Making Math Teaching Easier and More Effective

THE GAP BETWEEN OUR ADVANCED STUDENTS AND OUR STRUGGLING STUDENTS has probably widened during the distance learning that has been mandated by the recent pandemic. Though many students logged on faithfully and had good parental support during this time, our struggling students probably experienced the worst learning environments and fell further behind. Moreover, many of the students in the middle have been less faithful about fully engaging in the distance learning opportunity you provided.

As if that were not sad enough, the resources that we use to create rich learning environments in the classroom do not always lend themselves to an electronic format. Hands-on learning is pretty much off the table.

Many of these students will be returning to classrooms having fallen further behind their grade-level standards. This put added pressure on us as teachers at a time when we are doing more work than we have ever done.

The purpose of this training is to make this job more manageable for us and more effective for our students.

Part 1: Making skills practice engaging and effective

To help your students return to grade-level proficiency, some time will need to be devoted to skills practice. However, we all know that our minutes are valuable. We want to maximize the effectiveness of our time.

We also know that math skills practice is necessary, but typically not very engaging for students. Many who participate in class don't complete worksheets and textbook assignments. Here is where research can help us understand why this happens and how we can change it.

The brain loves moderate doses of challenge, discrepancy, and anomaly. A bit of confusion is good and keeps the brain engaged. A worksheet of 30 similar math problems may provide plenty of practice problems but it is not going to interest the student. Is there a way to get more students to engage in a lesson that will provide this necessary drill work? The answer is, "Yes!"

To understand how this works, let's take a moment to think about video games. The student who holds a math worksheet in one hand and a video game controller in the other is not likely to choose the worksheet. Aside from the cool graphics, what is it about video games that is so engaging?

Think about this: you have probably had a student who has said, "I'm not good at math. I've tried it, and I failed. My parents aren't good either." Yet I've never heard a student say, "I tried video games once, but it didn't work." A student will launch into level 1 of a video game and before too long, a zombie jumps out from behind a rock, eats his head, and he dies! His first response is, "I want to try that again!" Why does this happen?

It is because the brain likes ***moderate doses of challenge***. The brain is intrigued by discrepancy. We have a phrase for this: "I want to get my mind around that." Too much challenge and the brain becomes confused and discouraged. Not enough, and the brain is bored. In our classroom, a bored brain is a dangerous brain.

As I think about how to get my students their necessary skills practice time, I try to keep this in mind. There are three components that I require of all of my practice opportunities.

- Independent
- Engaging
- Self-assessing

Independent – This means two things. First, as much as possible, I want this done on their time, not classroom time. If my goal is to help students catch up, this will take a lot of concerted effort on my part. Since drill work tends to require less high-order thinking, I want to relegate that to their independent time. Secondly, I want this to be something that they can do independently. Many parents are struggling with Common Core and other state math standards. They may not be able to help their child do their work the way we want them to do it. Thus, I want the format of the assignment to be so simple that they can work on it independently.

Engaging – Here I am keeping video game design in mind. I want the assignment to provide some discrepancy and discovery. Rather than 30 problems randomly generated by software, each problem builds upon the one before. There is a continuum as the student progresses through the assignment. They are on a journey with purpose and a destination. I will give examples of assignments that do this.

Self-assessing – This is the most important component. Steve Marcy, the author of *Punchline Math*, did his doctoral research on self-assessment. He found that students who self-assess tend to engage longer, make fewer errors, and repair their errors as opposed to students who are teacher-assessed. For this reason, and to minimize some of my workload, all of my worksheets are self-assessing in some way. The following four examples will demonstrate this. Now let's explore four formats for skills practice that meet these conditions of independence, engagement, self-assessment.

- Foursquare Math
- Pyramid Math
- X Marks the Spot
- Leo's Pattern

Next, we will look at a powerful strategy that will help students who struggle with their multiplication facts and fraction operations. This clever approach will only take 5 minutes a day! "Fast Facts and Fractions" is a proven strategy being used by teachers throughout the United States.

We will also learn how to maximize our time with our students by focusing on the content that is most crucial. I'll show you how to study your grade level math standards to find where you need to invest the most minutes.

I'll show you how I have learned to address multiple standards at once through content integration. This not only helps me to get through more standards in less time, it also helps students make powerful connections.

Lastly, we will revisit a couple of concepts from earlier in the day to see how we can extend existing content to foster deeper understanding in our students.

By the end of the day, you'll have a full toolbox of ideas and strategies that will help your students accelerate their growth in mathematics.

OVERVIEW

Materials:

☒ paper

Optional:

☐ activity master

Foursquare Math

The commutative property is the foundation of these creative practice problems. Because students are solving each problem two ways, their matching answers allow them to self-assess their work.

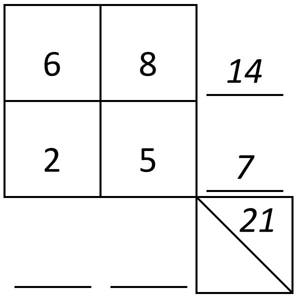
Vocabulary: commutative property, sum, addend, integer

PROCEDURE

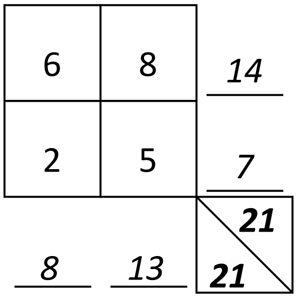
Skills:

- Addition and subtraction of whole numbers, integers, and decimals
- Finding patterns

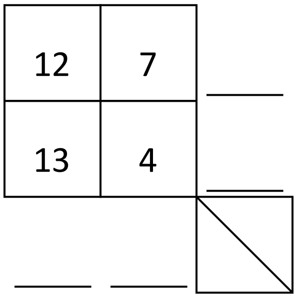
1. Write four numbers in the cells of a foursquare box. Add the six and the eight in the top row and write the sum to the right of the row. Repeat this for the two and the five in the second row. Add these two sums on the right and write the answer in the upper right triangle of the lower box as shown.



2. Now add the six and the two in the first column and write the sum in the blank below. Repeat this for the eight and the five in the second column. Add these two sums and write the answer in the lower left triangle of the lower box as shown. Voilà, the answers match!



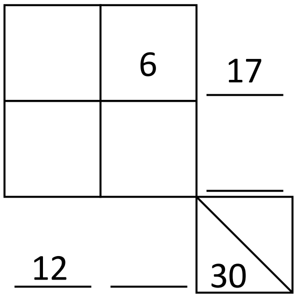
3. Your students will likely be surprised and will wonder if that always works. That's an invitation to try another one. Try one with four new numbers like the sample on the left. Your students will see that it works again.



4. Now try a third problem that uses the same numbers as the previous problem, but in a different arrangement. Not only do the numbers match, but you still get an answer of 36.
5. Ask your students to try to explain this mystery. It may occur to them that each triangle is the sum of the four numbers in the original four squares. This is a good time to introduce the

commutative property. This property states that a + b = b + a. It stands to reason that this will give the same sum.

6. Design further problems that challenge your students. You may wish to use two-digit numbers, decimals, or integers. Fractions can also be used, but since they are harder, you should assign fewer problems.
7. To introduce subtraction, put some numbers in the blanks and some in the cells like in the example on the right. Students must subtract to work backward and solve the puzzle. In most cases, only four numbers need to be given to solve the puzzle.



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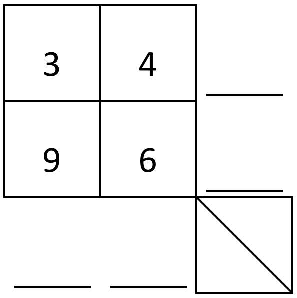
Journal Prompts:

Why does the same answer always appear in the two triangles?

What effect will switching the three and four have in the problem on the right? Will this always occur? Why or why not?



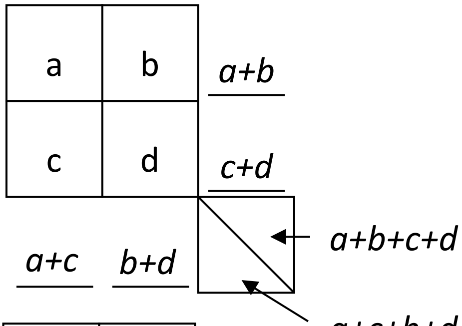
Homework:

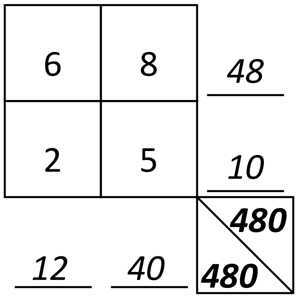
Assign one of the activity masters included or create your own using a blank master. You may wish to have students make up problems of their own which they write on the board for others to copy onto a blank master.

Taking a Closer Look:

Algebra students may wonder why these drills always produce matching answers. Substituting variables for the numbers shows that it is simply a matter of using the commutative property.

You may also have the students try multiplying the number pairs instead of adding. Since the commutative property also works for multiplication, matching answers will occur. However, multiplying the products





in the blanks can be tedious, so assign fewer problems. Oddly, this process also works for subtraction and division.

Good Tip!

To develop number sense, select number pairs that complement one another. For example, pick pairs that add to 100, decimals that add to one, or integers that are opposites.

Assessment: ☑

Since students should get matching answers in the two triangles, these drills are self-assessing for the most part. To be more certain, you may wish to write the answers to each worksheet randomly at the bottom of each page. Students can cross off answers as they find them to verify that they are doing the problems correctly.

Answer Key

(The answers in the lower triangles are given.)

