

## MODERATOR'S GUIDE

This document serves as a guide for the educator who wishes to conduct a Teacher to Teacher Press workshop featuring Bill Lombard and Brad Fulton. While you can't replace the interactive nature of a live workshop, this DVD format provides some distinct advantages for your school or district. By purchasing these DVD's you own this workshop and can conduct it over and over. As you hire new staff, they can benefit from the same training your veteran teachers have enjoyed. These DVD's are also inexpensive when you compare them to hiring a consultant. You can also take as much time as you want to proceed through the series. You may wish to view five or so of them in one day. On the other hand, some districts find these to be great assets for department meetings and collaboration time. It's your choice, and we encourage you to adapt these to fit the needs of your site and your staff.

Here is the format we suggest for viewing and using the materials.

1. First, we suggest that you, the moderator, view the DVD before showing it to the rest of your staff. This will help you direct discussions and planning time in the most effective manner possible
2. Prior to the workshop, print out any handouts on the CD so these will be ready for your teachers. You have purchased the rights to make as many copies as needed to distribute to the educators at your site or district.
3. When your staff gathers to view the DVD, begin with a discussion. List concerns and issues that your students face in dealing with the mathematical topic that will be addressed.
4. Watch the DVD, and then discuss the accompanying questions. This can be done informally or formally and in oral or written formats.
5. If you wish, allow planning time for your staff so they can discuss the implementation of these activities and teaching strategies.

If you follow this model, you will probably get through about five DVD's in a day of staff development. Each viewing, discussion, and planning cycle will take approximately one hour. Some will longer and some will be shorter.

And remember, Bill and Brad are always available if you have questions. You can reach us at these phone numbers and email address:

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Happy teaching,

*Brad and Bill*

## FAST FACTS AND FRACTIONS

Initial discussion:

1. What are the issues that our students face with their multiplication facts?
2. What factors cause them to struggle?
3. What have we tried so far?
4. What are the issues that confront them at each grade level regarding fraction, decimal, and percent representations?
5. How have we taught this in the past?
6. What worked well, and what didn't work?

Post-viewing discussion:

1. Which patterns in the multiplication table were new to you?
2. Are there other patterns you can suggest that will help students learn these facts?
3. How can we best implement this strategy for teaching the multiplication facts?
4. Can we let our students use grid paper as scratch paper for the state test so they can make their own multiplication table? (Check with your state department of education. In California, grid paper can be used.)
5. Which fraction, decimal, and percent strategies will work best with our students?
6. How will these be implemented at different grade levels? What are the standards for these concepts at each grade level? (Consult your state framework for this answer.)
7. How do we balance procedural teaching with conceptual approaches on fractions, decimals, and percents?

If desired, allow educators collaboration time to schedule the implementation strategies.

## HUNDREDS MAGIC

Initial discussion:

1. What algebra skills do students possess at each grade level currently?
2. What obstacles hinder their algebraic thinking?
3. How is algebra currently introduced?
4. What precursor skills are needed for later algebraic success?
5. At what grade level are the precursor algebraic skills introduced?

Post-viewing discussion:

1. Hundreds Magic has three levels of implementation:
  - a. Discovery of number patterns
  - b. Linguistic approach to rationalizing why these patterns exist.
  - c. Algebraic proofs showing why these patterns work.

How should this *conceptual layering* be distributed across the grade levels? (“Conceptual layering” refers to the incremental development of a concept so that all students can get enter the lesson and be taken up to an appropriate level of mathematical rigor. You might think of it as slowing down the train so all passengers can board safely and gradually taking them up to speed without losing passengers along the way.)

2. What arithmetic skills can be reinforced by exploring patterns in the hundreds chart?
3. Which chart is most appropriate for your students?
4. What trick or pattern most intrigues you? Which one will be most appropriate for your students?
5. Students will often be so engaged in this exploration that they discover tricks and patterns of their own. What algebra tricks can you find in the chart?
6. Can you explain linguistically and prove algebraically why your trick works?

If desired, allow educators collaboration time to schedule the implementation strategies. They may also want time to discover patterns that are appropriate the their grade level assignment.

## INTEGER STRATEGIES

Initial discussion:

1. What integer standards are taught at each grade level?
2. What percent of the state test requires proficiency with integers?
3. What struggles do students face with each of these concepts?
4. How are integers presented conceptually in our school?
5. What procedures have we tried?
6. What successes have we had with those procedures?
7. How do the separate integer procedures overlap in a student's thinking to cause problems? For example, many students apply the "two negatives make a positive" rule to all integer operations.

