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High interest discovery teaching
Generalizing arithmetic
Finding patterns
Using variables
Writing algebraic proofs
Practicing operations with integers

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Known throughout the country for motivating and engaging teachers and students, Brad has co-authored over a dozen books that provide easy-to-teach yet mathematically rich activities for busy teachers while teaching full time for over 30 years. In addition, he has co-authored over 40 teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

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♦ California Math Council and NCTM national featured presenter
♦ Lead trainer for summer teacher training institutes
♦ Trainer/consultant for district, county, regional, and national workshops

Author and co-author of mathematics curriculum
♦ Simply Great Math Activities series: six books covering all major strands
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References available upon request
Fast Facts and Fractions: Help students master their multiplication facts and learn simple strategies for taming fractions.

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ACTIVITY 5

Hundreds Magic

Overview: This powerful activity will keep your students engaged as they explore patterns, practice mathematical operations, and transition from inductive reasoning based on patterns to deductive reasoning supported by algebraic proofs.

Vocabulary: sum, product, adjacent, induction, deduction

PROCEDURE

Materials:
- paper
- transparency master
- activity master

Skills:
- Multiplying two-digit numbers
- Mental math
- Estimation
- Multiplying polynomials

1. The students will need to make a hundreds chart like the activity master. Alternately, you may wish to provide grid paper for this or simply distribute copies of the transparency master while you use the transparency on the overhead projector.

2. Ask the students to circle any four adjacent numbers which form a square. We will use 7, 8, 17, and 18 as an example. Tell them to add the two diagonals of the square and compare the results. They will notice that 7 + 18 = 8 + 17. Have them try the same process with a different set of four numbers.

3. Younger students will enjoy simply exploring the patterns in the chart without generalizing the relationships with formulas. More advanced students may be able to explain why the patterns occur, without using formal algebra. If your students are ready for the proof, this is the time to demonstrate it. Most students should be able to follow the explanation after their exploration of the chart. Notice that for any beginning number, the next number is n + 1. The numbers below these are n + 10 and n + 11. Thus the sums of the diagonals are:

\[(n) + (n + 11)\] and \[(n + 1) + (n + 10)\]

Combining terms gives us:

\[2n + 11 = 2n + 11.\]

4. Next, ask students to multiply diagonals and compare the results. They will see that the products are not equal. In our example, we get 7 x 18 = 126 and 8 x 17 = 136. However, when they try other locations, they will see that the second answer is always ten more than the first.

5. Once again, the reason can be explained fairly simply:

\[n(n + 11) = n^2 + 11n\]  \[(n + 1)(n + 10) = n^2 + 11n + 10\]

Journal Prompts:
Are the differences of the diagonals always equal? Explain why this is
or is not true.
Make up an arrangement of numbers other than the four number square. Describe any patterns or relationships that you find.

Homework:
Ask students to explore patterns found in other arrangements of numbers other than the four number square explained above. Some examples are shown in the pattern key on the following page. However, there are many more patterns and proofs for the students to discover.

Taking a Closer Look:
Ask students to explore these same relationships and others on any calendar page. An activity master is provided for this. What similarities and differences occur?
Advanced students can incorporate practice with negative numbers using the second activity master.

Assessment:
Allowing students to work in small groups will provide the opportunity for self assessment. Since all the patterns can be generalized, a single formula should result when students explore a given arrangement of numbers. Some sample patterns and proofs are offered on the following page.

Pattern Key:

Three-in-a-Row:
Pattern 1: The sum of the three numbers equals three times the middle number.
Proof: If “n” is the center then the left number is n – 1, and the right number is n + 1. Thus their sum is:
(n – 1) + n + (n + 1) = 3n – 1 + 1 = 3n

Pattern 2: The product of the left and right number is one less than the square of the center.
Proof: Their product can be written:
(n – 1)(n + 1) = n^2 + n – n + 1 = n^2 – 1

Five-Point: The average of the four corners is equal to the center.
Proof: If “n” is the center then the corners are n – 11, n – 9, n + 9, and n + 11. Thus the average is:
[(n – 11) + (n – 9) + (n + 9) + (n + 11)] ÷ 4 = (4n) ÷ 4 = n
Cross: The product of the top and bottom number is 99 less than the product of the left and right numbers.

