Developing Proportional Reasoning

By Brad Fulton
brad@tttpress.com

Teacher to Teacher Press
PO Box 233, Millville, CA 96062
Phone: (530) 547-4687
Fax: (530) 547-4317

There is at least one significant difference between a fraction and a ratio. A fraction *always* compares a part to a whole. A ratio is not limited by this restriction, and therefore, the rules and algorithms that apply to fractions do not necessarily apply to ratios. This is true even when a ratio behaves like a fraction by comparing a part to a whole. An example will illustrate this problem.

Let's assume two basketball players are comparing their shooting. Player A makes 5 of 8 shots, while player B makes 3 of 5 shots. They combined to make 8 of 13 shots. Yet when we express the ratios of the two players and show them combined, we seem to have violated a critical algorithm for adding fractions:

$$\frac{5}{8}+\frac{3}{5}=\frac{8}{13}$$

It seems we have added fractions by simply adding numerators and adding denominators. The ratios are all correct though. The first ratio expresses the shots of player A, and the second ratio shows the shots of player B. The final ratio shows the combination of their work. The problem is that expressing the ratios as we did makes it appear that they should obey the rules for fraction addition, and in this case, that is not true. The best solution is to avoid expressing the ratios of this problem in this format.

This example illustrates the confusion ratios may present to a learner. It also shows why we can't tell students that ratios are simply fractions. What we have is a single representation for two different concepts. It would be like an addition sign meaning "add" in one context and "divide" in another one. In this situation, it would be better to express the ratios as 5 out of eight, 5 to eight, or 5:8.

There are multitudes of ways to explore, think about, and solve proportional situations. Eight of these will be presented in this paper.

1. Factor of change
2. Unit rate
3. Fractional
4. Table
5. Graphical
6. Cross products
7. Factoring
8. Multiplication chart

The first six are described on the following pages in the activity called "Proportion Boxes." The other two are described below.

Some of these methods are more understandable while others are more abstract. However, some of the methods that are easier for students to understand apply to only a limited set of situations, while more abstract models apply to a wider variety of problems. You as the teacher will be the best judge of which methods are most appropriate for your classroom.

Solving proportions by the factoring approach:

Let's solve the proportion shown below.

$$\frac{18}{24}=\frac{x}{28}$$

Write the terms in prime factorization:

$$\frac{(2)(3)(3)}{(2)(2)(2)(3)}=\frac{x}{(2)(2)(7)}$$

Now cancel common factors:

$$\frac{\cancel{(2)}(3)\cancel{(3)}}{\cancel{(2)}(2)(2)\cancel{(3)}}=\frac{x}{(2)(2)(7)}$$

Next match up horizontal pairs of factors:

$$\frac{\cancel{(2)}(3)\cancel{(3)}}{\cancel{(2)}(2)(2)\cancel{(3)}}=\frac{x}{(2)(2)(7)}$$

Notice that the only remaining factors are the 3 and the 7. Put these in place of the x and the proportion is solved.

$$x = (3)(7) = 21$$

Solving proportions using a multiplication chart:
Let's solve the proportion shown below.

$$\frac{6}{x} = \frac{15}{25}$$

Find a rectangle in the multiplication chart that has the three numbers in three corners as shown.

x	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	***10***	15	20	25

Look what shows up in the fourth corner–the answer to our proportion!

"Building Mathematical Skill on a Foundation of Understanding"

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Proportion Boxes

The development of proportional reasoning is critical for intermediate and middle school learners. That is why we must provide many opportunities for students to apply and practice these skills. We also can help students by demonstrating numerous ways to solve ratios by filling their toolkit with a multitude of strategies.

Procedure:

> Required Materials:
> ☒ Paper
> Optional Materials:
> ☐ Transparency of Activity Master

1 There are at least six approaches to introducing ratios. Each will be explained briefly here, and then an approach for teaching word problems involving ratios will be shown.

2 Most students find the **factor of change** method the simplest place to start. This involves multiplying one ratio to get a second one. Consider the following problem:
Four widgets sell for six dollars. How much will 12 widgets cost?
It is easy to see that since 12 widgets are three times as many as six widgets, the cost will also be three times as much, or $18, as shown here.

x 3

$$\frac{4}{6} = \frac{12}{x}$$

x 3

3 The factor of change method works well until the factor is no longer a whole number. If we are trying to determine the cost of 10 widgets, other methods may seem easier. One is the **unit rate** method. In this, the unit price is established. For example, if four widgets sell for six dollars, dividing six by four shows that each unit sells for $1.50. Once this unit rate is found, it can be used as a multiplier in the second ratio as shown.

$$\frac{6}{4} = \frac{1.5}{1} \qquad \text{x 1.5} \quad \frac{6}{4} = \frac{x}{10} \quad \text{x 1.5}$$

4 A third method involves seeing the ratio as a **fraction**. In the last example, we could simplify the first ratio. The new ratio would allow us to apply the factor of change method. This method can often be applied to percent problems. If we want to know the percent for 60 out of 75, we can simplify the fraction first, then find a factor of change to make the denominator equal to 100.

x 5

$$\frac{4}{6} = \frac{2}{3} \qquad \frac{2}{3} = \frac{10}{x}$$

x 5

x 20

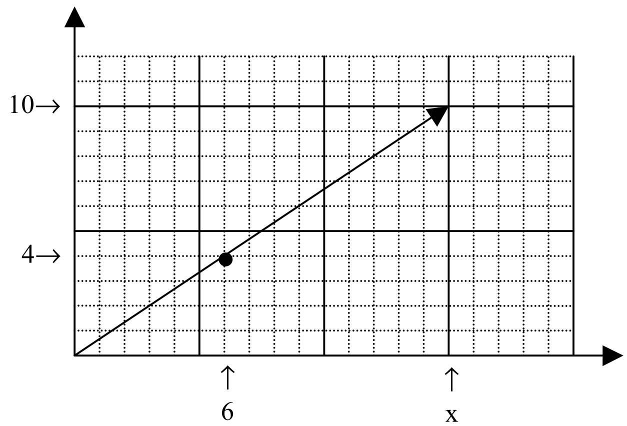
$$\frac{60}{75} = \frac{4}{5} \qquad \frac{4}{5} = \frac{x}{100}$$

x 20

5 In the fourth method, a **table** is constructed showing the initial ratio and multiples of it. Eventually this can lead us to determine a value for the second ratio as shown on the next page.