Foursquare Math Worksheet Number

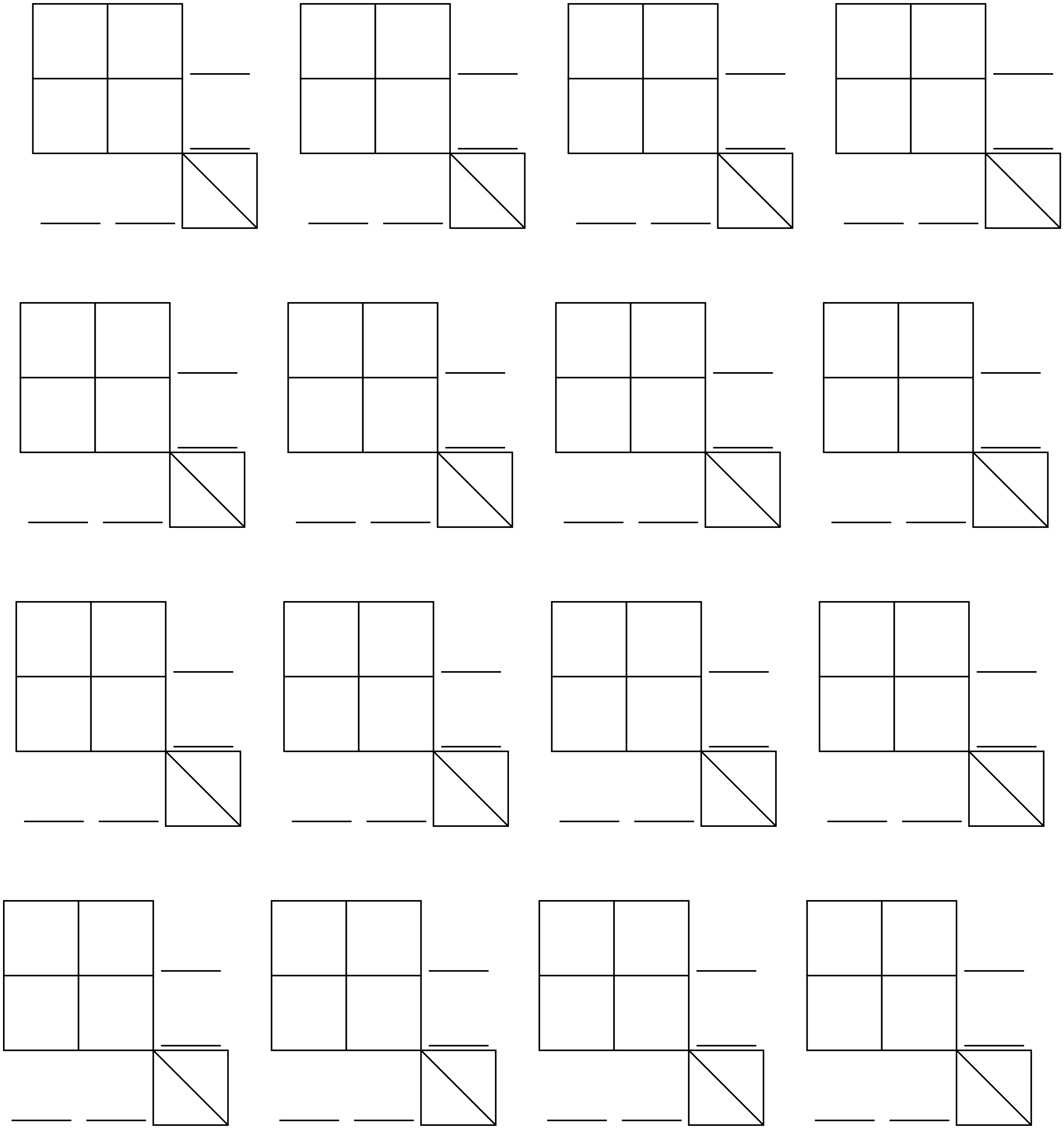
	1	2	3	4	5	6
Problem						
1	86	-34	6.5	47	31	1.4
2	135	8	1.63	given	given	given
3	121	-4	5.49	79	42	4.7
4	200	-16	99.99	given	given	given
5	89	-3	108.99	139	196	6.49
6	183	-21	22.2	given	given	given
7	163	0	55.09	97	97	3
8	163	0	9.26	given	given	given
9	145	-12	2.25	64	-10	2.89
10	180	15	7.2	43	26	.26
11	159	0	61.08	72	49	7.39
12	242	-1	9.03	given	given	given
13	100	13	24.6	given	given	given
14	258	-23	100.01	given	given	given
15	209	-63	20	94	-78	9.89

Activity master 1

Foursquare Math

Name________________________

Find the missing numbers by adding as you were shown in class. Do the sums in the triangles match?

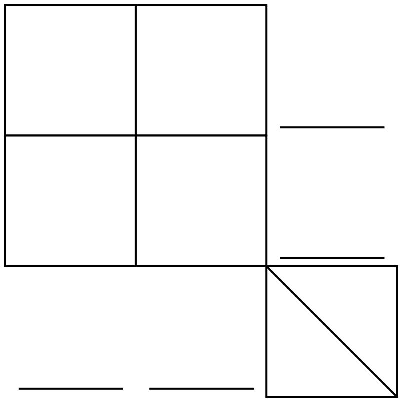


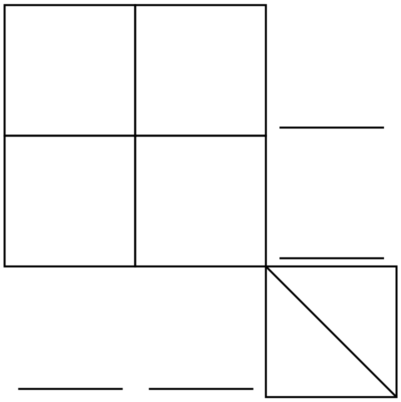
Activity master 2

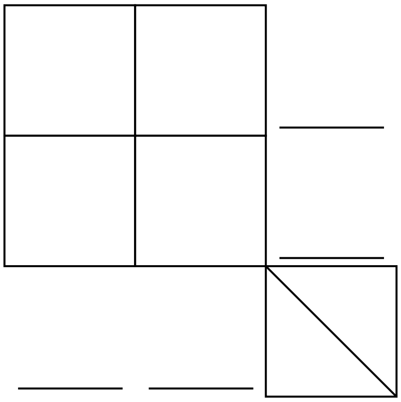
Foursquare Math

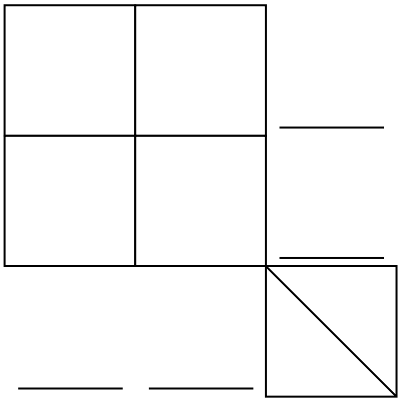
Name________________________

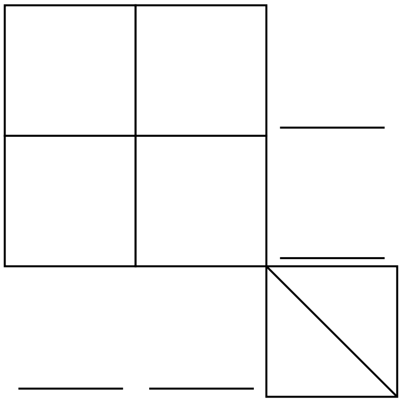
Find the missing numbers by adding as you were shown in class. Do the sums in the triangles match?

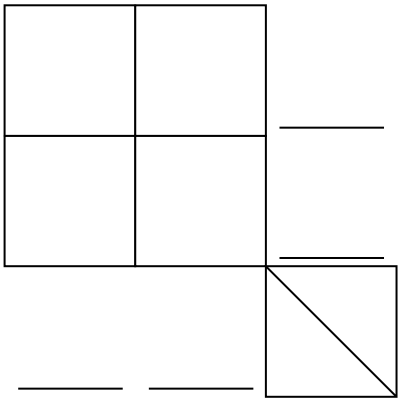












Activity master

Foursquare Math 1

Name________________________

Add across as in the example. Then add downward. Add the sums on the right side and write the answer in the upper triangle. Then add the lower sums and right the sum in the lower triangle. Do your answers match? Congratulations!

18	26	*44*
9	12	*21*
27	*38*	*65* *65*

1

19	19	_____
17	31	_____
_____	_____	

2

32	44	_____
7	52	_____
_____	_____	

3

11	35	_____
19	56	_____
_____	_____	

4

41	63	_____
59	37	_____
_____	_____	

5

61	12	_____
7	9	_____
_____	_____	

6

100	36	_____
42	5	_____
_____	_____	

7

19	45	_____
77	22	_____
_____	_____	

8

77	19	_____
45	22	_____
_____	_____	

9

88	19	_____
19	19	_____
_____	_____	

10

45	15	_____
55	65	_____
_____	_____	

11

38	54	_____
38	29	_____
_____	_____	

12

44	66	_____
77	55	_____
_____	_____	

13

28	26	_____
24	22	_____
_____	_____	

14

75	75	_____
54	54	_____
_____	_____	

15

26	62	_____
47	74	_____
_____	_____	

Foursquare Math 2

Name________________________

Add across as in the example. Then add downward. Add the sums on the right side and write the answer in the upper triangle. Then add the lower sums and right the sum in the lower triangle. Do your answers match? Congratulations!

-6	3	-3
-8	-12	-20
-14	-9	-23 -23

1

-8	-11	____
-5	-10	____
____	____	

2

-8	11	____
-5	10	____
____	____	

3

-8	-11	____
5	10	____
____	____	

4

4	-13	____
-6	-1	____
____	____	

5

14	0	____
-14	-3	____
____	____	

6

-25	6	____
12	-14	____
____	____	

7

-15	15	____
16	-16	____
____	____	

8

22	-11	____
11	-22	____
____	____	

9

-24	9	____
19	-16	____
____	____	

10

21	3	____
-3	-6	____
____	____	

11

24	-15	____
-4	-5	____
____	____	

12

18	-25	____
13	-7	____
____	____	

13

-18	25	____
13	-7	____
____	____	

14

-18	-25	____
13	7	____
____	____	

15

-18	-25	____
-13	-7	____
____	____	

Activity master

Foursquare Math 3

Name______________________

Add across as in the example. Then add downward. Add the sums on the right side and write the answer in the upper triangle. Then add the lower sums and right the sum in the lower triangle. Do your answers match? Congratulations!

