problem that I solved became a rule which served afterwards to solve other problems."
– René Descartes

"Most of the fundamental ideas of science are essentially simple, and may, as a rule, be expressed in a language comprehensible to everyone."
– Albert Einstein

"Mathematics is the lantern by which what before was dimly visible now looms up in firm, bold outlines. The old phantasmagoria disappear. We see better. We also see further."
– Irving Fisher

"The great book of nature can be read only by those who know the language in which it was written. And this language is mathematics."
– Galileo Galilei

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“Mathematics is the abstract key which turns the lock of the physical universe.”

– John Polkinhorne

“Even if you’re not going to make heavy use of math in your profession, you still need to know how to think logically.”
– Marilyn vos Savant

“As we move into the era of an information-based world, the value of mathematics as a former of our culture is going to become greater and greater.”

– Kevin Kelly, writer and publisher

“Once I realized that math was just a tool, it suddenly opened a whole world for me. It allows me to be creative and expressive in whatever domain I want.”

– Pattie Maes, computer scientist

“It’s remarkable to discover how many connections mathematics makes, how many lightbulbs it suddenly turns on.”
– Sylvia Spengler, biologist

“Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country.”
– David Hilbert

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Suppose a child wrote:

$$2^4 = 8$$

List what this tells you about this child's knowledge and understanding. What does the child know and not know?

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Now suppose the child had been asked to write or explain the meaning of 2^4, and had written this response:

> 2^4 is an example of an exponent.
> The two is called the "base" and the four is the "exponent." The exponent tells us how many times to multiply the base by itself.
> It means the two is multiplied four times like this:

$$2 \times 2 \times 2 = 8$$

Now take a minute to explain what this child knows and understands.

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"Answers alone often fail to reveal the nature of a student's thinking, the strategies used in the problem-solving process, or the level of understanding."

– Larry Buschman

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Reasons to Incorporate Language in a Mathematics Class

Just as computers use an operating system to function, language is the operating system of our brains.

Simply stated, to speak or write well is to think well.

Both oral and written language are effective in increasing depth of understanding.

It is probably a much better investment of a teacher's time to look at a writing sample as an assessment than to look at a homework paper.

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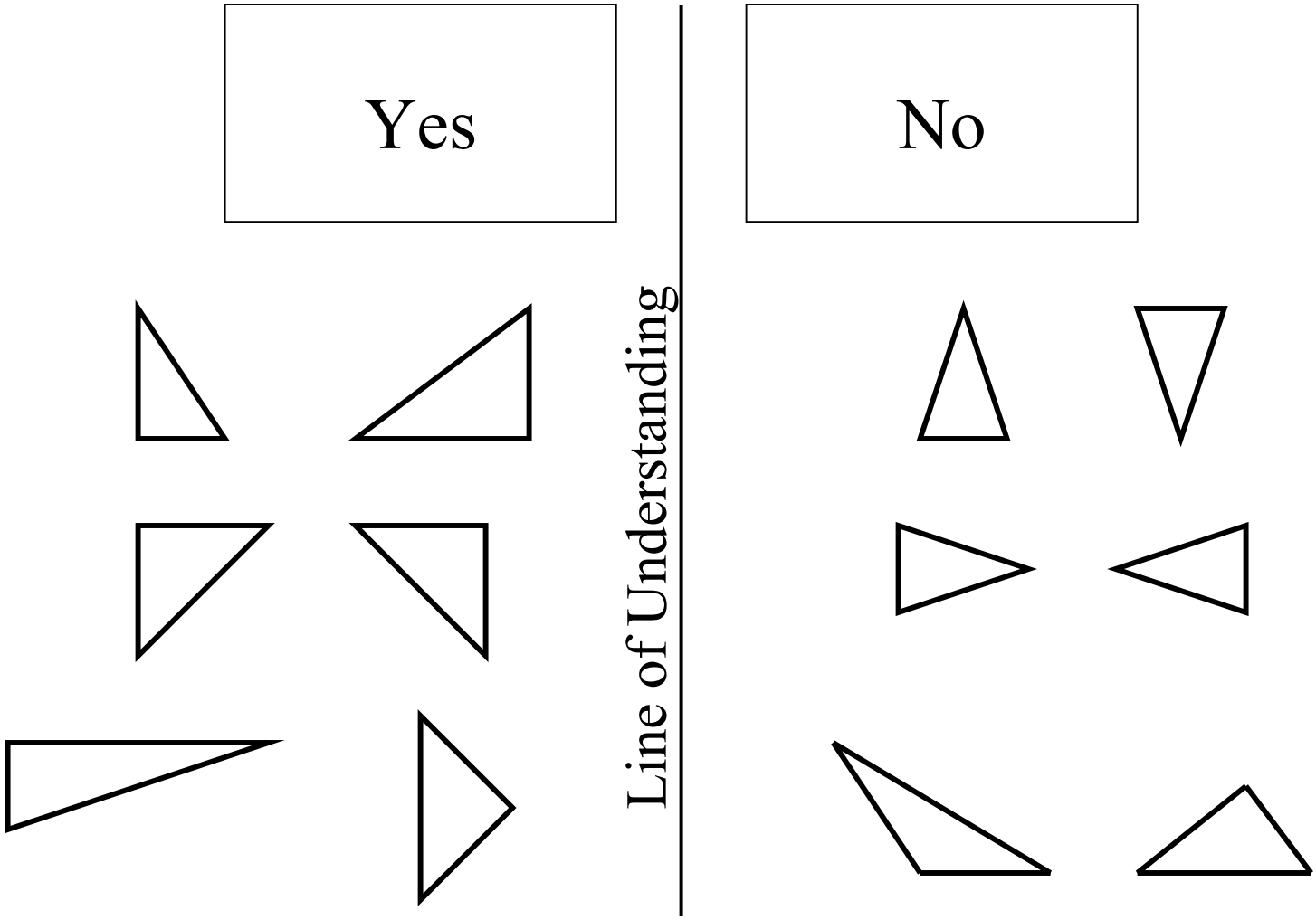
Building the Line of Understanding



Research shows that using this technique allows students to clearly see where the line of separation is, and results in large gains in comprehension and computation skills.