6 Another method uses a **graph** to solve the ratio. The initial ratio is graphed as a coordinate pair. A line is extended through this point from the origin. Next the second ratio is located on this line by finding the known part of the coordinate. In the previous example, the numerator, 10, is located on the vertical axis. This corresponds to a value of 15 on the horizontal or denominator's axis as shown. In fact, a graph provides a good visual definition of a proportion. When graphed, all proportions are straight lines, passing through the origin, and sloping up to the right. Any other graph is not a proportion.

	x	y	
	4	6	
	8	12	
10→			←x
	12	18	



7 The sixth method is to use **cross products**. Here the ratio is solved algebraically by multiplying diagonally. The advantage of this method is that it works universally.

$$\frac{4}{6} = \frac{10}{x} \qquad 4x = 60 \qquad x = 15$$

8 Now let's look at a way to analyze troublesome word problems. Keep in mind that a proportion compares two data pairs. In the previous example we are comparing the **quantity and cost** of widgets in a **small and a large** sample. Have the students make a sketch of the grid shown on the right. Label it as shown.

	small	large
quantity		
cost		

9 Next the students should fill in the known data and write an "x" in the fourth cell. Notice that this places the data in the form of a proportion.

$$\frac{4}{6} = \frac{10}{x}$$

	small	large
quantity	4	10
cost	6	x

Using cross products produces 4x = 60 as before. It even works if the labels are switched as shown on the bottom right. This produces a new proportion, but the cross products are the same.

	large	small
cost	x	6
quantity	10	4

$$\frac{x}{10} = \frac{6}{4} \qquad 4x = 60 \qquad x = 15$$

10 These ratio boxes are an excellent way to solve the three types of proportions involving percents. These three situations are shown here.

What percent is three out of five?

$$\frac{3}{5} = \frac{x}{100}$$

	fraction	percent
part	3	x
whole	5	100

What is 80% of 25?

$$\frac{x}{25} = \frac{80}{100}$$

	fraction	percent
part	x	80
whole	25	100

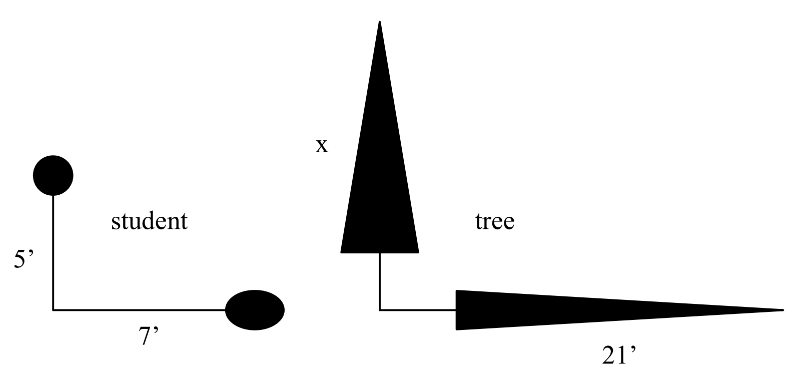
15% of some number is 6. What is the number?

$$\frac{6}{x} = \frac{15}{100}$$

	fraction	percent
part	6	15
whole	x	100

11 All percent problems can be solved using this ratio box and its four labels. Most other proportions involve a small and large sample. Many will use the labels "part" and "whole". Other common labels may include conversions from one unit to another, such as inches to centimeters or dollars to pesos.
These ratio boxes will help your students process the information in word problems and organize it so they will be successful in solving the proportion. **Check out next month's newsletter for a great problem that will allow your children an opportunity to use these skills in a challenging exploration.**

We always love to hear from our readers. Email us with your comments on this activity at either
brad@tttpress.com or **bill@tttpress.com**



Good Tip:

On a sunny day, take students outside to measure the heights of inaccessible objects by comparing the shadow lengths of the object to their own as shown here.

	student	tree
height	5'	x
shadow	7'	21'

What percent is 8 out of 25?

	fraction	percent
part	8	?
whole	25	100

Transparency Master 2

What is 40% of 20?

	fraction	percent
part	?	40
whole	20	100

Transparency Master 3

75% of some number is 15. What is the number?

	fraction	percent
part	15	75
whole	?	100

"When are we ever gonna use this?" is often asked in math classes, but when it comes to ratios and proportional reasoning, this should be an easy question to answer, as these concepts are frequently used in our daily lives.
If we drove 120 miles in two hours, how long will it take to drive to Sacramento? If we used four gallons of gas for that portion of the trip, how much gas will we use altogether? And if those two gallons of gas cost $6.28, how much will the trip cost?
If we earned $27 in three hours, how much will we earn in a 40-hour week? If taxes take 28% of each dollar, how much will I owe in taxes? How many of my 40 hours are spent earning money that pays taxes.
If my cell phone carrier charged me $5.76 for 72 text messages, how many can I send and stay under $5.00 for the month? How much will I spend on text messages in a year?
A very hands-on activity for using proportions involves measuring the heights of tall objects by measuring their shadows. On a sunny day, measure the height of a student of other reachable object and measure its shadow. Measure these in meters. Then measure the shadow of some tall objects such as a tree or gymnasium. This data can then be put into a proportion box as shown here:

	person	object
height	1.47	x
shadow	2.38	19.11

This can then be solve algebraically using cross products:

$$(1.47)(19.11) = 2.38x$$
$$\frac{(1.47)(19.11)}{2.38} = x$$
$$x \approx 11.80$$

Students enjoy getting outdoors and using mathematics to explore their world. They also see that mathematics is an empowering tool that gives them the ability to literally reach new heights.