Proof: If “n” is the center number, then the product of the top and bottom numbers is:

\[(n - 10)(n + 10) = n^2 - 100\]

The product of the left and right numbers is:

\[(n - 1)(n + 1) = n^2 - 1\]

and \[(n^2 - 100) = (n^2 - 1) - 99\]
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My 8th grade students enjoy discovering the algebraic patterns hidden within the hundreds chart. They are all familiar with this chart that they have seen since Kindergarten. One of our activities involves “Algebra Man.” I display a transparency of the hundreds chart and a second transparency of Algebra Man on top of it as shown.

I move the upper transparency around until a student yells, “Stop.” Then the students race me to find the total of the numbers inside Algebra Man. The total for the placement shown is 580. I always win this contest and boast that it is because I am so talented at math. The students insist there is a trick and demand a rematch. After a few times, some
of the students see a pattern and discover the trick. (Turns out I’m not a rocket scientist after all!) Once a few discover my secret, we explore the problem using algebra. The students notice that for any number on the chart, the number to the right is one more and the number to the left is one less. Also, the number beneath the given number is ten more and the number above it is ten less. This is always true no matter where Algebra Man runs. If we think of the number in his waist as $n$, then the following diagram shows the values of all ten numbers inside Algebra Man:

If we add the terms inside him, we get $10n + 20$. It’s a simple matter to look at the number in his waist (56), multiply it by ten (560), and add 20 (580). Many of my students say that they got the answer a different way. Often they simply look two spaces to the right of his waist and put a zero after the number. I express this algebraically on the board. The number two spaces to the right of his waist would be called $n + 2$. Putting a zero after it is the same as multiplying by ten. Thus their method is $10(n + 2)$. I show them that these are equivalent being the distributed and factored forms of the same expression:

$$10n + 20 = 10(n + 2)$$

Algebra Man works on any hundreds chart, even one that begins with -49 and ends with 50. He also works on a calendar, but the formula for his sum is slightly different. Challenge your students to find the formula when Algebra Man runs around on a calendar. This is just one of many algebraic explorations that can be made on the hundreds chart. If your students are like mine, they will want to explore all the “what if’s” they suggest. One time my students wanted to find out what happens to Algebra Man’s formula when he is standing on his head. Now I was curious.
Algebra Man Project Instructions

Getting ready:
1. Design a shape of ten cells on the hundreds chart.
2. Move it to three locations on your hundreds chart and calculate the sum. What pattern do you notice?
3. Find a centrally located cell (your key cell) and write a variable such as \( x \) or \( n \) in it.
4. Write the expressions for the other nine cells of your shape as shown in the top figure.
5. Combine like terms to find the total as shown in the second figure. You are now ready to begin your final project.
6. Show the results to your teacher

Designing your project:
7. Write the expressions neatly in the large grid cells and cut out the shape.
8. Neatly write the algebraic expressions in a column and total them. Simplify if necessary.
9. Neatly glue a hundreds chart onto your final paper.
10. Highlight your design on the hundreds chart.
11. Highlight your key cell.
12. Substitute the value of your key cell into the formula and calculate the sum.
13. Center your title and name at the top of the paper.
14. Your project should then look like the sample on the next pages.

Grade Sheet (cut out and tape to the back of your project)

Mathematics ......................................................... ____/25
- formula, combining like terms, example
Measurement ........................................................____/15
- centered title, alignment of terms, parallel and perpendicular edges
Presentation ......................................................... ____/10
- lettering, spelling, erasures creativity, coloring
Other ........................................................................

Total................................................................. ____/50
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