Post-viewing discussion:

1. What are the pitfalls of the chips approach to teaching integer operations?
2. Which strategies will be most beneficial to your students?
3. What are the strengths and weaknesses of each approach?
4. Which are most conceptual?
5. Which strategies are more procedural?
6. How will these be distributed among grade levels at your site?
7. Will you present one of these strategies to your students, or will you lean toward an eclectic blend that allows students to choose their preferred strategy?
8. Some of the strategies will appeal to visual and kinesthetic learners. Other will appeal more to students who tend to recognize patterns and favor rules and procedures. Can you identify students in your classes who will likely choose one strategy over another?

If desired, allow educators collaboration time to schedule the implementation strategies.

## LEO'S PATTERNS

Initial discussion:

1. What knowledge do your students currently have about the Fibonacci sequence?
2. What types of numbers or expressions are added and subtracted at your grade level: positive whole numbers, decimals, fractions, integers, monomials and binomials, or something else?
3. What struggles do students face with these addition and subtraction concepts?
4. What is your state or districts belief about teaching the “Guess and Check” strategy of problem solving?
5. How do students encounter practice in solving equations at your school or district?

Post-viewing discussion:

1. This activity is another example of *conceptual layering*. (Refer to “Hundreds Magic” for a definition of this term.) How are students gradually led from the simple task of adding  $1 + 1$  to solving equations in “Leo’s Patterns”?
2. Where would you begin this activity with your students? What types of numbers or expressions would you use?
3. Where would you exit this lesson for your students? How far would you take them?
4. How might your students view subtraction in the model where the sequence is begun on the right and developed toward the left? Would they think of this as subtraction or as a missing addend model?
5. Which of those approaches might be more intuitive for them?
6. What are the advantages and disadvantages of these two approaches?
7. What are your thoughts about using an answer bank with the worksheets? Do you think this would help or hinder your students’ learning?

If desired, allow educators collaboration time to schedule the implementation strategies.

## MATH PROJECTS

Initial discussion:

1. What projects are students currently doing in their math classes?
2. What role does art and visual representation play in our math curriculum?
3. What would be the advantages of such an approach.

Post-viewing discussion:

1. How do these projects illustrate high-level mathematical thinking?
2. How do they function as a teaching tool?
3. How do they function as an assessment tool?
4. How can these be incorporated into your teaching schedule?
5. Which projects are best suited to your students?
6. How will you address the issue of finding time to do them?
7. How will you address materials management?
8. Many of the projects shown to you in this two DVD set were made by students who had serious obstacles in their math education. Some were in the process of learning English; some didn't receive an 8<sup>th</sup> grade diploma; some had struggles at home or other issues that affected their education. On the other hand, some of these projects were made by our top students and our class president. The beautiful thing is that you can't tell one student from another by their work. What does this mean from an educational perspective?
9. It could be argued that you may not have the time to do projects. It could also be argued that we can't afford not to do them, given the quality work students do when allowed to express their learning this way. How was this issue addressed in the DVD?
10. Math projects allow students to demonstrate understanding in new and advanced ways. We call this *the emancipation of learning*. How do projects set students free to truly show what they know?

If desired, allow educators collaboration time to schedule the implementation strategies. Teachers can also use this time to think of ways to create new projects geared to the standards they teach at their grade level.

## MENU MATH

Initial discussion:

1. What algebraic concepts are addressed at the various grade levels?
2. What concepts are mandated by the state at each grade level?
3. How have we taught these concepts in the past?
4. What successes and failures have students experienced?
5. What struggles do they face in learning these algebraic concepts.

Post-viewing discussion:

1. Which of these activities is best suited to the students you teach?
2. How can these activities be adapted for upper grade students?
3. How can they be adapted for lower grade students? For example, the “Al Jibber’s Pet Store” approach has been used successfully with second graders. The prices vary, but the concepts taught are identical.
4. Once you introduce these lessons, where will you go next with your students?
5. How will you transition students from this intuitive approach to the way these concepts are presented in your textbook or on your state test?
6. Think of these activities as a foundation upon which we build a structure of algebraic thinking. As we compare this teaching model to the building of a structure, what role does the foundation play?
7. What extensions can you suggest for each of the seven activities modeled in this DVD?
8. Can you think of other algebra standards that can be introduced and taught with this menu model? For example, if we want to study linear functions, we can plot the cost of one hamburger and a medium soda, two hamburgers and a medium soda, three hamburgers and a medium soda, and so on. The slope of this function is the cost of the hamburger and the y-intercept is the cost of the medium soda. If we substitute cheeseburgers for the hamburgers, the slope increases. If we substitute a large soda for the medium one, the y-intercept increases.