"Building Mathematical Skill on a Foundation of Understanding"

Teacher to Teacher Press Newsletter

Volume 305 May, 2003

Lemons and Grapefruits

As our plane leaves San Antonio and the 81st annual NCTM conference, we put the finishing touches on May's newsletter. We were pleased to meet so many of you at our presentations and strolling along the meandering San Antonio River. We see that the mathematical community is a tightly woven fabric of teachers working together to support one another for the sake of our students. With this in mind, we offer you an intriguing activity to follow up last month's newsletter on teaching ratios. This problem will not only promot the diversity of thinking in your classroom, but the solutions we offer are just as unique.

Procedure:

Required Materials:
- ☒ Activity Master
- ☒ Super Citrus Solution

Optional Materials:
- ☐ Transparency of Answer Key

1 Give each student a copy of the Activity Master and explain the problem. As an example, ask them how many lemons can be shipped in a crate that contains 9 grapefruit. Though the problem is simple enough, the solution is challenging for students. If they have learned to solve proportions they may approach the problem this way:

$$\begin{matrix}\text{Grapefruit} \\ \text{Lemons}\end{matrix} \quad \frac{12}{20} = \frac{9}{x} \qquad 12x = 180$$

They might think this suggests that 15 lemons can be put in the case. However this solution cannot be possible since that puts a total of 24 mixed fruit in a box that can hold only 20 lemons at the most. Although the problem can be solved using proportions, it is worth exploring first using other means.

2 Ask the students to use the first t-table to explore possible combinations. Are there any combinations of grapefruit and lemon we know will fit in the box? In fact there are two obvious ones: 12 grapefruit with zero lemons and 20 lemons with zero grapefruit. Once these two solutions are put on the t-table, a third solution begins to become apparent. You could fill half the box with grapefruit (6) and the other half with lemons (10). Midway between these solutions, we have a box that is one-quarter grapefruit (3) and three-fourths lemons (15). There is also a solution that fills the box three-fourths full of grapefruit (9) and one-fourth full of lemons (5). These combinations are shown in the t-table.

G	L
0	20
1	
2	
3	15
4	
5	
6	10
7	
8	
9	5
10	
11	
12	0

3 Students may now notice the unit rate for this ratio. As the number of grapefruits increases from zero to three, the number of lemons goes down five. This is true throughout the table. That is, three grapefruit equal five lemons. Thus one grapefruit is equal to $^5/_3$ or $1^2/_3$ lemons.

$$\text{Grapefruit/Lemons} \quad \frac{3}{5} = \frac{1}{x} \qquad 3x = 5$$

4 Now we know how many lemons to deduct for each grapefruit added to the case. We must keep in mind though that when dealing with a fraction of a lemon the answer should be rounded down to the nearest whole number. The company would never ship a fraction of a lemon, nor would they try to overstuff a box and smash the fruit. In reality the remaining space would be filled with packing material. This results in the table shown here.

G	L	
0	20	
1	$18\,^{1}/_{3}$	≈ 18
2	$16\,^{2}/_{3}$	≈ 16
3	15	
4	$13\,^{1}/_{3}$	≈ 13
5	$11\,^{2}/_{3}$	≈ 11
6	10	
7	$8\,^{1}/_{3}$	≈ 8
8	$6\,^{2}/_{3}$	≈ 6
9	5	
10	$3\,^{1}/_{3}$	≈ 3
11	$1\,^{2}/_{3}$	≈ 1
12	0	

5 It seems that this table solves the problem. However this chart is dependent upon a customer ordering a specific number of grapefruit. But what if the customer orders twelve lemons? This is not shown in the table. For this situation, we need a second t-table that converts lemons to grapefruit. The same processes can be used to make this table as we used in the previous one. The unit rate is now 1 lemon = $^{3}/_{5}$ grapefruit. The table is shown below to the right. Notice that in both tables there is a pattern to the numbers of the rounded data.

L	G	
0	12	
1	$11\,^{2}/_{5}$	≈ 11
2	$10\,^{4}/_{5}$	≈ 10
3	$10\,^{1}/_{5}$	≈ 10
4	$9\,^{3}/_{5}$	≈ 9
5	9	
6	$8\,^{2}/_{5}$	≈ 8
7	$7\,^{4}/_{5}$	≈ 7
8	$7\,^{1}/_{5}$	≈ 7
9	$6\,^{3}/_{5}$	≈ 6
10	6	
11	$5\,^{2}/_{5}$	≈ 5
12	$4\,^{4}/_{5}$	≈ 4
13	$4\,^{1}/_{5}$	≈ 4
14	$3\,^{3}/_{5}$	≈ 3
15	3	
16	$2\,^{2}/_{5}$	≈ 2
17	$1\,^{4}/_{5}$	≈ 1
18	$1\,^{1}/_{5}$	≈ 1
19	$^{3}/_{5}$	≈ 0
20	0	

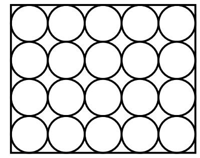
6 Another approach is to graph the problem. For this purpose we will consider the graph of the first t-table. If we graph the first point, (0, 20), and last point, (12, 0), on the table and connect them with a straight line, the line also passes through the points (3, 15), (6, 10), and (9, 5). The points that have been rounded off can also be shown on the graph, but they will not be on the line; they will be below it. On the answer key, these points have been connected with a dotted line and the area between it and the solid line has been shaded. You might consider the graph the following way. The solid line represents what the box is capable of holding. The dotted line represents the fruit that can actually be placed in the crate, and the shaded area represents packing material. The graph is a wonderful representation of the real-world aspects of the problem.

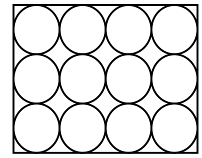
7 An even more visual approach to the problem is the Super Citrus Solution. Here a model of the 12 grapefruit and 20 lemons has been scaled so they are the same lengths. You might think of the space between the two dotted lines as a long box that can hold the 12 grapefruit, the 20 lemons, or any combination of the two. By sliding the joined strips between the dotted slits, students can see the different combinations that are possible.

Lastly, we can use proportions to solve the problem. If we consider the original problem posed in step one, it seems to suggest an impossible solution of 9 grapefruit and 15 lemons. However, what the problem tells us is that the 9 grapefruit *are equivalent to* 15 lemons. This means there is only room for five more lemons. This type of proportion is called an *inverse proportion*. This is demonstrated by the fact that the graph slopes downward instead of upward. Although this is a perfectly appropriate way to solve the problem, it is the least visual of the methods and should be saved for last.