1.8	2.6	4.4
.9	1.2	2.1
2.7	3.8	6.5 6.5

1

3.6	.9	____
1.2	.8	____
____	____	

2

.7	.02	____
.33	.58	____
____	____	

3

.09	.9	____
.5	4	____
____	____	

4

.09	.9	____
9	90	____
____	____	

5

.09	.9	____
9	99	____
____	____	

6

9.2	3.8	____
4.2	5	____
____	____	

7

.19	45	____
7.7	2.2	____
____	____	

8

5.6	.08	____
.08	3.5	____
____	____	

9

.54	.57	____
.58	.56	____
____	____	

10

4.5	1.5	____
.55	.65	____
____	____	

11

3.8	54	____
.38	2.9	____
____	____	

12

8.2	.07	____
.7	.06	____
____	____	

13

4.5	5.6	____
6.7	7.8	____
____	____	

14

.01	99	____
.96	.04	____
____	____	

15

6.3	3.7	____
2.8	7.2	____
____	____	

Activity master

Foursquare Math 4

Name______________________

Find the missing addends to solve each problem as in the example. You will need to work backwards to be successful.

18	*26*	44
9	*12*	21
27	38	65 *65*

1

	8	15
12		32

2

24		
	36	
28		47

3

19		64
52	27	

4

	35	
		39
36		74

5

	56	99
34		
	62	

6

1		
		45
	87	101

7

	19	73
		24
	48	

8

	17	41
	28	60

9

29		38
45	19	

10

		26
0		17
	19	

11

23		55
	16	
24		

12

	17	
		0
59		59

13

	46	
46		
72		109

14

11		44
	38	65

15

	33	35
41		
	51	

Activity master

Foursquare Math 5

Name______________________

Find the missing addends to solve each problem as in the example. You will need to work backwards to be successful.

18	*26*	44
9	*12*	21
27	38	65 *65*

1

	8	-1
12		32

2

24		
	36	
28		20

3

19		64
15	27	

4

	35	
		39
36		32

5

	-1	99
34		
	62	

6

-1		
		45
	87	101

7

	-19	73
		24
	48	

8

	17	41
	28	-11

9

29		38
-29	19	

10

		26
17		0
	19	

11

23		55
	16	
1		

12

	17	
		0
70		59

13

	-46	
46		
72		0

14

-11		-44
	-38	-65

15

	33	-35
41		
	-51	

Activity master

Foursquare Math 6

Name________________________

Find the missing addends to solve each problem as in the example. You will need to work backwards to be successful.

.8	*2.1*	2.9
.7	*1.5*	2.2
1.5	3.6	5.1 *5.1*

1

	.5	.6
.2		.8

2

1.4		
	1.6	
2.8		5.1

3

.9		1.9
2	2.7	

4

	3.5	
		6.9
3.6		13

5

	.9	.99
.2		
	6.2	

6

3.8		
		4.5
	9.7	12.4

7

	.1	.7
		2.3
	4.2	

8

	.09	4.1
	2.8	9.6

9

.09		2.8
.99	1.9	

10

		.26
.17		0
	.19	

11

.36		5.53
	1.22	
1		

12

	.23	
		0
.75		1

13

	.46	
0		
.48		3

14

1.01		2.1
	2	3.1

15

	3.21	5
4		
	4.1	

Pyramid Math

Where Skill Practice and Number Sense Combine

Overview:

This creative activity facilitates discovery of number patterns and develops number sense while providing critical skills practice. The activity works great with both positive and negative numbers and decimals, fractions, and even variable expressions in algebra. Because it can be designed to be self-checking, it is easy for the teacher and engaging for the students.

Required Materials:

☒ Paper

☒ Blank master

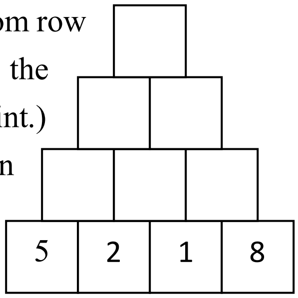
Optional Materials:

☐ activity master

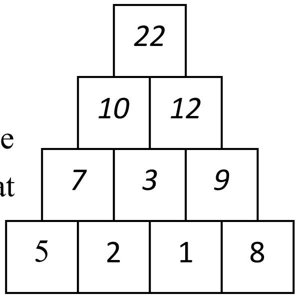
☐ calculators

Procedure:

1 Display the transparency master and enter four numbers in the cells of the bottom row as shown. (Use single-digit whole numbers at first so students can focus on the structure of the problem instead of struggling with the computation at this point.) To solve the pyramid, an adjacent number pair is added. The sum is written in the box above the number pair. This is repeated for the other number pairs in the bottom row. Then this process is repeated for the second row to fill the third row. Finally, the number pair in the third row is added to get the final top number as shown.



2 Since each sum is based on the sums below, all students should get the same answer in the top cell. **Thus ,they only need to check the top answer.** If that is correct, all other cells are likely correct too.

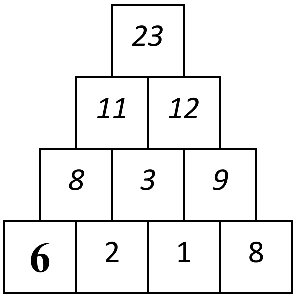


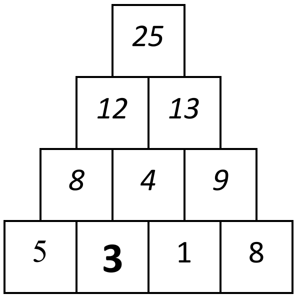
3 Now try another pyramid using new numbers. Students will catch on to the process quickly and will be eager to check their answer with those of their classmates. (No more correcting papers!)

4 As students understand how the problems work, introduce appropriate numbers. If you are studying decimals, throw in a few decimal points. If you have covered integers, use some negative numbers. Fractions make these problems much more difficult. Try one yourself before asking the students to do so. I suggest beginning with like denominators. Or you could

use fractions that have a fairly small common denominator. For example, ¼, ½, ½, and ¾ can all use fourths for a common denominator.

5 Try to make slight variations in the arrangement and values of the numbers to help children focus on the number sense involved. For example, in the first problem if we increase the five by one, making it a six, the top number also increases by one, but if we change the two to a three, the top number increases by three.





Is this always true when we add one to a cell? What would happen if we added two to the first or second cell of the bottom row? What happens when we do this to a five-row pyramid? As students answer these questions they will develop number sense.

6 Ask students to change the order of the numbers in the bottom row of a pyramid. How does this affect the top cell? Is the result always the same? How does the commutative property affect this result?

7 If everyone puts the same number in the top cell of a blank pyramid, will everyone get the same bottom row by working backwards? Why or why not?

8 Introduce subtraction by using Pyramid Math 6 in which other cells are filled in. You can create one of your own easily, or have students create them for their classmates to solve.

9 Explore what happens when all odd numbers or all even numbers are used. What patterns occur when all four cells in the bottom row contain the same number?

* * *

Journal Prompts:

If you rearrange the numbers on the bottom row of a pyramid, will you always get the same numbers on top? Why or why not?

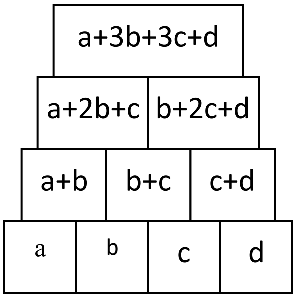
What can you predict about the number on the top of a four-row pyramid if all the starting numbers are equal? Does the number of rows in the pyramid affect this? In what way?

Homework:

Use one of the accompanying activity masters or tailor one to your students' needs using one of the blank masters.

Taking a Closer Look:

There is a way of predicting the top of the pyramid without solving all the rows. This leads students into the algebra involved in the process. For example, let's assume that we are going to solve a four-row pyramid. The bottom cells contain four numbers called a, b, c, and d as shown. It follows that the second row contains three sums which are a + b, b + c, and c + d respectively. The third row contains these two sums:

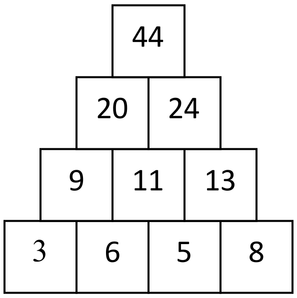


$$(a + b) + (b + c) \text{ and } (b + c) \text{ and } (c + d)$$

These simplify into a + 2b + c and b + 2b + c. Adding these to get the top row gives a + 3b + 3c + d.

Now let's start with four numbers: a = 3, b = 6, c = 5, and d = 8. Using the formula, the top answer should be:

$$a + 3b + 3c + d =$$
$$3 + 3(6) + 3(5) + 8 =$$
$$3 + 18 + 15 + 8 = 44$$



Assessment: ☑

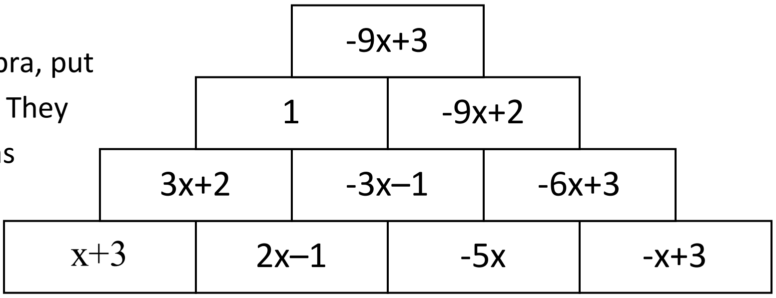
These activities can be made self-assessing by writing the answers at the bottom of the page. As students solve each pyramid, they can cross off the answers. If they get an answer that is not listed, they know they have made a mistake and can try again.

