Birthday Magic

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Brad

This activity is part of a fuller handout on the teaching of exponents that is available in my Teachers Pay Teachers store. That activity is called *Teaching Exponents for Greater Understanding*. The full unit contains five activities including a culminating project.

In the full unit, students will come to understand the *why* behind such abstract concepts as a number to the zero power (n^0) and fractional and negative exponents.

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Teaching Exponents for Greater Understanding

The activities in this handout are from a publication titled *More Power^2 You*. That book was born out of my masters research on the teaching of exponents, exponential growth, and how students conceptualize these topics.

Too often students encounter exponents in middle grades without any conceptual understanding of them. They have no number sense for exponential growth. Textbooks often add to this problem by oversimplifying the concepts. Many textbooks include statements like, "Exponents are simply repeated multiplication." Not only do students not see them as simple, this definition is faulty and leads to faulty conclusions in our students.

Such definitions are often accompanied by examples such as $2^3=2\bullet2\bullet2=8$. This works fine for positive whole number exponents. However, it leads students to conclude that 2^0 means no twos exist and that 2^0 must be equal to zero. That is a logical conclusion based on that definition. Moreover, by that definition and example, a student would be fair to mistakenly conclude that $2^{-3}=2\div2\div2=½$.

One textbook tried to prove that $2^0=1$ by this process:

$$\frac{7^3}{7^3}=1$$

$$\frac{7^3}{7^3}=7^{3-3}=7^0$$

$$Therefore, 7^0=1$$

This hardly constitutes a "proof" as it is only demonstrating that $7^0=1$. Worse, it requires students to understand the transitive property of equality, a mathematical proof, *and* the laws of exponents!

One publisher went further by stating that mathematicians had *invented* the idea that negative exponents are fractions, so there is "nothing to understand."

What exponents really represent are a multiplicative growth as opposed to an additive growth. For example, a base of two means that we are *doubling* something, and this doubling usually happens repeatedly. The idea that doubling something three times gets us to 8 is only true if we begin with something. Doubling zero never gets us anywhere. So, what do we start with in our growth? It turns out, that though we don't write it this way, we always begin with the *multiplicative identity*.

Let's back up and consider addition. If we model 3+2 on a number line, we could start at zero, ***the additive identity***. Then we take three steps to the right followed by two more steps to the right to land on five. However, we don't write the problem as 0+3+2. We omit the implied identity of addition.

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Similarly, in multiplication, the identity is implied. 3•2 could be written as 1•3•2 meaning, start with 1, triple it, then double that. This lands us on six.

What this suggests in regard to exponential notation is this: "Two to the third power" means "Start at the multiplicative identity of one, and double it three times: 1•2•2•2=8." This means that 2^2 implies 1•2•2=4. Similarly, 2^1 is 1•2, and thus 2^0 is 1 with no doubling. Likewise, 2^{-3} would imply we are considering 1÷2÷2÷2 which is $^1/_8$.

The activities in this handout lead students to discover these concepts themselves through kinesthetic, visual, and conceptual applications. They are high-interest, engaging activities that have been proven to provide students with a deep understanding of exponential growth and a fluency with the exponential notation we use to represent it.

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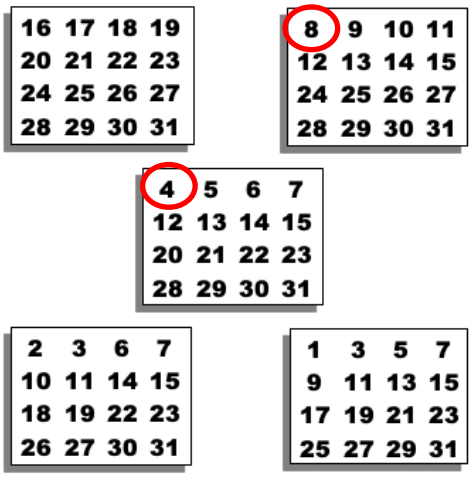
Birthday Magic

1. Show a student the five cards. It is even more impressive if the cards are revealed one at a time.
2. Ask them if their birthday (the day of their birth month) is on each card. After they respond "yes" or "no", you immediately predict their birthday.
3. For example, if they say "yes" to the second and third cards, and "no" to the others, you tell them that their birthday is on the 12th.
4. How do you do it? You simply add the numbers in the upper left corners of the "yes" cards. In this case, it is 8+4=12.



5. Essentially, the student is converting their birthday into binary where "No" corresponds to 0, and "Yes" to 1.

16	8	4	2	1
0	1	1	0	0

6. My students thought that I had memorized their birthdays because I had access to that on the computer attendance. Then I had them think of the birthday of someone else such as a family member and whisper that to a partner. They were amazed that I was still able to predict their day.
7. I print the cards on fuller 8½ by 11 inch stock so that I can hold them up in front of the class, but you could also use the accompanying version with a document camera.
8. Once I show them how the trick works as the culmination of our unit on exponents, the students make their own Birthday Magic cards.
9. The trick is even harder to figure out if the numbers that are *not* in the upper left corner are scattered randomly on each card instead of in ascending order. Then the victim can't recognize the pattern as easily.

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16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31

8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31

4	5	6	7
12	13	14	15
20	21	22	23
28	29	30	31

2	3	6	7
10	11	14	15
18	19	22	23
26	27	30	31

1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31

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Birthday Magic!

16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31

8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31

4	5	6	7
12	13	14	15
20	21	22	23
28	29	30	31

2	3	6	7
10	11	14	15
18	19	22	23
26	27	30	31

1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31

16	8	4	2	1

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Similar activities include:

- *Algebra Man: The tantalizing extension of Hundreds Magic. Students design their own project integrating number sense, algebra, and the mathematical practices.*
- *Take Your Places: Two versions for younger or older students help them transition from arithmetic to algebraic reasoning.*
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Feel free to contact me if you have questions or comments or would like to discuss a staff development training or keynote address at your site.

Happy teaching,

Brad

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