Activity Master

Name________________________





The Sunshine Citrus Company sells cases of lemons or grapefruits. Each case holds 20 lemons or 12 grapefruit. To increase sales, the company decides to ship cases of mixed fruit. They have hired you to help them decide how many of each type of fruit can be shipped in a case.

G	L
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

L	G
0	
1	
2	
3	
4	
5	
6	
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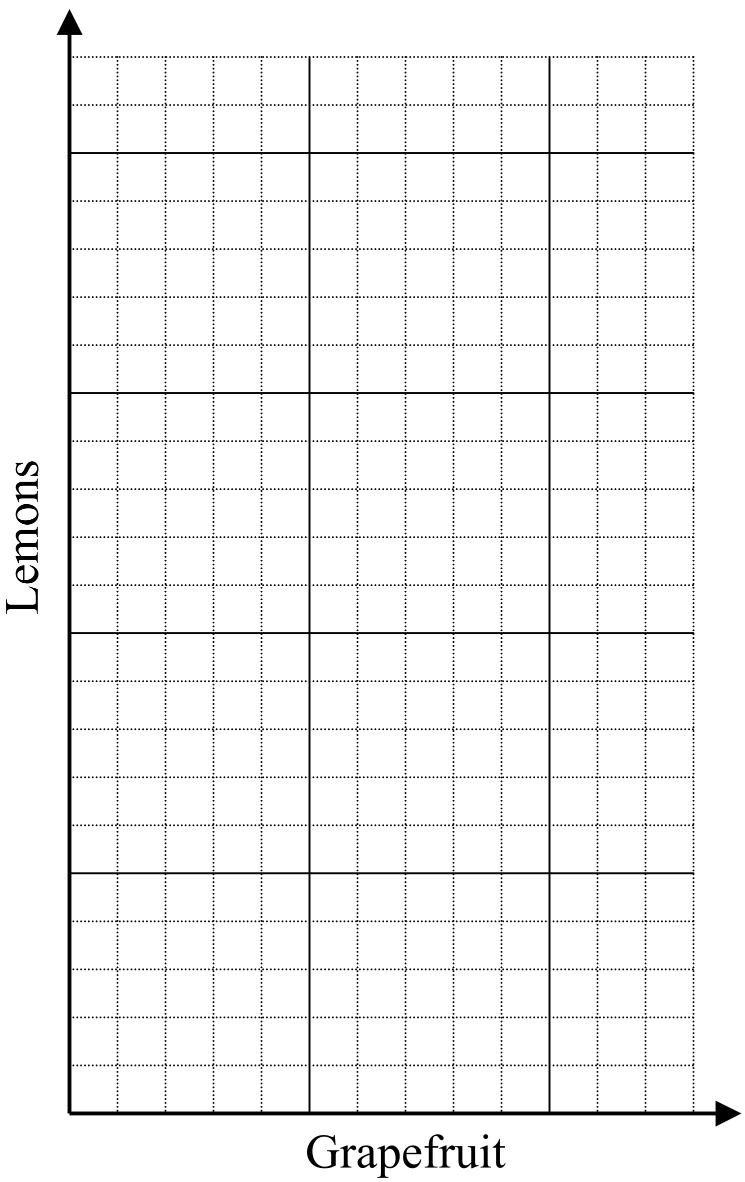
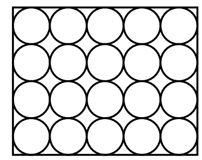
17	
18	
19	
20	

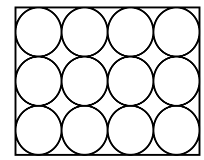


Answer Key

Sunshine Citrus

Name________________________





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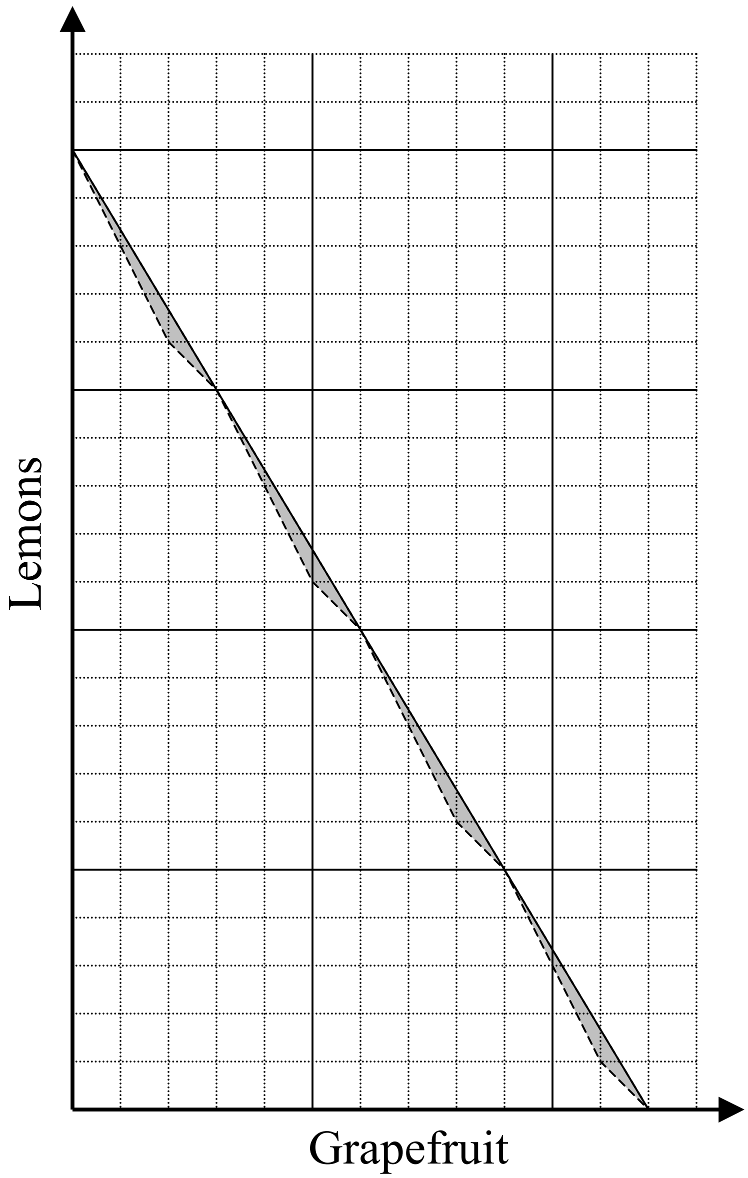
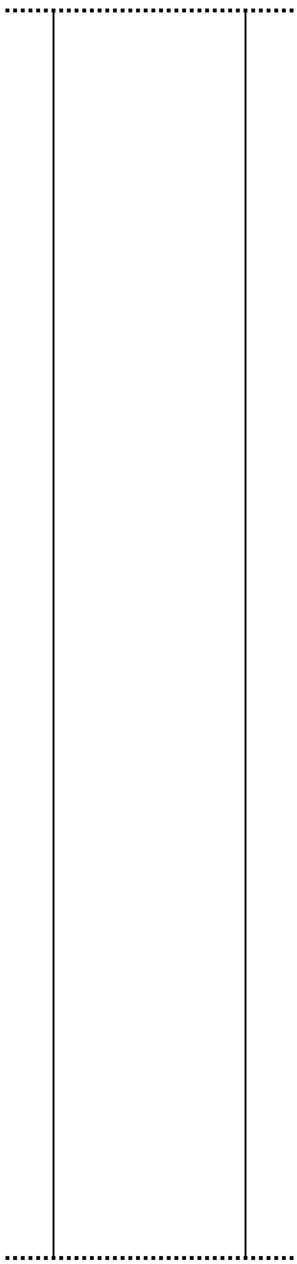
G	L
0	**20**
1	18
2	16
3	**15**
4	13
5	11
6	**10**
7	8
8	6
9	**5**
10	3
11	1
12	**0**