Answer Key:

	Pyramid Math Worksheet Number				
	1	2	3	4	5
Problem					
1	33	16.5	21	267	$4\,{}^{2}/_{5}$
2	24	10.2	-21	224	$3\,{}^{3}/_{7}$
3	40	4	11	377	
4	32	3.2	-20	264	
5	60	2.05	-8	312	
6	68	23.86	0	332	
7	80	6.84	24	222	
8	23	16	-34	308	
9	23	21.81	0	332	
10	100	3.08	100	363	
11	106	1	1	528	
12	188	1.76	9	2000	

Great Tip!

For students who are preparing for algebra, put variable expressions in the bottom cells. They can then practice combining like terms as shown. Or put them in some of the upper cells so they can subtract binomials by working downward.



For sets six through eight, these are the answers to the bottom row of each pyramid.

	Pyramid Math Worksheet Number		
	6	7	8
Problem			
1	8, 3, 2, 2	.3, .1, .7, .6	6, -4, 5, -2
2	3, 4, 4, 2	.5, .6, .2, .5	8, -12, 6, -8
3	11, 3, 2, 10	.6, .2, .5, .5	8, 6, -12, -8
4	1, 0, 11, 23	3, .6, .9, 1	-3, -6, -9, -14
5	7, 12, 10, 11	.7, .9, .9, 2	-8, 0, 19, -20
6	19, 0, 6, 26	.9, 0, .5, .9	-7, -6, 16, -16
7	27, 0, 0, 27	1.2, 6.3, 2.7, 3.1	21, -9, -19, -8
8	8, 19, 10, 9	17, 1.9, .8, 6.4	11, 30, -16, -14
9	31, 26, 19, 17	8.7, 7.6, 6.5, 5.4	11, -30, -16, 14
10	18, 2, 0, 27	.29, .16, .5, 1.2	-17, -3, 3, 17
11	5, 38, 19, 31	7.1, .99, .09, .5	-15, -5, 6, 13
12	13, 6, 34, 9	7.7, .77, .55, 5	18, -9, 6, -3

The Common Core Connection

In addition to providing skills practice in addition of all number types, these pyramid problems cab also help students think mathematically and develop number sense. They are a great way to develop the **eight mathematical practices** of the Common Core Math Standards:

1. Make sense of problems and persevere in solving them

This practice applies when students are developing an understanding of the problem and its underlying mathematics. Students also practice this when they make conjectures about what will happen in a problem. That is why it is important to pose questions to the students such as, "What do you think will happen if we rearrange the numbers?" "If the bottom numbers in a pyramid are all odd, will the top number be even or odd?"

2. Reason abstractly and quantitatively

As students move past the arithmetic of a problem and begin to think about the fact that the numbers can vary and what happens when they do, they are thinking abstractly. That is why it is important to help your students to move toward exploring what happens when the bottom row contains variables instead of specific numbers.

3. Construct viable arguments and critique the reasoning of others

When students are asked to make conjectures, we should also ask them to explain their reasoning. "Why do you think that the top number will be even?" Students will need to learn the skills of explaining their thinking and of disagreeing with the conjectures of others in an appropriate manner.

4. Model with mathematics

This practice involves using mathematics to represent the problems they see in the real world. At first it might seem not to apply to the pyramid problems because they are skills practice. However, using a formula as a model to explain why the pyramids behave the way they do is an example of this practice.

5. Use appropriate tools strategically

In this activity students might employ pencil and paper as their primary tools. However, the teacher may wish to allow calculators on some of the more challenging problems. A good guide in knowing when to switch from paper to calculator is this: **when the arithmetic is impeding the mathematical thinking, students are only getting skills practice and are not likely to develop number sense**.

Students in a computer class could also create a spreadsheet that would solve pyramid problems. Designing such a spreadsheet would require the students to know the *how and why* behind the mathematics that governs the pyramid problems.

6. Attend to precision

When students self-assess either by comparing their answers to others or by checking their solution against an answer bank, they are more likely to amend their errors (Marcy). Working separately, they often lack the number sense to know if the result of their calculation is right or even reasonable.

7. Look for and make use of structure

As students explore and come to understand how the structure of these problems affects the number at the top of the pyramid, they are developing this skill. That is why it is important not only to provide them with random numbers in practice problems, but also with examples that illustrate what happens when the same numbers are rearranged or changed slightly.

8. Look for and express regularity in repeated reasoning

As students notice the effects of these changes in the bottom numbers, they begin to notice shortcuts. For example, adding one to either of the two squares on the outside of the bottom row *always* increases the top box by one, but adding one to one of the interior cells of the bottom row *always* increases the top number by three.

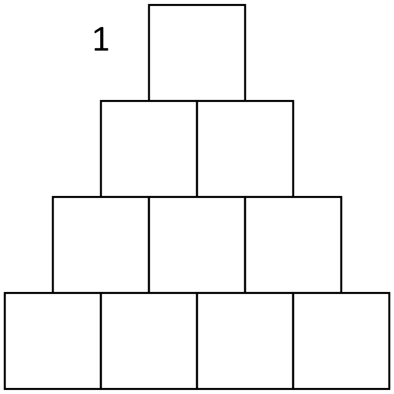
Activity Master 1

Pyramid Math

Name_______________________________

Add pairs of adjacent numbers and write their sums in the box above them. Keep going until you reach the top of the pyramid.

1



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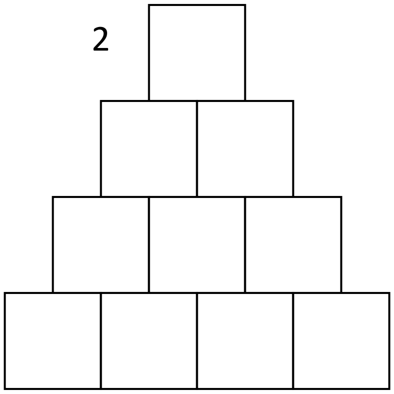

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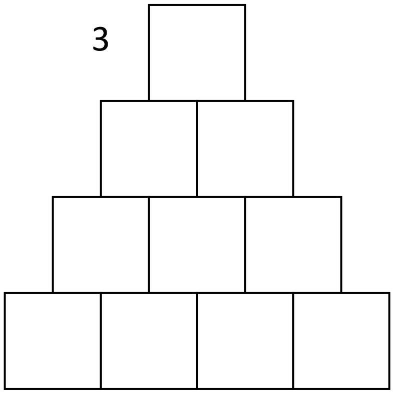

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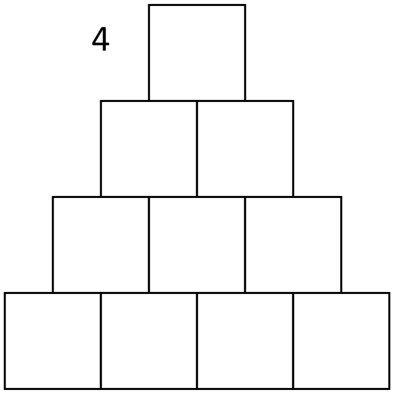

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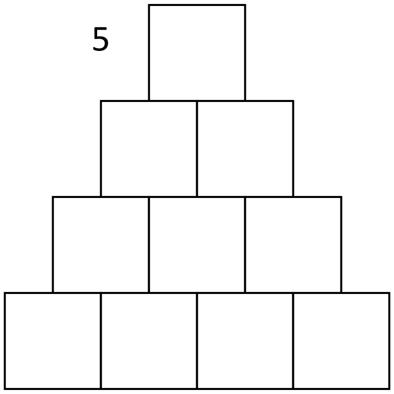

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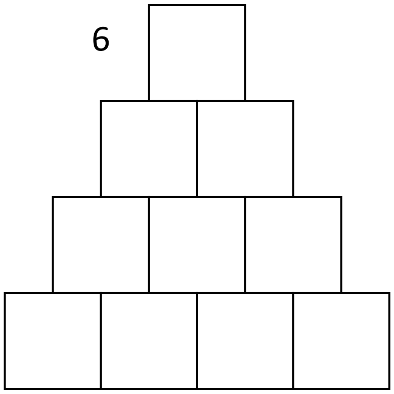

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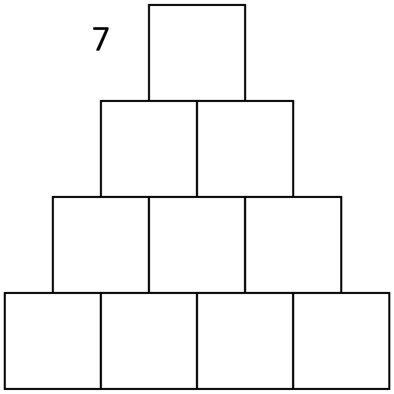

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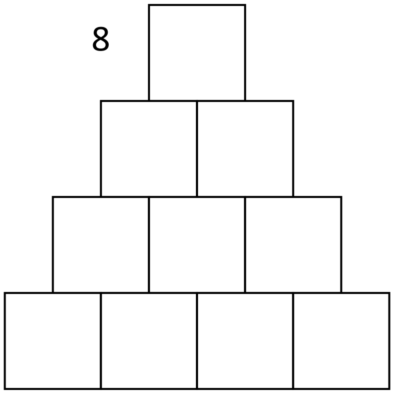

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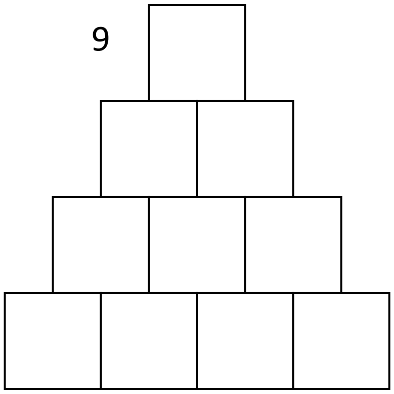

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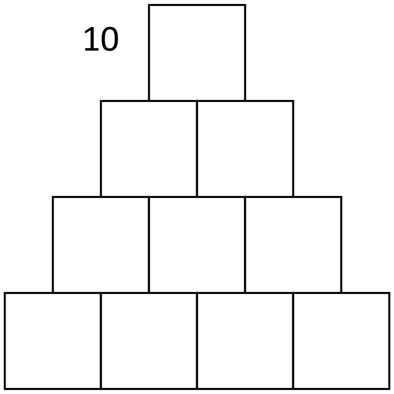