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The Power of the D.A.E.O.P.

Due At the End Of Period

Teachers of mathematics must flesh out concepts by good examples. The DAEOP is used to verify concept and procedure understanding before students leave the classroom.

- ✓ Good for written process
- ✓ Students show all needed steps
- ✓ Students see teacher with their solutions
- ✓ If correct, students get stamp of approval
- ✓ If incorrect, students must find the mistake
- ✓ Causes students to focus on their work
- ✓ Best feedback comes from students themselves

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Promoting Good Discussions

Strategies to include more students in discussions:

- ✓ State the question
- ✓ Pause for 5 (or more) seconds
- ✓ Ignore hands raised for awhile
- ✓ Call on the student of your choice
- ✓ Avoid repeating what the student said
- ✓ Then call on other students
- ✓ Ask students to paraphrase

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Thoughts on Question Methodology

After a student has offered a suggestion, refer to it by the student's name. Then ask if anyone else had a similar idea.

- ✓ Did anyone else solve the problem Sarah's way?
- ✓ Who found a different way to solve the problem?
- ✓ Encourage divergent thinking that enhances good discussion.
- ✓ Communicating respect for student thinking will cause students to feel safe in expressing their thoughts.

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Asking a Question in the Right Way

Key Elements of a Good Question

- ✓ Audience
- ✓ Voice
- ✓ Format
- ✓ Specific Topic

"Tell me what you know about fractions"
or
"Make two lists, one list of the skills you understand about adding fractions, and one list of the things you need to study more."

Simply including these four elements in your question is all that is needed to promote effective responses.

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Examples - Four Key Elements

"As a worker bee, write a letter to the queen explaining why the hexagon is the best shape for the hive cells."

"Pretend you are the teacher and write a note to a parent explaining what we are covering in class this week."

"Write a dialog between two students one of whom was absent explaining what we learned in class today."

"Write an instruction manual for young children telling them how to draw a parallelogram."

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Format, Voice, Audience Ideas

Write a (format) from (voice) to (audience) about (specific topic)

Format	Voice	Audience
note	teacher	younger student
paragraph	younger student	sibling
song	older student	the President
list	alien	employee
award	architect	customer
dialogue	parent	boss
speech	principal	teacher
poem	news reporter	the PTA

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Oral or Written Language?

Written:	Oral:
♦ More formal	♦ Less formal
♦ Higher-level thinking	♦ High-level thinking
♦ Deeper understanding	♦ Deep understanding
♦ Audience of one	♦ Audience of many
♦ Time intensive	♦ Requires less time
♦ Can be refined	♦ Usually not edited
♦ More precise	♦ Less accurate
♦ Management issues arise	♦ More easily managed

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Language in the Math Classroom
Journal prompts

- ✓ Language takes students to a deeper level
- ✓ To speak or to write well is to think well
- ✓ Language shows us what students are thinking
- ✓ Language offers realistic and accurate assessment
- ✓ The English teachers will like you
- ✓ Promotes communication
- ✓ Journal prompts are easily created
- ✓ Helps teachers correct misunderstandings
- ✓ Brain uses language to process information

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"Reading makes a full man.
Speaking makes a ready man.
Writing makes an exact man."

– from Francis Bacon

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Grading Math Journals versus Correcting Homework

Let's face it, correcting homework papers is boring. There are even more reasons why this task is not the best use of our time.

- Wrong answers don't tell teachers what students don't understand.
- Correct answers don't tell what students ***do*** understand.
- Some students use calculators, and others don't.
- Some parents help with homework, others don't.
- Students often copy homework.
- Dogs eat homework.

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Why Language Matters in the Math Classroom

1

Aside from the fact that the English department will appreciate it, there are numerous good reasons why we should encourage more use of language in our math classrooms. The first is that written and oral language allow us to know what a student is thinking. Quite often when a student misses a math problem, we may not understand what it is that he or she does not understand. In fact, the student may understand the concept perfectly, but has made a casual error. One example would be if we asked a child to find the value of 2^4. The child may write:

$$2^4 = 8$$

Before reading further, take a moment to list what you believe the student may or may not understand about exponents.

At first it may seem that the child does not understand exponents at all. The student may believe that you are supposed to multiply the base by the exponent. Although this is a common misconception, the incorrect answer only tells you one thing–the child wrote the wrong answer. It tells us nothing about the child's level of understanding. However, if we ask the child, "*Explain* the meaning of 2^4." The child could write:

> *2^4 is an example of an exponent. The two is called the base, and the four is the exponent. It means two multiplied four times. That is 2 x 2 x 2 which is eight.*

We can see that the student has an excellent understanding of what exponents are and how they work along with a familiarity of the vocabulary involved. In fact the only error the child made was a simple miscounting. Had we not asked for a written or oral explanation to this problem, we would have assumed the child needed much more instruction in this concept. Language is the most accurate and reliable assessment.

"Answers alone often fail to reveal the nature of a student's thinking, the strategies used in the problem-solving process, or the level of understanding."
Larry Buschman

Language also takes a student to deeper levels of understanding. We often hear teachers say, "I really understood algebra when I had to teach it." This is true of many concepts in mathematics and other curricular areas. This occurs because using language to explain mathematics to another person requires us to both formalize and refine our understanding of it. The younger the learner or the more the learner struggles in math, the more sophisticated and detailed our teaching and consequently our understanding must be. While in college, I watched a Chinese man tutor students in mathematics. When he tutored calculus, he had no problem getting his point across to the student. When a student needed help in more basic arithmetic, however, the man's mastery of the English language was not sufficient for the two to communicate.

That is why I often ask my students, "Assume I really don't understand this concept. Pretend I just moved here from another school and math

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is difficult for me. How would you explain this concept to me so I can understand it?" When I do this, I can see the sophistication of the students' understanding develop. Both oral and written language are effective in increasing depth of understanding. Because it is more formal, written language is more effective than speech, but both are valuable tools for the teacher. If oral language is being used, frequently ask the student for clarification. When the student refers to a quotient as a "thing," ask him or her to be more specific.