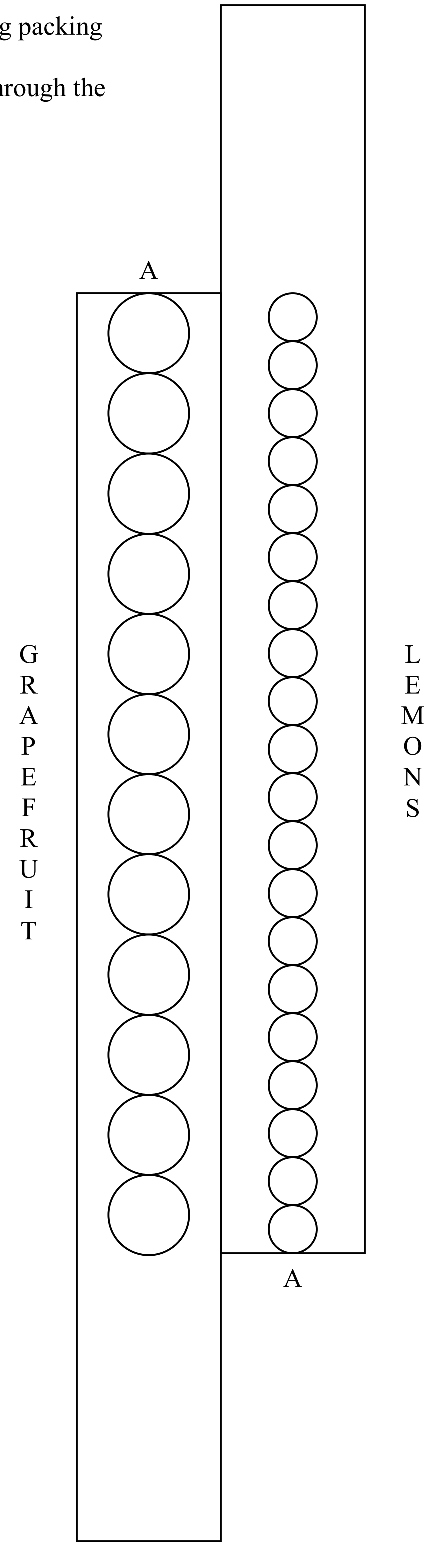
L	G
0	**12**
1	11
2	10
3	10
4	9
5	**9**
6	8
7	7
8	7
9	6
10	**6**
11	5
12	4
13	4
14	3
15	**3**
16	2
17	1
18	1
19	0
20	**0**



The Super Citrus Solution

1 Color the lemons yellow and the grapefruit pink.
2 Cut out the two rectangles and tape them together so end A of the lemon rectangle is joined with end A of the grapefruit rectangle.
3 Slit the two dotted lines on the left rectangle which represents a long packing case.
4 Slide the lemon and grapefruit strip up through one slit and down through the other to solve the Sunshine Citrus problem.