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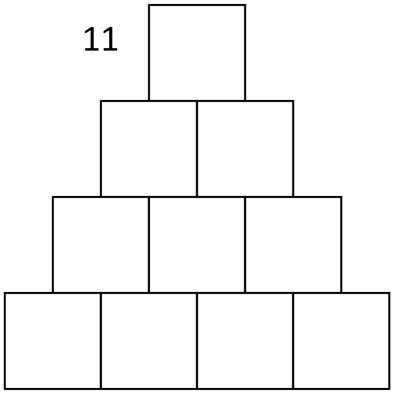

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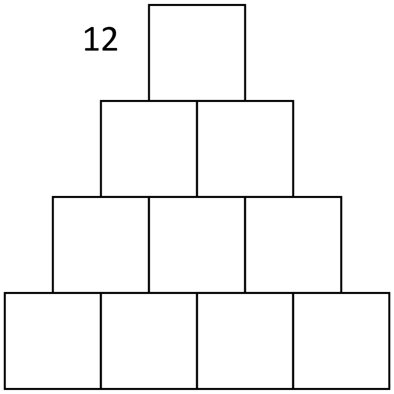


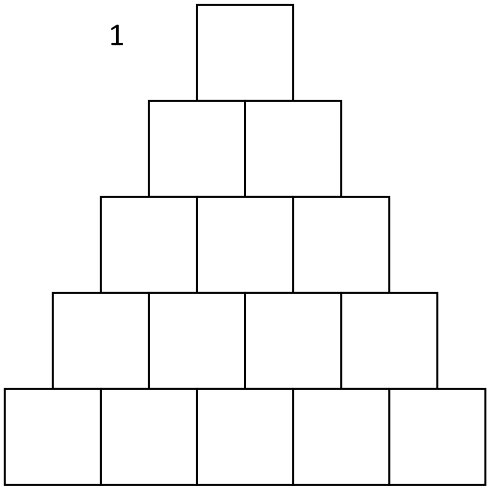
Activity Master 2

Pyramid Math

Name______________________________

Add pairs of adjacent numbers and write their sums in the box above them. Keep going until you reach the top of the pyramid.

1



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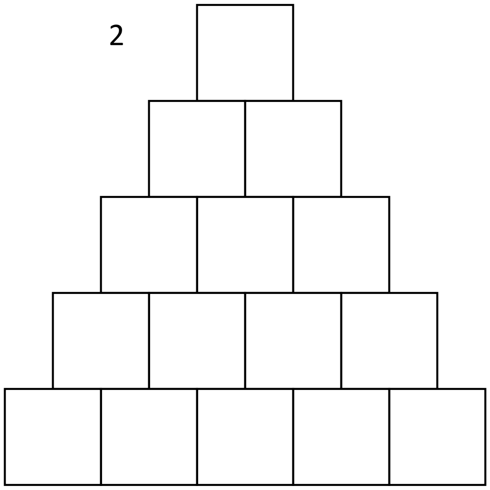

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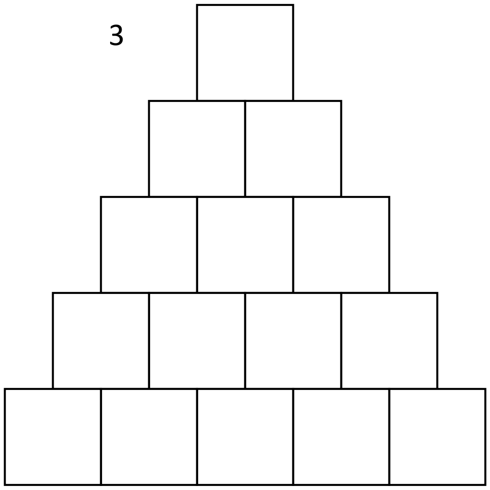

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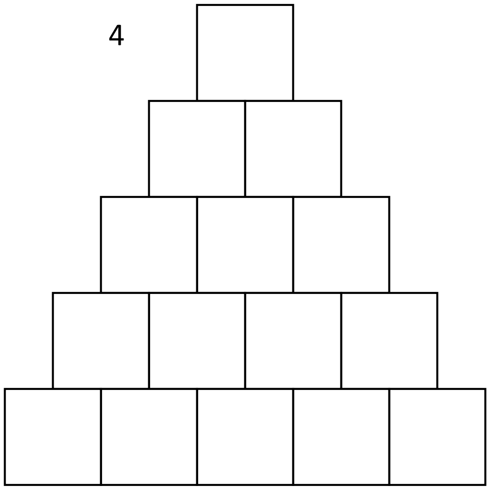


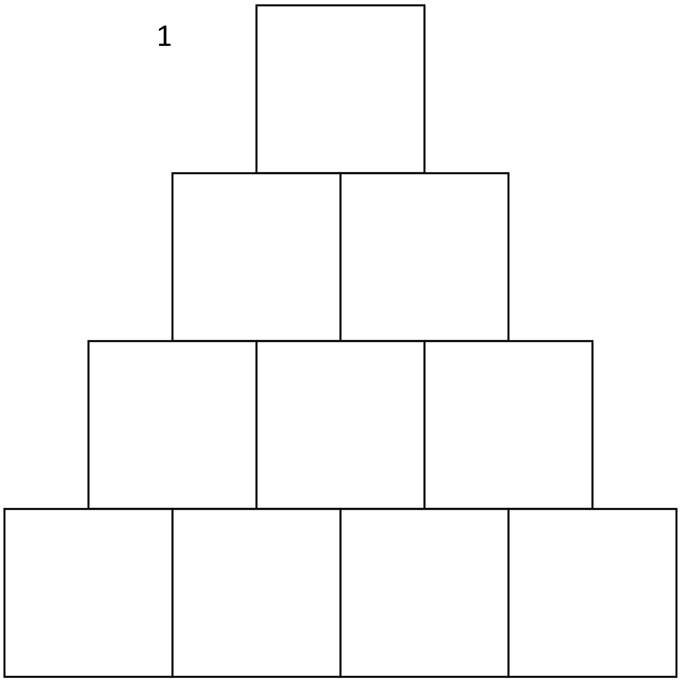
Activity Master 3

Pyramid Math

Name______________________________

Add pairs of adjacent numbers and write their sums in the box above them. Keep going until you reach the top of the pyramid.

1



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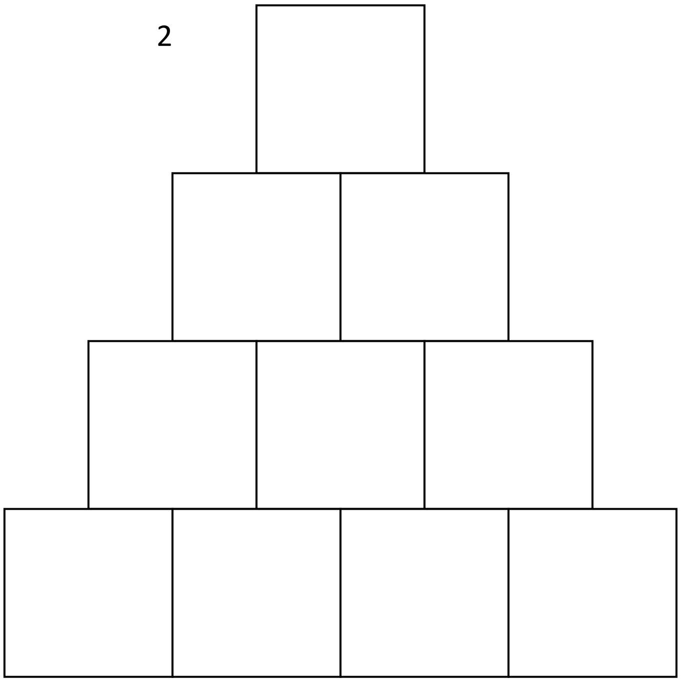


Activity Master

Pyramid Math 1

Name______________________________

Add pairs of adjacent numbers and write their sums in the box above them as in the first example. Keep going until you reach the top of the pyramid.

1

		9	
8	4	2	7

2

6	0	3	9

3

2	4	6	8

4

1	3	5	7

5

3	6	9	12

6

4	7	10	13

7

4	8	12	16

8

8	0	5	0

9

0	5	0	8

10

20	9	12	17

11

11	14	14	11

12

22	23	24	25

Activity Master

Pyramid Math 2

Name______________________________

Add pairs of adjacent numbers and write their sums in the box above them as in the first example. Keep going until you reach the top of the pyramid.

1

		.9	
.2	5	.2	.7

2

.7	.16	3	.02

3

.2	.4	.6	.8

4

.1	.3	.5	.7

5

.25	.11	.09	1.2

6

.06	7	.5	1.3

7

2	.06	1.5	.16

8

.01	5	0	.99

9

8.4	.67	3.2	1.8

10

.22	.33	.44	.55

11

.06	.19	.06	.19

12

.22	.22	.22	.22

Activity Master

Pyramid Math 3

Name______________________________

Add pairs of adjacent numbers and write their sums in the box above them as in the first example. Keep going until you reach the top of the pyramid.

1

		-2	
-4	7	3	-5

2

4	-7	-3	5

3

4	7	-3	-5

4

-1	-2	-3	-4

5

-1	2	-3	-4

6

1	-2	3	-4

7

8	-6	15	-11

8

-13	-5	0	-6

9

11	-21	31	-41

10

40	-15	40	-15

11

8	2	-14	29

12

9	3	-13	30

Activity Master

Pyramid Math 4

Name______________________________

Add pairs of adjacent numbers and write their sums in the box above them as in the first example. Keep going until you reach the top of the pyramid.