"Brain-based education involves ensuring that students process experiences in such a way as to increase the extraction of meaning."
Renate Nummela Caine and Geoffrey Caine

If we do not ask students to use language to verify their understanding, we will be unable to differentiate between those who truly get it, and those who have simply memorized the steps. Research has shown, and our own good intuition as teachers verifies, that if students lack conceptual understanding of a subject, all they can do is memorize information. However, memorization of information that lacks meaning is very short term.

It has been said there are three levels of understanding. The first is when you are taught something. The second comes when you have to teach it to another. The deepest level of understanding occurs when you have to try to get the point across to a computer. Computers are the most reluctant of learners in that they don't make any assumptions but do exactly what they are told. Math journals and mathematical discussions provide students with the forum to begin to move through these deeper levels of understanding.

This leads us to the foremost reason for incorporating language in a mathematics class. Whether your brain is in an English class, a history class, an art class, or a math class, it uses language to process the information. Just as computers use an operating system to function, language is the operating system of our brains. Simply stated*, to speak or write well is to think well*. A prime example can be found in the famous story of Helen Keller. Her thinking was rudimentary and undeveloped until her teacher, Annie Sullivan, put into her hand that first word. In that moment, she had handed Helen language and the ability to think effectively. Although your brain can do many things, its primary mode of operation is through language.

"Reading makes a full man. Speaking makes a ready man. Writing makes an exact man."
Francis Bacon

Interestingly, the history of algebra moved through three distinct stages. The first stage could be called the linguistic stage. During this time period, algebra began to appear in the form of essays by mathematicians. Letters were used to make words, not as symbols for numbers. For example, a mathematician might write a paper explaining why the sum of two odd numbers always had to result in an even number. It might look something like this:

> *All odd numbers follow even numbers. Thus it can be said that an odd number is an even number plus one. If we are to add two odd numbers, we are adding two even numbers and two ones. Since all even numbers represent sets of numbers that can*

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be formed into pairs, both even numbers can be used to create pairs. The two ones can also form a pair. Thus the sum of the two odd numbers will form a set of pairs and is an even number.

Much later, as in hundreds of years, letters were used to temporarily hold the place of an unknown number. It was always the purpose of the task to find the value of this mysterious letter as when we solve equations.

$$3x + 23 = 80$$
$$3x + 23 - 23 = 80 - 23$$
$$3x = 57$$
$$3x/3 = 57/3$$
$$x = 19$$

It was much later again that letters began to represent variables. Here a letter could be taken to be *any* value. An odd number could be defined by the formula $2n + 1$ where n is any integer.

There is research that suggests that students learn algebra in a sequence that reflects this historical development. Notice that the first stage involved the preeminent use of language.

There are other reasons to encourage the use of language in the classroom. One reason is simply to allow students the chance to talk. Sometimes I ask the students, to write a note completing this idea: "Something I have wanted to talk to you about for some time is…"

This writing prompt has helped students communicate problems with the curriculum, with the classroom or fellow classmates, or even with myself. Although I don't promise them I can fix all the problems, I believe they were as glad to be listened to as I was to listen. This is not a mathematical prompt, but it does improve the classroom climate and student management, making my teaching of math go a little easier.

Consider this fact too: many of us put a great deal of emphasis upon assessing homework. Yet how accurately do the answers to a homework assignment reflect what a child knows? Some students use calculators on their homework and some do not. Some parents help their children with the assignment while other students don't seek such help. And of course, some students copy homework. Is this to say we should not grade homework? Not at all. It is worthwhile to give students credit for *completing* homework assignments as this fosters a work ethic. However, it may not be a good use of time to invest hours of your time in correcting the answers under the assumption that it is an accurate assessment. It is probably a much better investment of a teacher's time to look at a writing sample as an assessment than to look at a homework paper. A subsequent section will deal with ways to do this without creating an additional workload for the teacher.

"The development of a student's power to use mathematics…is best accomplished in problem situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural."

Margaret E. McIntosh

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To Write or To Speak?

2

One of the questions that must be asked is whether to focus on written language or to encourage discussion instead. What are the benefits of each? Should both mediums be used? These are important questions, and the answers will vary from classroom to classroom and from teacher to teacher depending upon your goals. However, let's consider some of the advantages and disadvantages of each.

Because written language is more formal and requires more thought, it fosters deeper understanding than oral language, and if that were the only factor, the choice would be easy. But oral language is easier for the teacher to manage to manage than written language. This is not an insurmountable problem, and a future section will be dedicated to simple ways to read, score, and record written work.

"Using oral or written communication as a tool with which students can reflect their understanding of mathematics helps them make connections and personalize mathematics concepts."
Larry Buschman

One of the advantages of oral language is that during a discussion, the whole class becomes the audience and can engage in the thinking of the speaker. This is a significant benefit over written work in which the only audience is the teacher. However, even written work can be shared orally. Interestingly, many students who are asked to share their writing with the class will paraphrase their writing. They do this because they see that their written work may not have fully captured their thinking or may have errors. Asking them to read their work word for word will encourage them to edit and improve their writing.

There are other ways to increase the audience of written work. You can ask students to share their writing in small groups of students. Another option is to have students exchange papers and write a commentary on the first student's writing. A noticeable advantage of the oral language model is that the thinking of one student will often spur another student. This may not happen with written work unless it is shared.

As you can see, both formats have their advantages: oral language boasts a larger audience; written work promotes more sophisticated thinking. You may wish to incorporate both mediums in your classroom, emphasizing one more than the other. Keep in mind the advantages of each by first considering your goals. Do you wish to help students really cement their understanding? A journal write might be the best choice. Are you trying to get students to look at new ways of considering a concept? A discussion would serve the purpose best here. Remember that using oral and written language is like painting a room. Sometimes a roller is used to cover a wider area, and at other times a fine brush is used for detail work. Oral language addresses a wider audience than the finer quality we get from written work.

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Promoting Good Discussions

3

The strategies that a teacher employs to encourage students to *speak* mathematically will do more for improving the quality of mathematical *thinking* than any other single factor. The elements that make this happen are simple habits any teacher can develop. There are a few strategies and a few specific questions that can be easily learned and quickly applied.