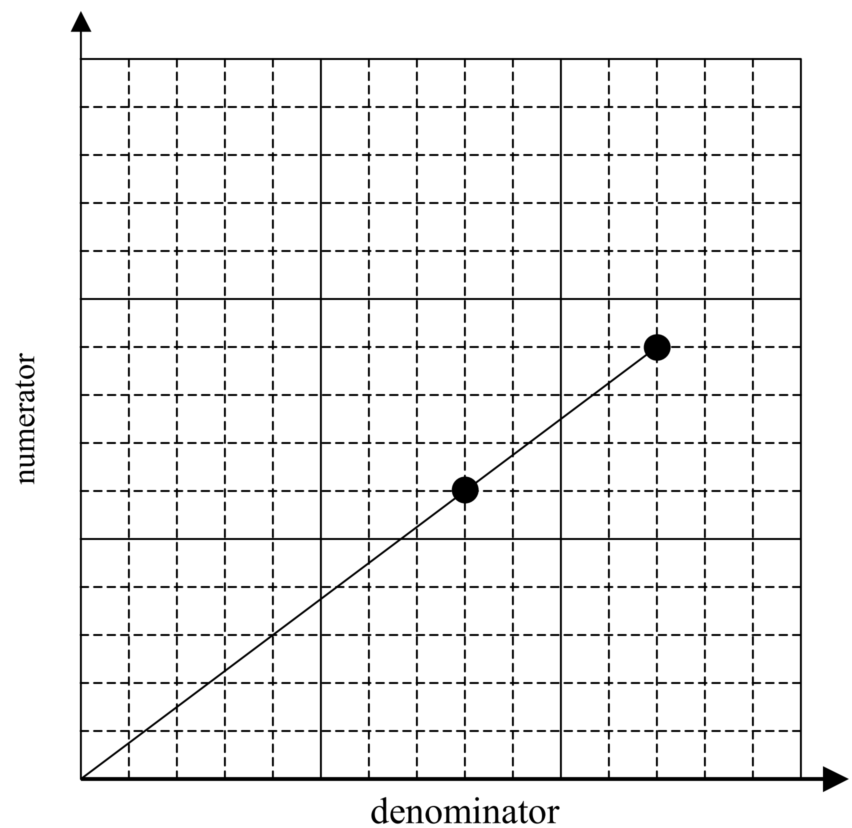




What is a proportion? The answer to this question can be somewhat technical. While a proportion can be defined simply as an equality of two ratios, how can we tell which of the following are proportions and which are not?

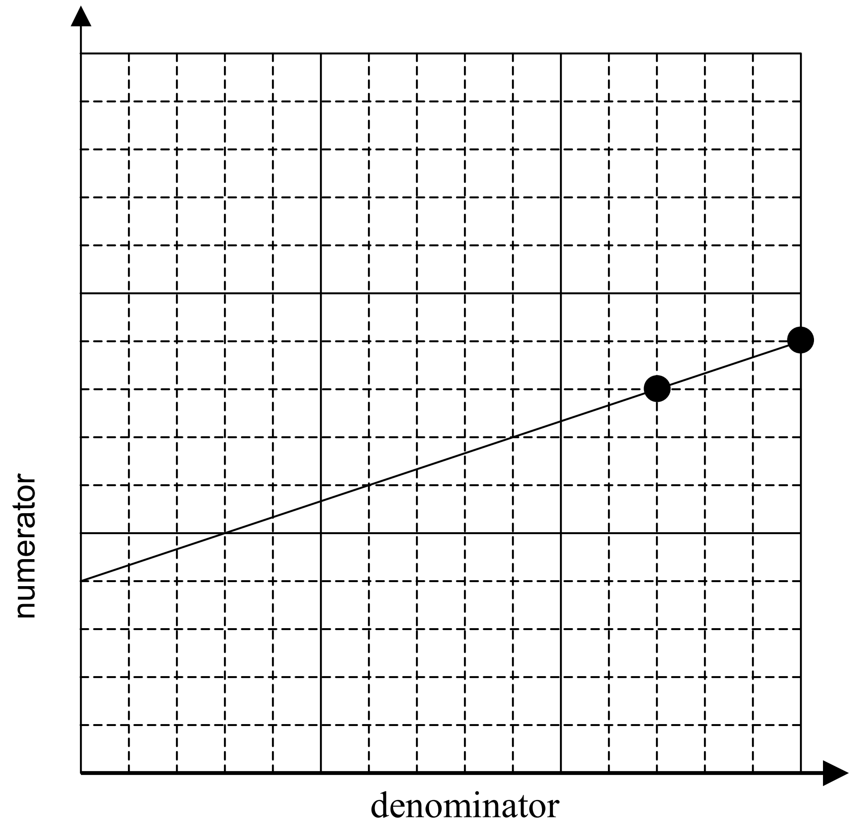
$$\frac{8}{18} = \frac{20}{45} \qquad \frac{9}{15} = \frac{8}{12} \qquad \frac{6}{8} = \frac{9}{12}$$

One way is by comparing cross products. Proportions have equal cross products. While this is a reliable method, it relies on a student's ability to multiply numbers correctly…twice. A graph however offers us a visual way to identify what is a proportion and what is not.
Let's graph the third proportion by plotting the denominator on a horizontal axis, and the numerator on the vertical axis. When we do this to both of the ratios in the example, we see that a line passing through them will also pass through the origin. *This is true for all proportions.*



Proportional ratios

However, When we try this for the second example, we see that the two ratio points are not contiguous with the origin, proving that they are not proportional.



Nonproportional ratios

A Day in My Life

Project Instructions

1. Students chose five activities that are not done concurrently, such as eating, sleeping and going to school. (Hopefully they are not asleep at school. Activities that can be performed concurrently will not work for this project. For example, the students should not choose watching television and spending time with their family if these are done at the same time.
2. Next they are to estimate the number of hours they typically spend on each activity each day. After totaling these five activities, they subtract this from 24 (hours in a day) to find the amount of time they spend doing other activities.
3. Next they set up a proportion using their age expressed as a decimal as shown below. To find their age as a decimal, they must first find how many months it has been since their last birthday. For example, if a student turned 12 three months ago, their age is 12 $^{3}/_{12}$. By dividing 3 by 12, they find that their age is 12.25 years. In this example, a student sleeps 7 out of every 24 hours.

	day	life
part	7	x
whole	24	12.25

4. By solving this proportion, they can find an estimate of the number of years of their life spent doing this activity.
5. Once this is done for every activity and for the "other" category, students convert the data to angle measures to make a pie chart. This is done using proportions also. Here the number 360 (degrees in a circle) is substituted for the child's age.

	day	circle
part	7	x
whole	24	360

6. Lastly, the students label their pie chart with the titles of the activities and with their percents. Again a proportion is used to convert the original ratios as shown here.

	day	percent
part	7	x
whole	24	100

7. A final copy is then made on another paper. (11” by 17” works well, or two 8.5” by 11” papers can be taped together.) The project can then be colored or decorated if you wish.