1

		54	
55	42	16	38

2

27	19	31	47

3

58	65	34	22

4

18	28	38	48

5

54	44	34	24

6

37	37	46	46

7

59	29	9	49

8

43	34	43	34

9

82	28	28	82

10

63	37	45	54

11

18	82	82	18

12

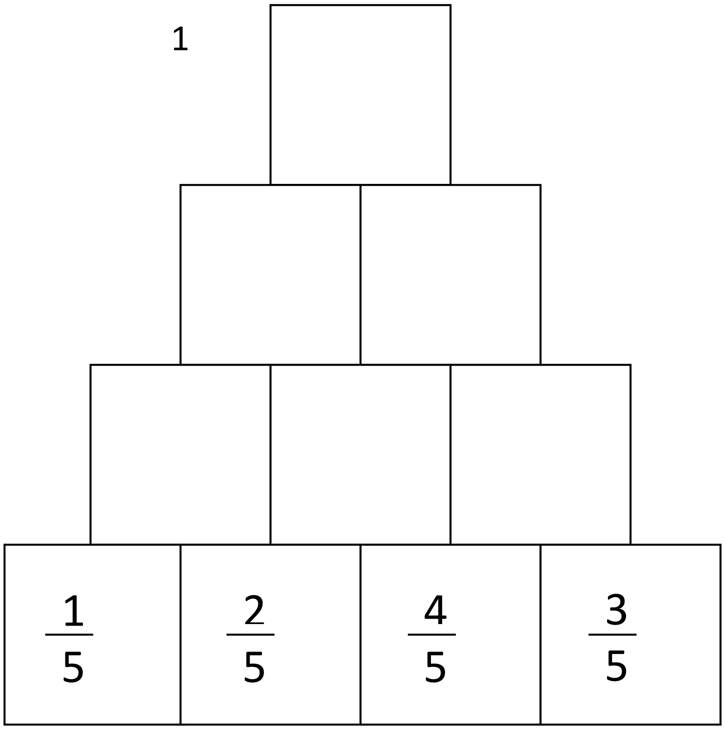
100	200	300	400

Pyramid Math 5

Name______________________________

Add pairs of adjacent numbers and write their sums in the box above them. Keep going until you reach the top of the pyramid.

1



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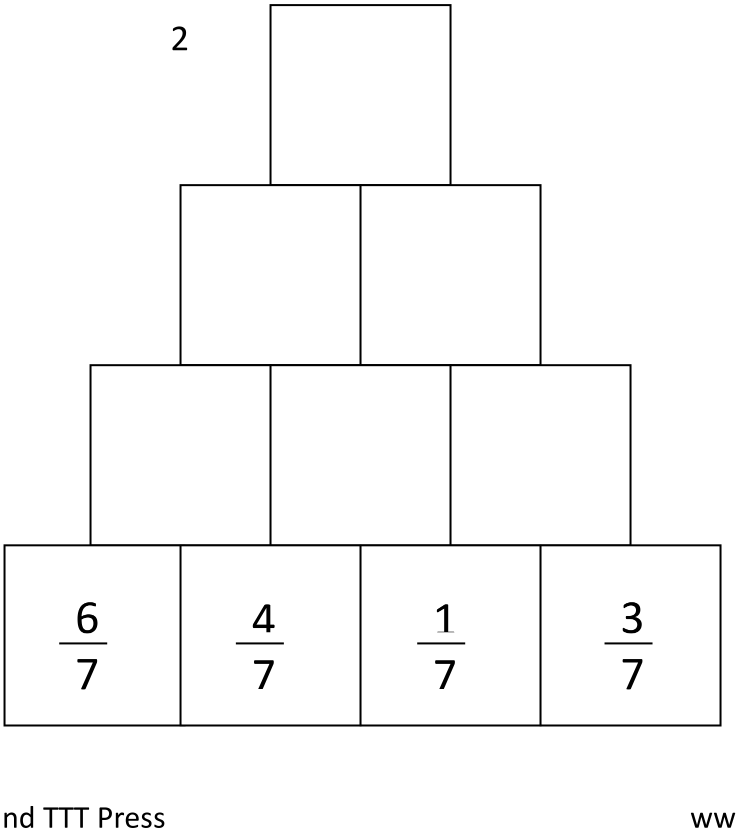


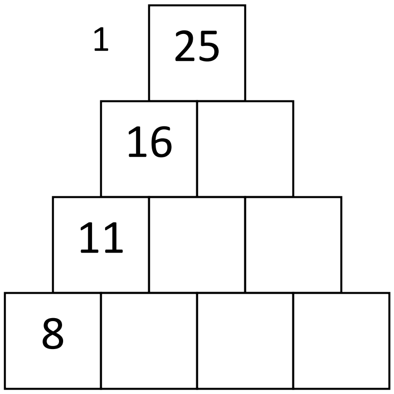
Activity Master

Pyramid Math 6

Name______________________________

Each number is the sum of the two numbers below it. Work backward to fill in the bottom row.

1



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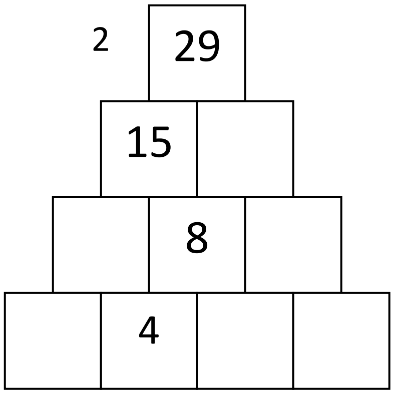

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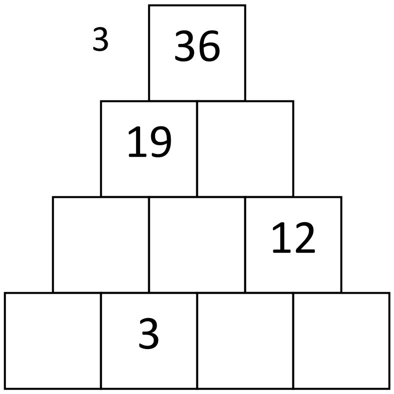

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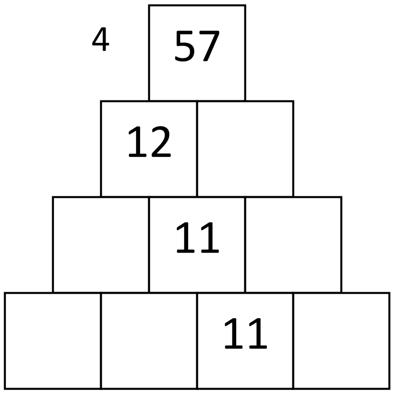

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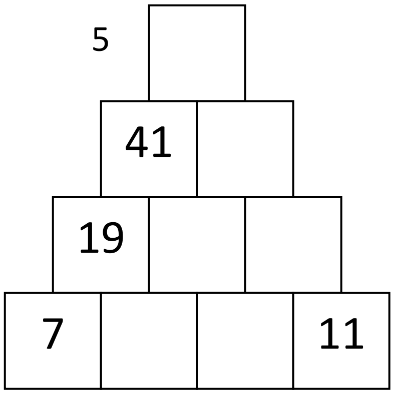

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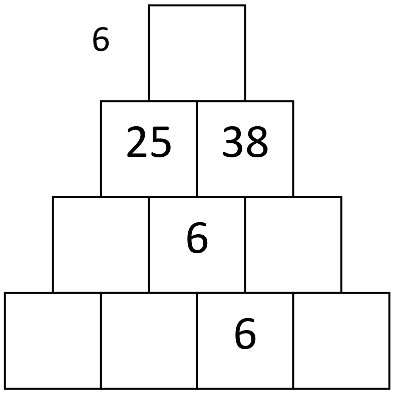

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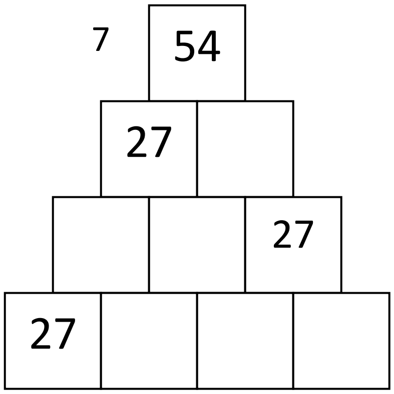

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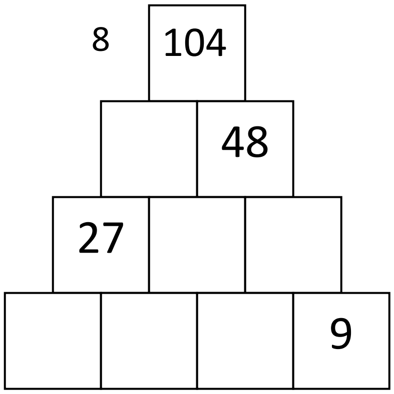

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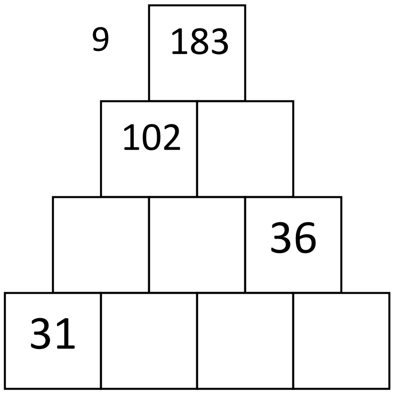

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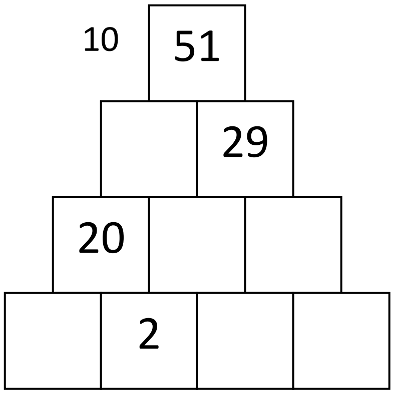