Often we teachers have developed the habit of calling a student's name and then asking that student the question. This is the opposite of what is most effective. When a student's name is called, other students can relax since they realize the thinking task does not involve them. We want every student engaged in the thinking process. The best way to do this is to first state the question. Then pause for five seconds. Mentally count these seconds to give every student a chance to begin thinking. During this time, which will seem like an eternity, many hands will go up. Ignore them for now. Then call on a student of your choice (even if that student's hand is not raised) for the answer. After that student has responded, feel free to call on others who had raised their hand. If we get in the habit of calling on only students who raise their hands, we know that discussions will engage only five or six eager students, and those who need to participate in discussions the most will fade quietly into obscurity.

"A better understanding of students' thinking in mathematics is needed if mathematics education is to be improved."
Karen S. Norwood and Glenda Carter

Create a climate that fosters discussion by giving students this valuable think time before calling on a respondent. For more intensive questions, more than five seconds of wait time may be required. I have sometimes told students, "I'm going to give you 60 seconds to think about this question. Don't discuss it with anyone else until the time is up. Then you can discuss it with a partner or with the class." Sometimes I find that after five seconds a student is still not ready. I ask them if they want more time to think about the question. If they do, I simply tell them I'll check back with them, then I pose a different question for the class to consider and provide wait time for it. After that question has been handled, I go back to the original student. My students have learned that it's okay to say, "I don't know. I need more time," but it isn't okay to say, "Um…huh?…What's the question?"

When a student responds, there are some rules to follow which will help develop the climate of mathematical discussion you want to create. Try to avoid repeating what the student says. This inadvertently communicates to other students that it's not necessary to listen to the student–the teacher will repeat it if it was important. A better plan is to ask another student to paraphrase the idea of the first student. This is a difficult task for most students, but their skill will improve with time. Also, this request helps other students realize the need to listen attentively to classmates just as they would for the teacher.

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I try to call on every student every day in no particular order. My students know that being called upon once does not mean you are finished answering questions. As nearly as possible, I try to make my selections randomly. It is just as important to do everything possible to ensure that a student is capable of answering the question. For this reason, I try to avoid calling on students merely for the purpose of pointing out that they aren't paying attention. If the question is one that many students may find difficult, I make sure to say something like, "I'm going to ask a difficult question, and I'm interested to know your thoughts about this. Make sure you listen carefully, and ask me for clarification if you need it."

"Mathematics is the science which uses easy words for hard ideas."
Edward Kasner and James Newman

There are a couple of related strategies that are helpful. After a student has offered a suggestion, such as a method of solving a problem, refer to it by the student's name. Then ask if anyone else had a similar idea. For example, you might say, "Did anyone else solve the problem Sarah's way?" This validates Sarah's thinking and also shows who did and did not use such a strategy. Then ask, "Who found a different way to solve the problem?" Suggesting the importance of other ways to solve problems encourages the divergence of thinking that will enhance good discussion. These solutions also can be labeled with the name of the student who offers them. Soon you will notice students referring to Ray's method, or Rashawn's procedure.

To summarize, here are strategies that will foster mathematical discussions in the classroom. This is not an exhaustive list, and you will likely discover more tricks that help your students.

- Use wait time of at least five seconds.
- Call on as many students as possible.
- Avoid repeating a student's words
- Ask fellow students to paraphrase.
- Encourage students to respond to one another instead of only to the teacher.
- Ask for alternate ways to solve problems.
- Label thinking with the name of the student.

Overall the goal is simple: we are trying to communicate respect for a child's thinking in all we say and do. Accomplish this, and students will see that you value their ideas and will feel safe in expressing them.

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Quick Questions That Promote Good Discussions

- How did you solve the problem?
- Did anyone else solve the problem this way?
- Can you solve the problem a different way?
- Who solved it a different way?
- Is this problem similar to another you solved?
- Can you write a rule or formula for your problem?
- What patterns do you notice?
- Does that always work?
- How did you know that?
- Why does that work?
- Why is that true?
- Can you think of a situation in which that wouldn't work?
- Why did you decide to do it that way?
- What pattern(s) do you see?
- Do you agree of disagree with what that student said? Why?
- How would you convince me that you are right?
- Does that seem like a reasonable answer?
- Can you paraphrase what that student said?
- What information is important in this problem?

4

"Mathematics is the only universal language there is, Senator."
Jodie Foster in the film Contact

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Promoting Good Writing

5

Few strategies will do more for eliciting good thinking from students than asking a question in the right way. If a student is told, "Tell me what you know about fractions," the teacher will get vague and disjointed writing that offers little insight to what the student truly knows. Rephrasing the question will result in greatly detailed and high-level thought. Asking such questions is not a difficult task. We simply have to remember the four key elements of a good question. First of all, students are usually not told who their audience is. Most simply think they are writing this for the teacher. No great writer picks up a pen without first considering the **audience**. Thus one of the important elements of a well worded writing prompt is that it offers a unique audience.

"What humans do with the language of mathematics is to describe patterns. Mathematics is an exploratory science that seeks to understand every kind of pattern – patterns that occur in nature, patterns invented by the human mind, and even patterns created by other patterns."

Lynn Steen

Great novels never start with, "Hi. I'm an author and I have something to say." Instead, the writer creates a character or **voice** from which to speak. Although the need for this may not seem apparent at first, it separates creative and stimulating writing from the mundane. Even though the setting of the writing is a mathematics classroom, our goal is to stimulate rich thinking.

Every good writer also chooses an appropriate **format**. Will the writing be a letter? …a note? …an essay? …a poem? …a list? …an argument? The example used above, "Tell me what you know about fractions," could be improved by saying, "Make two lists, one list of the skills you understand about fractions and one list of the things you want to study more." The teacher will get much more usable information with this type of question.

Lastly, to write well, a writer needs a narrowly focused, **specific topic**. The difference in the responses between general and specific writing prompts is phenomenal. "Tell me what you know about fractions," will never generate the valuable thinking we would receive if we were to say, "Explain how to add two fractions when one is thirds and the other is fourths."

Thus the four components of a good writing prompt are audience, voice, format, and specific topic. Simply including these elements in your question is all that is needed to promote effective responses. Look at the following examples and see if you can identify the four elements.