If desired, allow educators collaboration time to schedule the implementation strategies.

## MULTIPLYING AND FACTORING POLYNOMIALS

Initial discussion:

1. What struggles do students face when learning to operate with polynomials?
2. To what degree are multiplying and factoring polynomials tested in your state?
3. How do you currently teach these concepts, and what do you do to foster connections between multiplying and factoring?
4. How is factoring (of composite numbers) taught at the grade levels that precede your own? For example,  $24 = 2 \times 2 \times 2 \times 3$
5. What future algebraic concepts are contingent upon an understanding of and fluency with multiplying and factoring polynomials?

Post-viewing discussion:

1. How do the methods in this DVD compare to your current method of instruction on multiplying and factoring polynomials?
2. Describe how the individual components of this process connect to one another.
3. What purpose does teaching students lattice and area multiplication provide? Would these steps be necessary with your students?
4. Where would you begin this process with your students? How far would you have to take them through the stages shown in the DVD?
5. Do you feel it is necessary to have a strategy for factoring polynomials such as  $6x^2 - x - 12$  or is a guess-and-check strategy sufficient?
6. How will you use these techniques to enhance your current instruction?
7. How will you use the materials in the PDF handout to supplement or replace your current materials?
8. Can any of the steps in this process be implemented across grade levels at your school site? For example, can lattice and area multiplication be taught one year, and multiplying polynomials be introduced the subsequent year?

If desired, allow educators collaboration time to schedule the implementation strategies.

## NUMBER LINE

Initial discussion:

1. What issues do students face in comparing fractions?
2. What strategies do they use to compare them?
3. What types of errors do you see them commonly making at your grade level?
4. What errors do students make with decimals? With percents?
5. Research has shown that only division of decimals is more difficult for students than placing decimals in order. Why do you think this is so?
6. How do the problems students face with fractions, decimals, and percents impede their success in future mathematics?

Post-viewing discussion:

1. What surprise you most about the strategies and thinking of these eighth grade students in Brad's classroom?
2. What strategies did you see them using? Are any of these new ideas to you? (Keep in mind that these students were not prompted in any of their observations or thinking prior to the filming of the video.)
3. How did Brad use the strategy of asking the students to stand effectively? How does this more effectively engage the students? How does it help the teacher assess the progress of the lesson and the thinking of the students?
4. What role does the discussion and the use of oral language play in the development of this lesson? Why is it important to ask the students to convince a majority of the classroom in order to change the number line?
5. Do you see value in covering all three part-whole representations (fraction, decimal, and percent) in one day, or should they be addressed on separate days?
6. How would this activity help students in thinking mathematically on state tests and other assessments?
7. At the conclusion of the video, we see Brad implement this same lesson in an algebra 1 class. What topics came up for discussion?
8. How does this activity foster clearer algebraic thinking in his students?
9. What extensions and applications can you envision for this lesson?

If desired, allow educators collaboration time to schedule the implementation strategies.

## THE POWER OF TWO

Initial discussion:

1. What roadblocks prevent students from understanding exponents and exponential notation?
2. How is exponential growth modeled in the world of a child?
3. How have we taught exponents in the past with our students?
4. What procedural models have we used?
5. What conceptual models have been implemented in their learning?
6. What concepts about exponents are taught at each grade level?
7. Why do students think the zero power of a number is zero?
8. Often we say that exponents are simply repeated multiplication. Using this model, what struggles would a student encounter in thinking about  $2^0$ ? How might they view  $2^{-3}$ ? How would they explain  $2^{3.76}$ ?