SLEEPING

	DAY	LIFE
PART	7.5	X
WHOLE	24	39

7.5 X 39 ÷ 24 ≈ 12.2

EATING

	DAY	LIFE
PART	2	X
WHOLE	24	39

2 X 39 ÷ 24 ≈ 3.3

SCHOOL

	DAY	LIFE
PART	6.5	X
WHOLE	24	39

6.5 X 39 ÷ 24 ≈ 10.6

TELEVISION

	DAY	LIFE
PART	.5	X
WHOLE	24	39

.5 X 39 ÷ 24 ≈ .8

EXERCISE

	DAY	LIFE
PART	.5	X
WHOLE	24	39

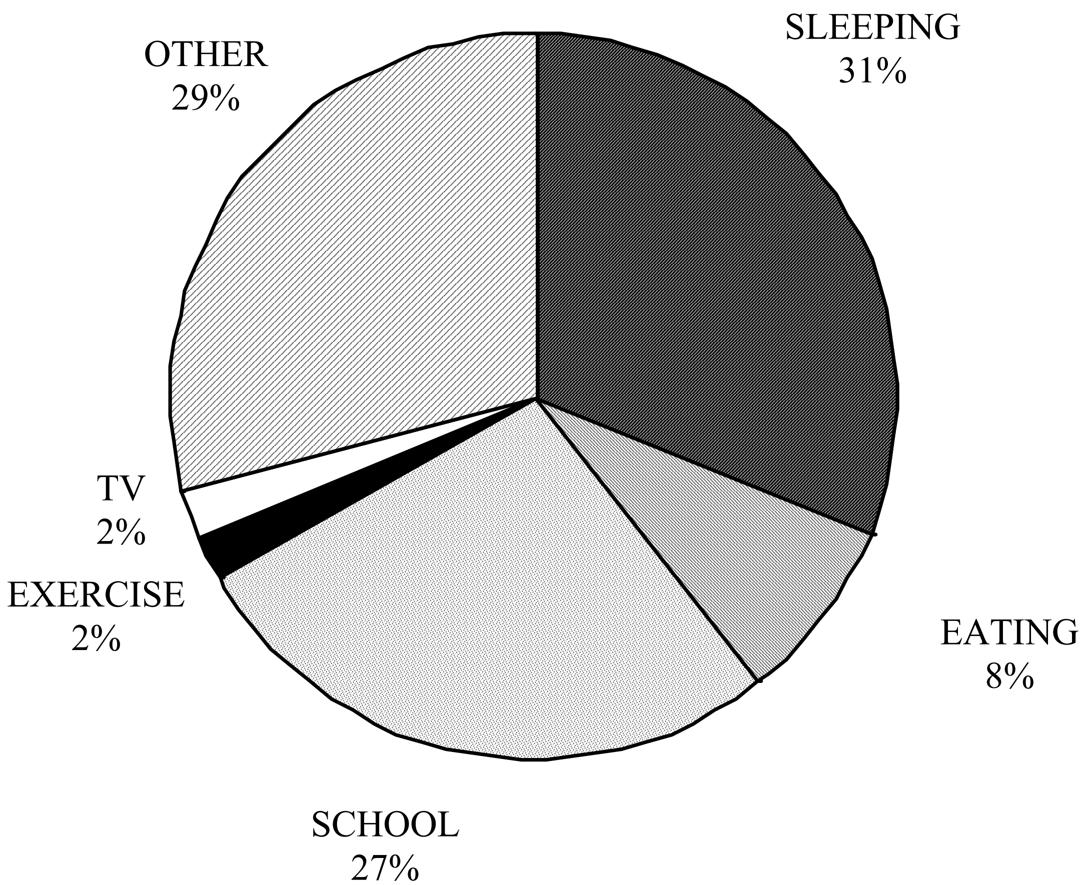
.5 X 39 ÷ 24 ≈ .8

OTHER

	DAY	LIFE
PART	7	X
WHOLE	24	39

6 X 39 ÷ 24 ≈ 39.8

THE LIFE OF MR. FULTON

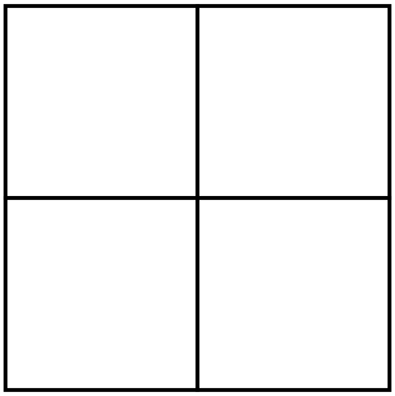


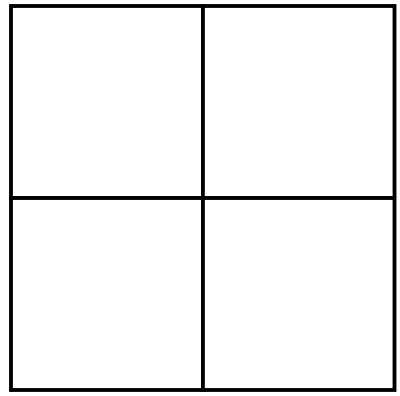
Grading criteria for proportion project

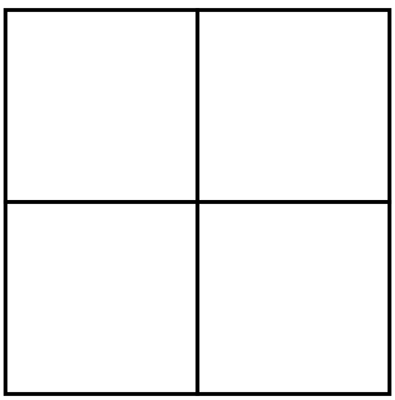
MATH............................... ____/50 Proportions, calculations, percents, angles MEASUREMENT............ ____/25 Parallel & perpendicular lines, proportion boxes, straight lines PRESENTATION........... ____/25 coloring, shadow box, lettering, spelling, erasures Other__________________ TOTAL: ____/100	MATH...............................____/50 Proportions, calculations, percents, angles MEASUREMENT............____/25 Parallel & perpendicular lines, proportion boxes, straight lines PRESENTATION...........____/25 coloring, shadow box, lettering, spelling, erasures Other__________________ TOTAL: ____/100