As a worker bee, write a letter to the queen explaining why the hexagon is the best shape for the hive cells.

Pretend you are the teacher and write a note to a parent explaining what we are covering in class this week.

Write a dialog between two students one of whom was absent explaining what we learned in class today.

Write an instruction manual for young children telling them how to draw a parallelogram.

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Write directions for blind people telling them how to walk in the shape of an isosceles right triangle.

In fact, writing good prompts can be as simple as filling in the blanks. Use the following sentence as a template and substitute items from the list below to create effective prompts. Remember that for the topic, you will simply use whatever you are studying currently, but keep it focused. Ask the student to summarize a specific skill or concept but not the entire unit of study.

"When students communicate mathematical information, they remember it, understand it, and use it to uncover and find even more information. (Perkins 1992)"
Larry Buschman

Write a (format) from (voice) to (audience) about (specific topic)

Format	Voice	Audience
note	teacher	younger student
paragraph	younger student	sibling
song	older student	the President
list	alien	employee
award	architect	customer
dialogue	parent	boss
speech	principal	teacher
poem	news reporter	the PTA

Obviously these lists could go on and on and the combinations are extensive.

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If a discussion seems to be bogging down, try these sure-fire questions:

Why is that true?
Why does that work?
Does that always work?
How did you know that?
What pattern(s) do you notice?
Who solved it a different way?
How did you solve the problem?
Why did you decide to do it that way?
Does that seem like a reasonable answer?
Can you paraphrase what that student said?
Can you solve the problem a different way?
Did anyone else solve the problem this way?
Is this problem similar to another you solved?
What information is important in this problem?
How would you convince me that you are right?
Can you write a rule or formula for your problem?
Can you think of a situation in which that wouldn't work?
Do you agree or disagree with what that student said? Why?

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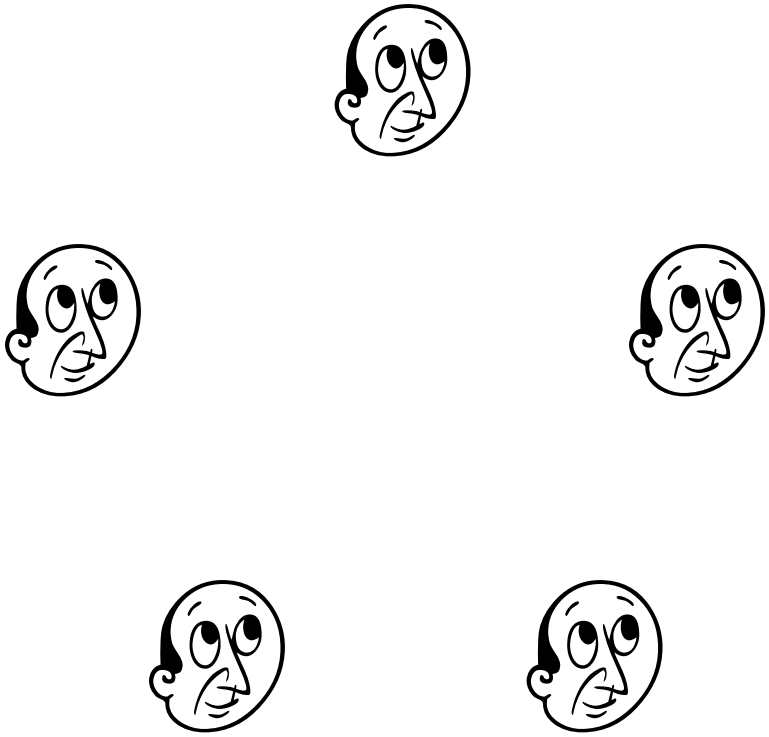
4. Five friends meet on the street and shake hands. How many handshakes will be required? Explain how you got your answer.



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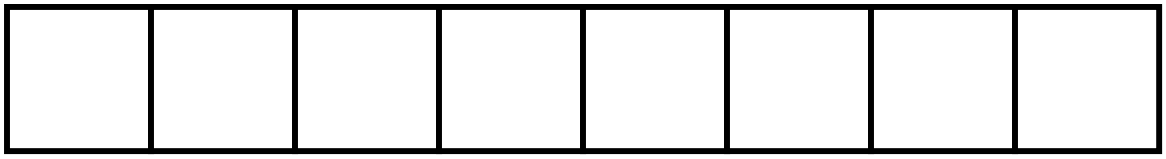
7. How many people can be seated at this arrangement of tables? People can sit on each side and both ends. What if eleven tables were used? What if 100 tables were used?



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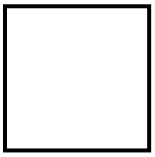
7. How many people can be seated at these tables? People can sit on each side and both ends.

1



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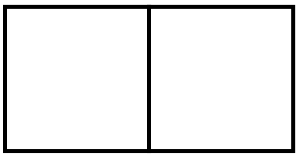

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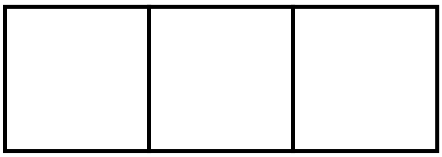

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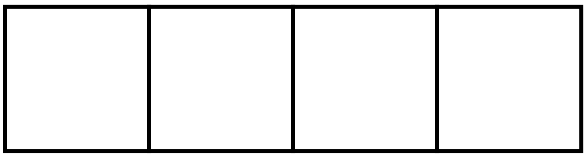

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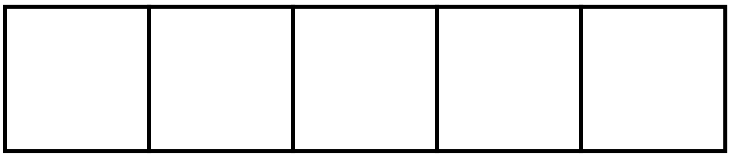

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"The development of a student's power to use mathematics . . . is best accomplished in problem situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural."