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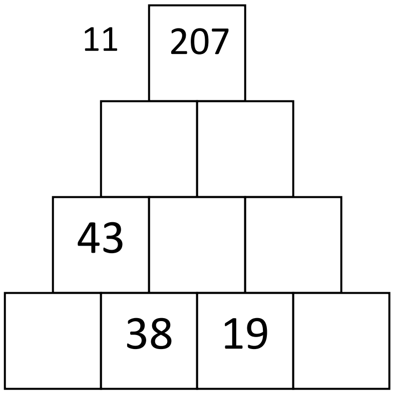

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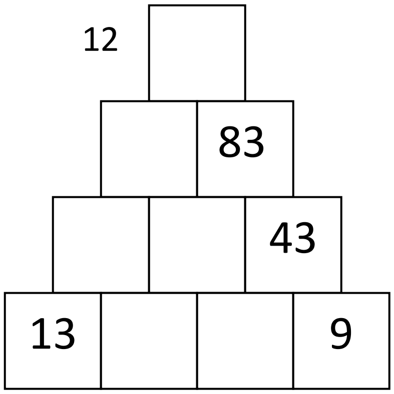


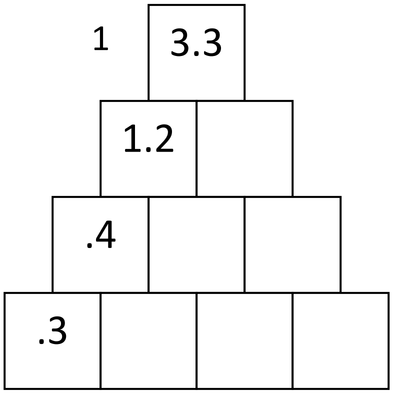
Activity Master

Pyramid Math 7

Name______________________________

Each number is the sum of the two numbers below it. Work backward to fill in the bottom row.

1



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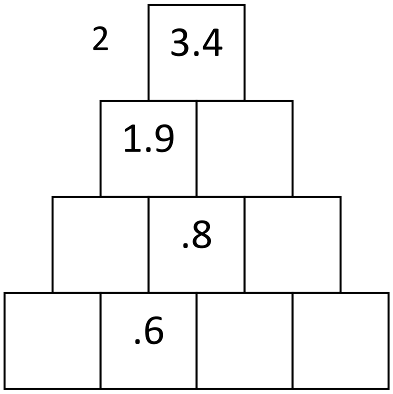

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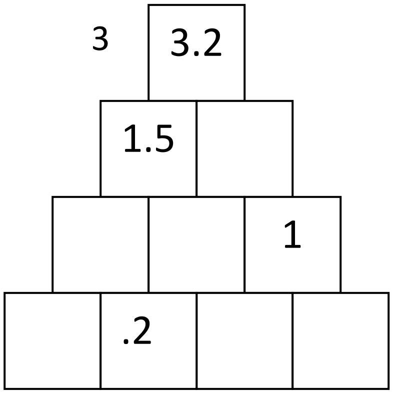

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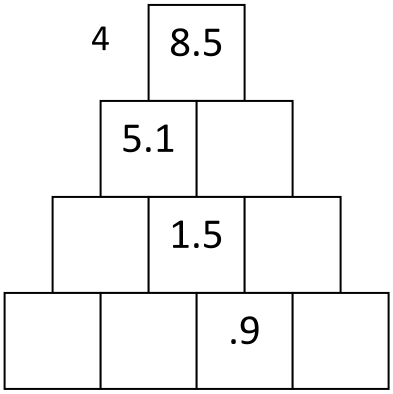

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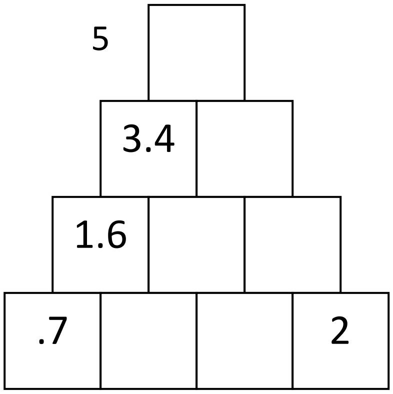

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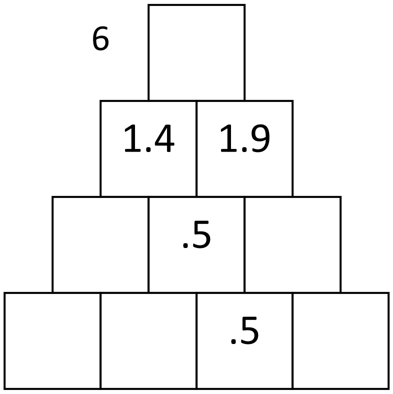

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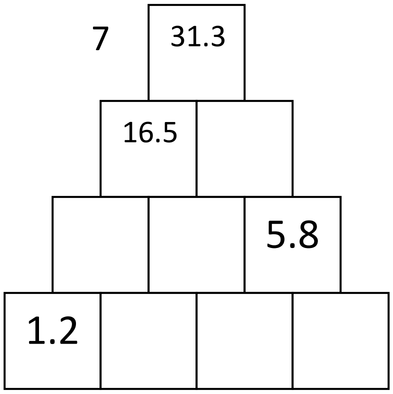

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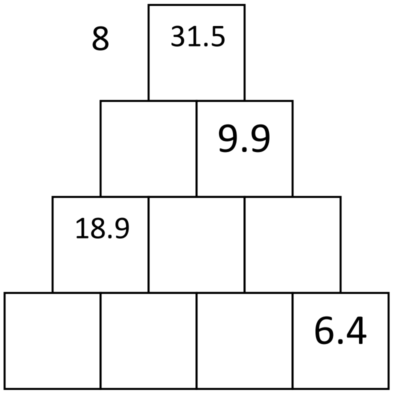

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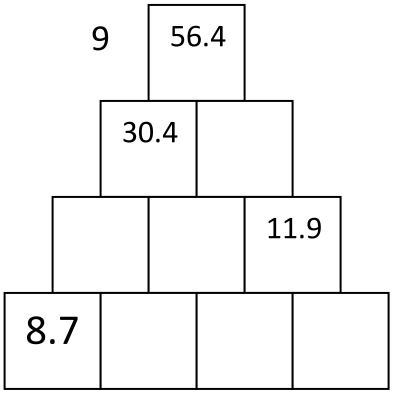

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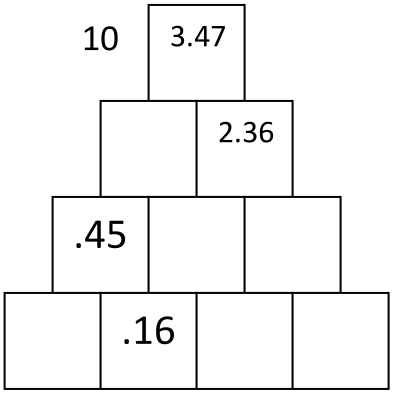

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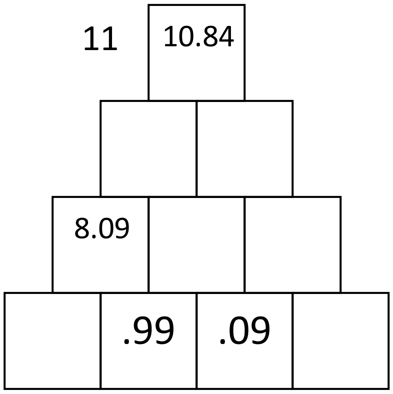

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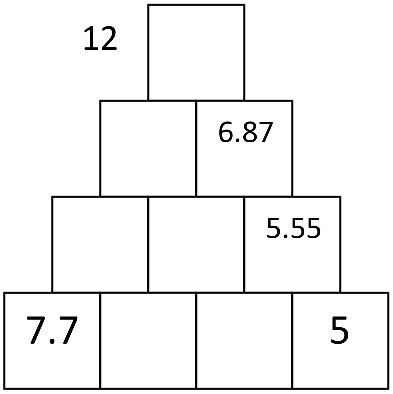


Activity Master

Pyramid Math 8

Name______________________________

Each number is the sum of the two numbers below it. Work backward to fill in the bottom row.

1

			7			
		3				
	2					
6						

2

			-18			
		-10				
			-6			
		-12				

3

			-18			
		8				
					-20	
		6				

4

			-62			
		-24				
			-15			
				-9		

5

		11				
	-8					
-8						-20

6

		-3		10		
			10			
				16		

7

			-71			
		-16				
					-27	
21						

8

			39			
				-16		
	41					
						-14

9

			-113			
		-65				
					-2	
11						

10

			0			
				20		
	-20					
		-3				

11

			1			
	-20					
		-5		6		

12

				0		
					3	
18						-3

X Marks the Spot
Developing Skill in All Operations

Overview:

The rules are so simple you can teach them without saying a word! Yet the math is rich and abundant. You can use these simple drills to reinforce basic addition, subtraction, multiplication, and division facts and develop number sense without boring your students. Fractions, decimals, and negative numbers can also be used. You can even factor polynomials using this simple method!

Required Materials:

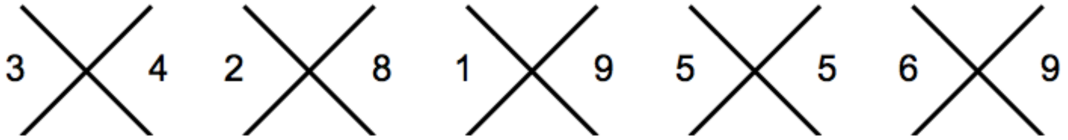
☒ Pencil and paper

Optional Materials:

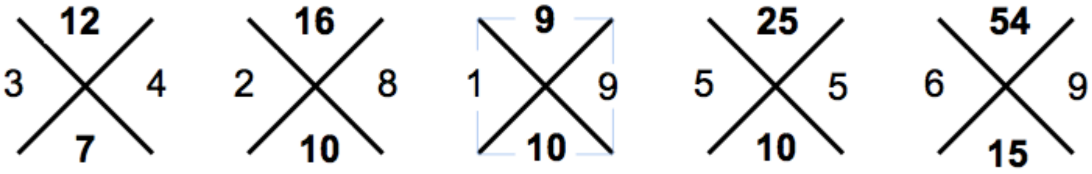
☐ Calculators

Procedure:

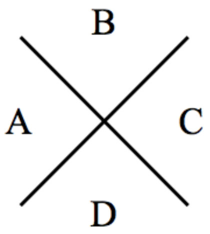
1. Tell the class that this game has only two simple rules...but you won't tell them what they are. They will have to figure out the rules by themselves. As soon as a student knows how to play, he or she can come up to the board and write down the answer.
2. Have them copy these five problems onto a piece of paper as you write them on the board.