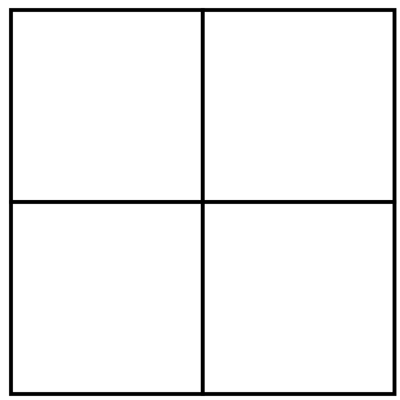
MATH............................... ____/50 Proportions, calculations, percents, angles MEASUREMENT............ ____/25 Parallel & perpendicular lines, proportion boxes, straight lines PRESENTATION........... ____/25 coloring, shadow box, lettering, spelling, erasures Other__________________ TOTAL: ____/100	MATH...............................____/50 Proportions, calculations, percents, angles MEASUREMENT............____/25 Parallel & perpendicular lines, proportion boxes, straight lines PRESENTATION...........____/25 coloring, shadow box, lettering, spelling, erasures Other__________________ TOTAL: ____/100

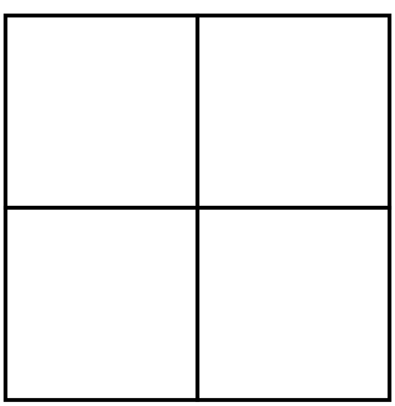
Master copies for the project are provided on the following pages.

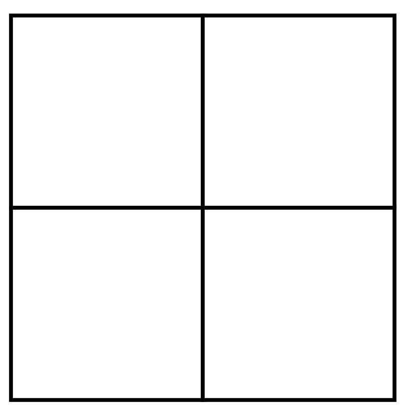


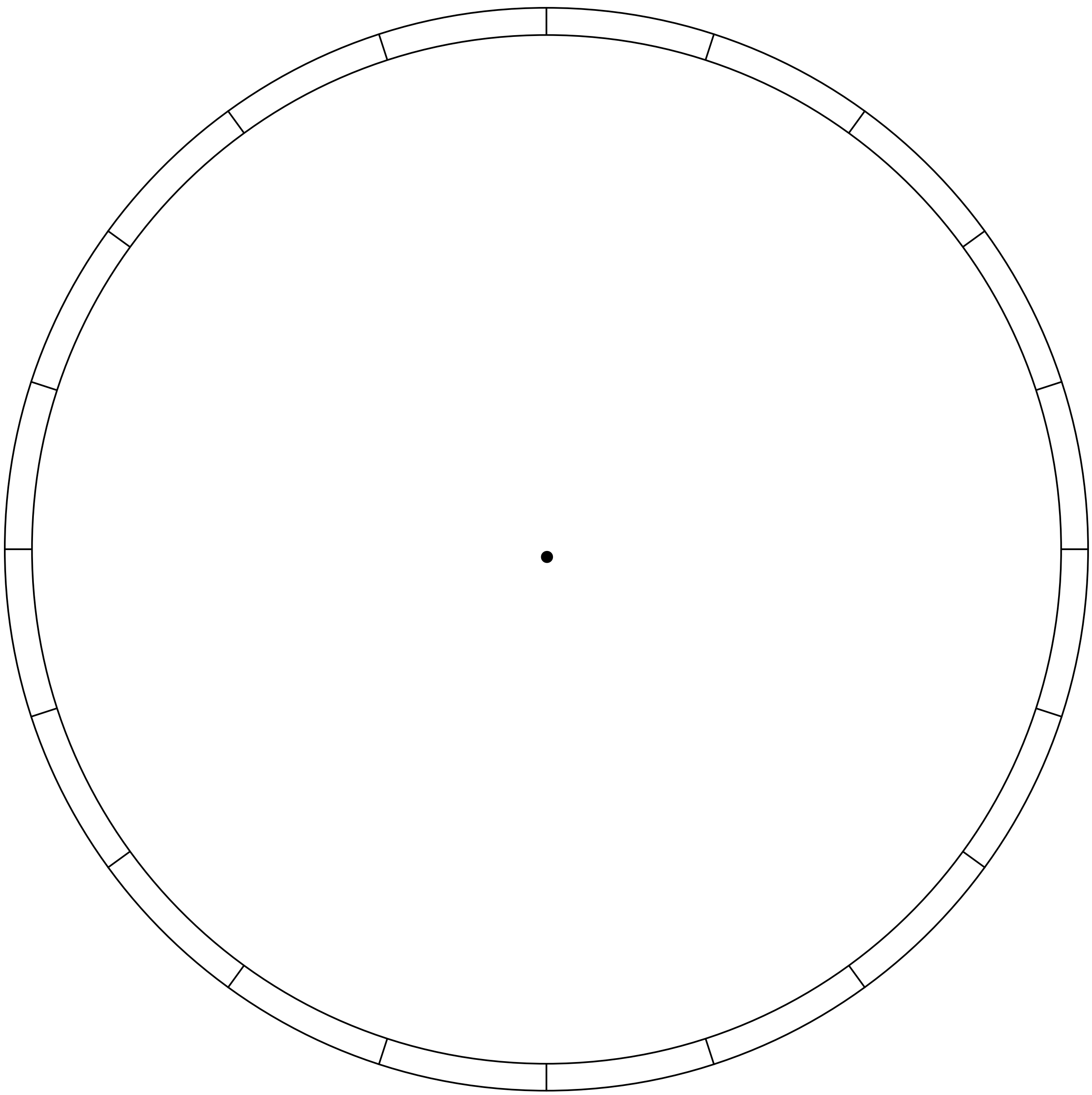












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Bill Lombard

PO Box 233, Millville, CA 96062
(530) 547-4687 brad@tttpress.com

5885 Avery Way, Redding, CA 96003
(530) 243-2064 bill@tttpress.com

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