– Margaret E. McIntosh

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P. Find a fraction that is between these two fractions. Do not use a calculator, and don't use decimals. Explain how you know it is between them.

$$\frac{2}{3} \qquad ? \qquad \frac{3}{4}$$

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34. Arrange these fractions in order from least to greatest. Explain how you know you are correct.

$$\frac{2}{5} \qquad \frac{3}{4} \qquad \frac{1}{5} \qquad \frac{3}{8} \qquad \frac{1}{3}$$

These journal prompts are from our new book "*The Language of Math: Helping Students Speak, Write, and Think Mathematically*" which contains over 100 journal prompts and over 70 ready-to-use transparency masters, as well as a complete treatment on the use of oral and written language in the math classroom.

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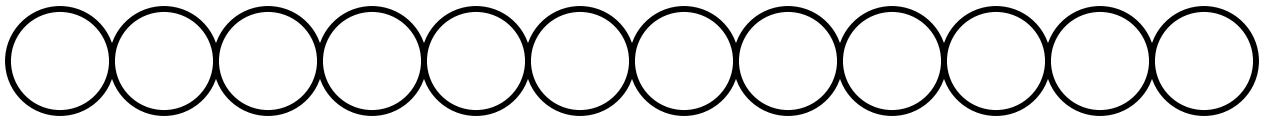
18. Twelve marbles are in a row. One third of them are yellow. There are an equal amount of red and blue. There are two less green marbles than yellow. Explain how to find out how many of each color of marble are in the row.



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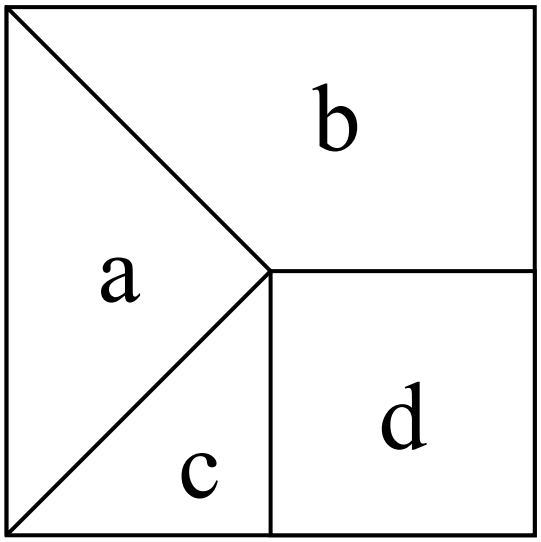
22. This tile costs $2 when it is whole. Explain how to find a reasonable price for each of the pieces.



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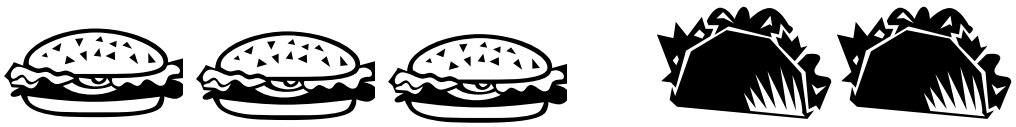
75. Sasha and Josh are at the food court in the mall. They noticed these prices. Explain how to find the price for the burger and for the taco.



$16

$14

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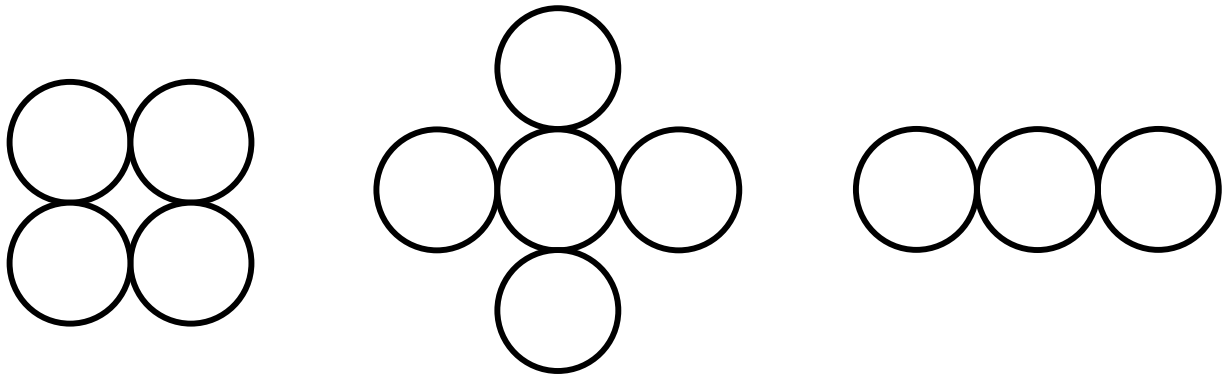
Hundreds Magic: 1 – 100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



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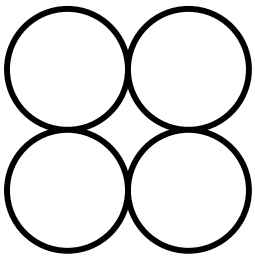
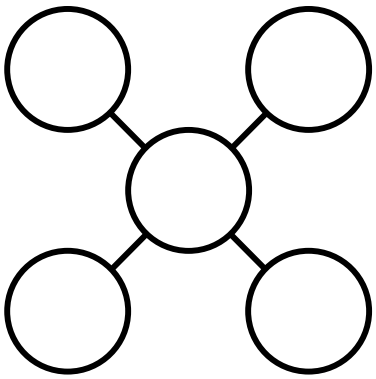
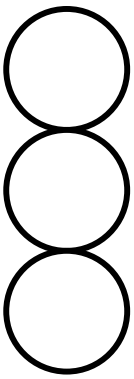
Hundreds Magic: ¯49 – 50

-49	-48	-47	-46	-45	-44	-43	-42	-41	-40
-39	-38	-37	-36	-35	-34	-33	-32	-31	-30
-29	-28	-27	-26	-25	-24	-23	-22	-21	-20
-19	-18	-17	-16	-15	-14	-13	-12	-11	-10
-9	-8	-7	-6	-5	-4	-3	-2	-1	0
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50







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Hundreds Magic

a	b
c	d

Are the differences in the diagonals always equal? Explain why it *is* or *is not* true.

a	b
c	d

These are numbers from a calendar. Are the differences in the diagonals always equal? Explain why this *is* or *is not* true.

a	b	c

Using algebra, explain why *b* is the average of *a* and *c*.