Post-viewing discussion:

1. Often students fail to see how math applies to the real world. How does this activity address this problem?
2. Having seen the DVD, how would you explain *why*  $2^0 = 1$ ?
3. Would this model work for other folding patterns such as tripling or quadrupling folds?
4. Does this suggest that any nonzero number raised to the zero power is one?
5. How could a student explain negative exponents using the exponential graph project?
6. Having seen this experiment, student Brittney Gallivan set out to exceed the known limits on paper folding. See her world record fold on the web at:  
[www.osb.net/pomona/12times.html](http://www.osb.net/pomona/12times.html)
7. The authors have often said that no matter what we need to teach, there is probably some object or experience that will allow us to connect that to the real world. Do you agree or disagree with this? Why or why not?

If desired, allow educators collaboration time to schedule the implementation strategies.

## TAKE YOUR PLACES

Initial discussion:

1. What issues trip up your students' thinking about place value?
2. What errors do students make in thinking about and applying operations on numbers? For example, do they assume that multiplication *always* creates a larger product?
3. How much time do your students spend "playing around" with numbers and operations to see their affects?
4. Do you tend to address specific standards and skills in your instruction? To what degree are concepts interconnected?
5. How much language and discussion are used in typical classroom instruction?
6. You are about to watch a DVD in which students think about two-digit multiplication. On a scale of one to ten, how well would you say your students understand this process?

Post-viewing discussion:

1. Look back at question six above. Would your answer to that question change after watching this DVD? Do you think these students came away with a more sophisticated understanding of the process because of this lesson?
2. What errors and insights did the students make as they explored this activity? How do you see their thinking progressing during the lesson?
3. How did Brad integrate different mathematical strands in this lesson? What value does this have for the student?
4. Look back at question five above. What role do you see language playing in fostering deeper understanding in this lesson?
5. How could you integrate this type of instruction into your typical or text-based instruction? What amount of time should be dedicated to this type of instruction?
6. Notice that Brad took a fairly simple elementary concept, two-digit multiplication, and presented it in a way that challenged his eighth graders. How might they have responded if the foundation of the lesson involved a more rigorous subject such as exponents or radicals?
7. Look at the many applications of this activity in the PDF handout that accompanies this DVD. Which of them are of value in your classroom?

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## TEACHING TWO-DIGIT MULTIPLICATION THROUGH CONCEPTUAL LAYERING

### Initial Discussion:

1. Do you think your students “get it” when they are learning mathematics?
2. What portion of your class would you say is mimicking steps in solving a problem versus actually understanding those steps?
3. To what degree do you find yourself backtracking or re-teaching concepts already taught?
4. What different strategies do you employ when you re-teach? How would you measure the success of those strategies?
5. Do you have a typical plan for leading into a lesson or a new concept?
6. To what degree do you feel it is important to start at a pre-abstract level when introducing new concepts at higher grade levels?
7. Do you sometimes suspect that some of your students have maxed out and learned about as much math as they ever will?

### Post-viewing discussion

1. How would you define *Conceptual Layering* for someone who had not seen today’s video?
2. What are the characteristics unique to *Conceptual Layering*?
3. Are you currently employing any of these techniques, and how have they worked?
4. What do you see as the strengths and advantages of the *Conceptual Layering* approach?
5. Do you foresee problems in implementing *Conceptual Layering* in your instruction?
6. Does the fact that this technique was used to help 4<sup>th</sup> and 5<sup>th</sup> graders understand and perform high-level algebraic thinking lend credence to this instructional strategy?
7. How might *Conceptual Layering* be integrated into other math concepts?
8. How might you implement some of the components of *Conceptual Layering* into your instruction?

If desired, allow educators collaboration time to schedule the implementation strategies.

## X MARKS THE SPOT

Initial discussion:

1. How do students currently practice the four operations at each grade level?
2. What types of numbers are used: positive whole numbers, decimals, fractions, integers, or something else?
3. What has worked well with your current format for drill and practice?
4. How much is number sense and mathematical reasoning fostered during this practice?
5. What struggles do students currently face with the four operations?

Post-viewing discussion:

1. To what degree will this drill and practice model supplant, replace, or supplement your current approach?
2. Which of these many versions of X Marks the Spot is most appropriate for your students?
3. How will you divide these versions among the grade levels at your site?
4. How does the simultaneous practice of two operations, such as addition and multiplication, foster mathematical thinking?
5. In what other ways does this approach foster number sense and mathematical reasoning?
6. How do you see the algebraic applications of X Marks the Spot fitting into your teaching assignment? For example, do you plan to use the activity to help students factor quadratics (of the form  $ax^2 + bx + c$  when  $a = 1$ )?

If desired, allow educators collaboration time to schedule the implementation strategies.