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Explorations and Journal Prompts:

For the square pattern of numbers we started with, are the differences of the diagonals the same? Why or why not? Justify algebraically.

For the horizontal three in a row pattern, multiply the largest and smallest numbers. Compare this product with the square of the middle number. Make a conclusion and justify algebraically.

Make up an arrangement of numbers other than those on the Hundreds Magic charts, and explore the patterns you find there. Justify your conclusions.

Do the patterns you have found still work when you have negative integers? Explore relationships using the Hundreds Magic chart ¯49 – 50.

Using a page from a calendar, explore these same relationships. What similarities and differences are there? Can you justify these?

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"The most important mode for students' writing about mathematics is . . . as a follow-up to a lesson. Writing is especially effective when if follows hands-on activities, since in those exercises the most active learning takes place."

Learning to Write About Mathematics,
Arithmetic Teacher,
S. Wilde

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6. Resources for Teaching Mathematics

TTTPress website www.tttpress.com

- Books
- Activities
- Newsletters
- Conference handouts
- Websites
- Quotations

ENC – Eisenhower National Clearinghouse

- Website http://www.enc.org/

NCTM – National Council of Teachers of Mathematics website http://www.nctm.org/

- Journals and conferences

CMC – California Mathematics Council

- Communicator and conferences
- Website http://www.cmc-math.org/

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A Note from Brad and Bill

Teacher to Teacher Press produces curriculum written by teachers, to teachers, and for students. We know how busy you are. In addition to the literally hundreds of duties and responsibilities you already have, you may be teaching a class without a good textbook or other source of curriculum. You simply don't have the time to find all the material you need. That's where we hope to help.

Our books are based on specific and important goals that we think you'll appreciate. First of all, we are schoolteachers. We know first hand that many texts and units are either too symbolic or solely concrete and manipulatively based. Students are in transition as they learn. They learn from concrete experiences, but they are capable of moving into the abstract. All of our materials are designed to help them make a successful transition.

Secondly, we believe that the N.C.T.M. Standards are filled with great ideas. Now it is up to those of us in the field to develop curriculum to meet these great standards.

Thirdly, we know that your time is precious. You have much to teach and only 180 days or so to do the job. Your students need more experience with a new subject than the two-page spread offered by the typical textbook. At the same time, you can't afford to spend six weeks on a unit that only covers a single topic. That is why our activities are designed to take shorter periods of time or to be broken up into smaller increments of time. Sure, you can spend more time on an activity or unit if you have some spare days, and your students want it–we offer plenty of extensions–but our activities are designed to be finished before your students become bored.

Lastly, you didn't go to college to spend your life in front of a copy machine, or to chase down the obscure supplies required for teaching some units. That is why our units and their materials lists are simple. We teach what we write, and if it isn't simple and effective, we find something that is.

If you are interested in our other units, books, materials, workshops or inservices, you may contact the authors at this address:

Teacher to Teacher Press
P.O. Box 233, Millville, CA, 96062
Phone: (530) 547-4687 Fax: (530) 547-4317
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Brad and Bill

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Bill Lombard

Teacher to Teacher Press

"Building Mathematical Skill on a Foundation of Understanding"

- **Consultant**
- **Educator**
- **Author**
- **Seminar leader**
- **Teacher trainer**
- **Conference speaker**

Bill Lombard

5885 Avery Way, Redding, CA 96003
(530) 243-2064 bill@tttpress.com

Known throughout the country for motivating and engaging teachers and students, Bill has co-authored over ten books that provide easy-to-teach yet mathematically rich activities for busy teachers. In addition, he has co-authored six teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

Seminar leader and trainer of mathematics teachers

- California Math Council featured speaker, NCSM, and NCTM national conference presenter
- Lead trainer for summer teacher training institutes
- Trainer/consultant for district, county, regional, and national workshops

Author and co-author of mathematics curriculum

- *Simply Great Math Activities* series: five books covering all major strands
- *Math Discoveries* series: four books bringing math alive for students in middle schools
- Teacher training seminar handbooks for elementary, middle, and secondary teachers

Workshops, conference sessions, and keynote addresses

All workshops provide participants with complete and ready-to-use activities. These activities require minimal preparation, use materials commonly found in classrooms, and give clear and specific directions and format. Participants will also receive journal prompts, homework suggestions, and ideas for extensions and assessment.

"Excellent…relevant…useful…practical…engaging."
Tizoc Tirado, kindergarten teacher

"I have learned more about math in these five days than in all my career."
Annie Jackson, 5th grade teacher

"Brad and Bill's math activities are the best I've seen in 30 years of teaching!"
Wayne Dequer, 7th grade math teacher

"The best and most practical and relevant workshop I have ever attended. Thank You!"
Leslie Lowman, high school teacher

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TESTIMONIALS FROM TEACHERS

"*Brad and Bill's math activities are the best I've seen in 30 years of teaching middle school! These activities are engaging and meaningful but simple to use. They are invaluable in infusing algebraic concepts and processes into lower grades in preparing all students for the demands of formal algebra. Many other math teachers at my school site are finding them equally useful*."
– Wayne Dequer, 7th grade math teacher, Arcadia, CA

"*The most informational workshop I have ever attended. This is a workshop everyone should attend. There is information available for use in any grade or classroom."*
– Melba VanPelt, RSP Special Education, Palmdale, CA

"*A fun and innovative way to teach math. I have used Brad and Bill's activities with middle school and high school students with great success. I have also presented them to teachers in workshops and in an algebra standards course at the college level as an excellent way to actively involve students in learning important algebra concepts*."
– Terry L. Nevin, Algebra Teacher/El Dorado High School, Placentia, CA

"*If we want students to engage in mathematics, we need to have engaging, rigorous curriculum to share with them. These units do that better than anything else I have ever used. The activities can be modified to use at many grade levels to introduce or review important mathematical concepts. Students love the activities, and because of the connections to the world outside of school, parents are impressed with what their students are learning to do*."
– Judy Ferrarri, 4th/5th grade teacher, Springfield, OR

"*Your methods should be taught to all math teachers and in all math books!! What a wonderful way to teach outside the box*."
– Nancy Heim, Adult Education, Dana Point, CA

"The content was great! Very captivating and energetic. The whole staff was very involved with the activities."
– Michelle Wehrhan, Title 1 Teacher, Santa Ana, CA

"Absolutely loved the step-by-step process of leading the students to complex algebraic processes while providing a deep understanding for the students at every step."
– Kathy Meiser, 4th grade/GATE, Palmdale, CA

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TESTIMONIALS FROM ADMINISTRATORS

"*Teacher To Teacher Press materials and program workshops have provided Orange County teachers with a rich and important set of resources, models, and instructional tools in mathematics. These resources have been successfully applied and used to boost teachers' and students' understanding of mathematics. The use of these materials, coupled with our ongoing staff development program, has positively impacted student learning. The Teacher to Teacher Press materials and presenters are both adept and talented in their approach, understanding, and use of modeling to facilitate concept development. These program materials continue to provide a rich resource of ideas and strategies for the classroom teacher.*"

Ed Rodevich
Coordinator, Math/Science
Orange County Department of Education
Costa Mesa, California

"*I am a mathematics consultant with the Los Angeles County Office of Education. We have had Brad Fulton present numerous times to many elementary, middle, and high school teachers who provide mathematics instruction in Los Angeles County. Brad has presented workshops in all the strands, and on a regular weekday he commands an audience close to 200 participants. Many of them are second or third time attendees who want more of his strategies to make math accessible.*

These strategies are taken from the eight books written by Brad and his partner, Bill Lombard, and published by Teacher to Teacher Press. From their partnership, innovative activities have been developed and extended into week-long explorations to give students an opportunity to deepen their mathematics. At the same time, teachers work on these activities to increase their own mathematics content knowledge at Brad and Bill's workshops. Used with research-based lesson plans, these strategies support the California Mathematics Content Standards and help to strengthen conceptual understanding and mathematical reasoning while increasing students' motivation. Brad Fulton and Bill Lombard have provided materials and methodologies, and have helped both teachers and students in Los Angeles County strengthen and enjoy their mathematics learning."

Yuki Mio
Mathematics Consultant
Los Angeles County Office of Education
Downey, CA 90242

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Teacher to Teacher Press

"Building Mathematical Skill on a Foundation of Understanding"

Simply Great Math Activities: Number Sense

Eleven extensive ready-to-teach and mathematically rich activities will captivate your students' interest. Many of the activities can be extended into week-long explorations. The book includes homework masters, transparency masters, journal prompts and simple directions.

Simply Great Math Activities: Fractions, Decimals, and Percents

A dozen incredible and innovative activities will captivate and educate your students. They will learn creative and clever tricks that make fractions less frightening. The book includes homework and transparency masters, journal topics, easy-to-follow directions, and much more.

Simply Great Math Activities: Algebra Readiness

These motivating activities will work for young students just beginning to work on algebra concepts, while ideas for extensions make them just as appropriate for older students in formal algebra classes. The book includes homework ideas, transparency masters, journal prompts and simple directions.

Simply Great Math Activities: Geometry Explorations

Students use geometry as a tool to explore unique mathematical situations. Area formulas, volume, surface area, compass and straightedge constructions, angle measurement, the Pythagorean Theorem and more are covered in unique ways that promote deeper understanding. Algebraic principles and fraction concepts are embedded.

More Power^2 You!

A unit that actually makes the concept of exponents and exponential growth tangible! Lead your students on a journey traveling from physical and manipulative models to graphical and symbolic formats. They will even be able to explain why $n^0 = 1$. The book includes student worksheets, homework, teacher lessons, journal prompts, and ideas for extensions.

The Language of Math: Helping Students Speak, Write, and Think Mathematically

Everything you need to incorporate oral and written language into your classroom lessons, complete with easy tips for leading rich mathematical discussions, and great ideas for **easily** managing written work. There are over 70 transparency masters, 100 journal prompts, and masters for making your own math journals. Writing and speaking mathematically has never been easier!

The Pattern and Function Connection

This three-week unit is the easy and effective way to introduce students to linear functions. Students will move from physical and manipulative models to pictorial and graphical representations, then finally to symbolic expressions for linear equations. The book includes student worksheets, homework, teacher lessons, journal prompts, and ideas for extensions.

24 Pattern Cards ..... **New Product!**

A fantastic supplement and great time saver! These full-color 11" by 17" cards are printed on heavy matte-finish card stock that is easy on the eyes and will stand up to years of student use. Students will enjoy working with the patterns so much, they won't notice how much they are learning about functions and algebra. They are a great companion to go with either of our titles: *The Pattern and Function Connection* or *Simply Great Math Activities: Algebra Readiness*.

Transparencies of Pattern Cards ..... **New Product!**

Another great addition to *The Pattern and Function Connection* or *Simply Great Math Activities: Algebra Readiness* family of products. These brilliant transparencies are full-color representations of the Pattern Cards listed above. Use them for whole-class instruction to introduce the powerful mathematics of either book. The bright and clear colors will capture your students' interest, introduce them to the beauty of patterns, and send them well on their way to algebraic learning. Can be ordered separately or at a package discount with other Pattern and Function products.

A Blueprint for Geometry

In the two- to three-week project presented here, students read and draw floor plans and elevations and compute building costs. As they solve the problems inherent in drawing house plans, students learn measurement, scale, and architectural symbols. Students also use their problem-solving skills and creativity. The book contains blackline masters for 17 plans and homework sheets, as well as student outcomes, vocabulary, and procedures.

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Teacher to Teacher Press

"Building Mathematical Skill On a Foundation of Understanding"

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_____	Simply Great Math Activities: Fractions, Decimals, & Percents	$16.50	_____
_____	Simply Great Math Activities: Algebra Readiness	$16.50	_____
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_____	More Power^2 You!	$16.50	_____
_____	The Language of Math	$16.50	_____
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_____	24 Full–Color Double–page Pattern Cards ***New Product***	$39.00	
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_____	A Blueprint for Geometry	$21.00	_____
		Subtotal	_____
		Please add 10% shipping and handling per order	_____
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