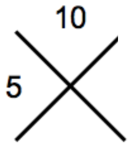
Then begin writing in the answers by adding the numbers on the left and right to get the bottom number and multiplying them to get the top number.



3. If you work slowly, pausing as if to ponder before writing each answer, some of the students will soon catch on. After a majority of the class has discovered the rules of the game, allow a student to explain them.
4. Then you can continue to play the game by varying the format.

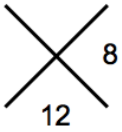


- Placing numbers in sections A and B will require students to divide first and then add.



5 + 2 = 7

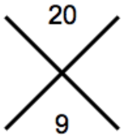
- Placing numbers in sections A and D or C and D will require students to subtract first and then multiply.



12 – 8 = 4

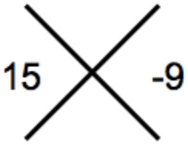
4 x 8 = 32

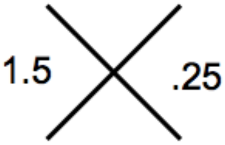
- Placing numbers in sections B and D will require students to study the various combinations of sums and products that satisfy the given answers.

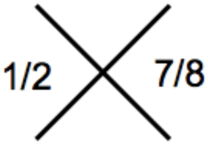


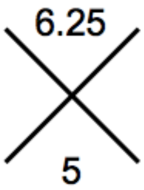
1 x 20 = 20	1 + 20 = 21
2 x 10 = 20	2 + 10 = 12
4 x 5 = 20	**4 + 5 = 20**

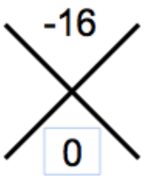
5. Eventually, you may wish to increase the difficulty through examples like these.

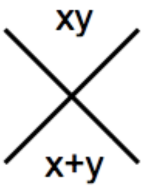


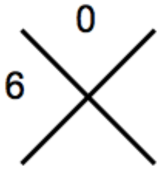


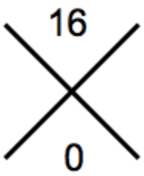




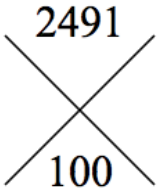








6. You can also use the guess and check method to solve complex puzzles. Research has shown that the guess and check method is not only a valuable skill, it helps children transition to solving equations in algebra. Here is how to solve problems like the one on the right using this method.



Pick a pair of numbers that add up to 100 such as 50 and 50. Write them in columns *a* and *b*.

Then multiply them to find the product. In this case, it is 2500, which is too high. We mark our check with an "H" to signify that this is too high. This tells us that the number in column *a* is too high.

Let's adjust our guess by trying 40 and 60. Remember that our guesses must add to 100. *It is also very important to note that the smaller of the two numbers should go in column a*.

The product of these two numbers is 2400, which is too low. This can be marked with an "L" because it is too low.

Our next guess for column a must be greater than 40 but less than 50. Let's try 45. This makes b = 55. Our new product, 2475 is too low also.

a	*b*	check
50	50	2500 H
40	60	2400 L
45	55	2475 L
48	52	2496 H
47	53	2491 ☺

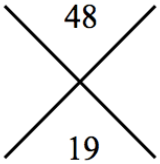
Our fourth guess will be 48. Now b = 52, and our product is 2496. Although this is too high, it is very close.

For our next guess, we try 47 for *a*, and 53 for *b*. This gives us the product we wanted.

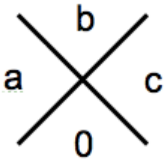
* * *

Journal Prompts:

Explain to a student how you would find the solution to the problem on the right.



What could you tell about the value of a and c in the example below? What can you tell about the value of b? Explain



Homework:

Assign one of the accompanying activity masters.

You can make a homework worksheet by placing numbers in a copy of the blank activity master. Alternately, the students can copy down problems as you write them on the board.

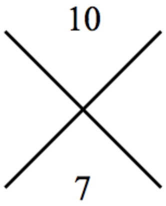
Taking a Closer Look:

The difficulty of these drills can be varied by the numbers chosen and their placement. Using decimals, fractions, or negative numbers can also increase the complexity.

Algebra students can practice factoring polynomials this way too. For the polynomial $x^2 + \mathbf{7}x + \mathbf{10} = 0$, students would construct the problem shown to find the solutions 2 and 5. The expression factors into the following binomials:

$$x^2 + 7x + 10 = (x + 2)(x + 5)$$

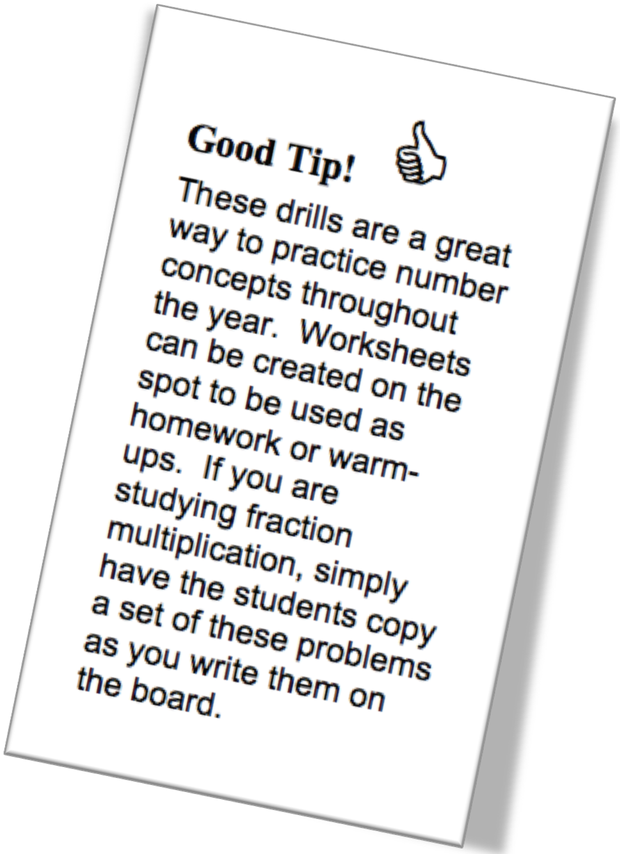
The solution to the equation then is $x = \{-2, -5\}$.



To help them remember which term goes on the top and which goes on the bottom, I tell them that the *c* term goes in the *ceiling* and the *b* term goes in the *basement*. This technique works well when the *a* term is 1. A fuller treatment of factoring polynomials when the *a* term does not equal 1 can be found in my DVD *Multiplying and Factoring Polynomials.*

Assessment: ☑

These drills can be spot checked for accuracy or students can exchange papers to check them. You may also use the answer keys for the accompanying activity masters.



Good Tip!

These drills are a great way to practice number concepts throughout the year. Worksheets can be created on the spot to be used as homework or warm-ups. If you are studying fraction multiplication, simply have the students copy a set of these problems as you write them on the board.

Answer Key:

Set 1	top	bottom	Set 2	side	top	Set 3	side	bottom	Set 4	side	side
a	14	9	a	5	35	a	8	10	a	4	7
b	45	14	b	6	24	b	7	14	b	4	8
c	12	7	c	0	0	c	6	14	c	3	12
d	8	9	d	7	49	d	7	13	d	6	8
e	36	12	e	12	60	e	9	13	e	6	6
f	36	13	f	9	54	f	12	15	f	0	12
g	48	14	g	11	22	g	6	12	g	4	9
h	12	8	h	9	108	h	4	16	h	7	9
i	0	7	i	12	96	i	6	10	i	8	8
j	9	6	j	10	10	j	8	15	j	6	10
k	20	9	k	12	48	k	7	11	k	5	12
l	25	10	l	12	120	l	0	9	l	8	11
m	33	14	m	2	12	m	3	8	m	10	12
n	60	16	n	12	132	n	2	11	n	11	12
o	24	14	o	0	0	o	7	10	o	7	12
p	77	18	p	6	42	p	12	21	p	12	12
q	72	20	q	9	72	q	11	21	q	8	9
r	60	16	r	12	0	r	12	23	r	9	11
s	60	17	s	12	144	s	1	9	s	9	9

Set 5	top	bottom	Set 6	top	bottom	Set 7	side	side	Set 8	side	side
a	1	7.2	a	-14	-5	a	-36	-1	a	47	53
b	0.45	1.4	b	45	-14	b	-1	36	b	23	63
c	1.2	4.3	c	-12	1	c	-6	6	c	50	60
d	0.008	0.18	d	8	-9	d	2	14	d	211	289
e	3.6	6.6	e	-36	0	e	-14	-2			
f	0.36	1.3	f	36	-13	f	-7	4			
g	0.048	0.68	g	-48	2	g	-4	7			
h	0.12	0.8	h	-12	4	h	-9	-2			
i	0	0.7	i	0	-7	i	-6	3			
j	0.9	3.3	j	9	-6	j	-6	-3			
k	0.002	0.09	k	-20	1	k	-18	-1			
l	0.025	0.55	l	-25	0	l	-9	-5			
m	0.33	1.4	m	33	-14	m	-15	-3			
n	6	10.6	n	-60	-4	n	-15	3			
o	0.24	2.12	o	24	-14	o	-1	45			
p	0.077	0.81	p	77	-18	p	-8	-6			
q	0.96	1.28	q	-96	-4	q	-6	8			
r	0.6	10.06	r	60	-16	r	-12	4			
s	6	12.5	s	-60	-7	s	-2	24			

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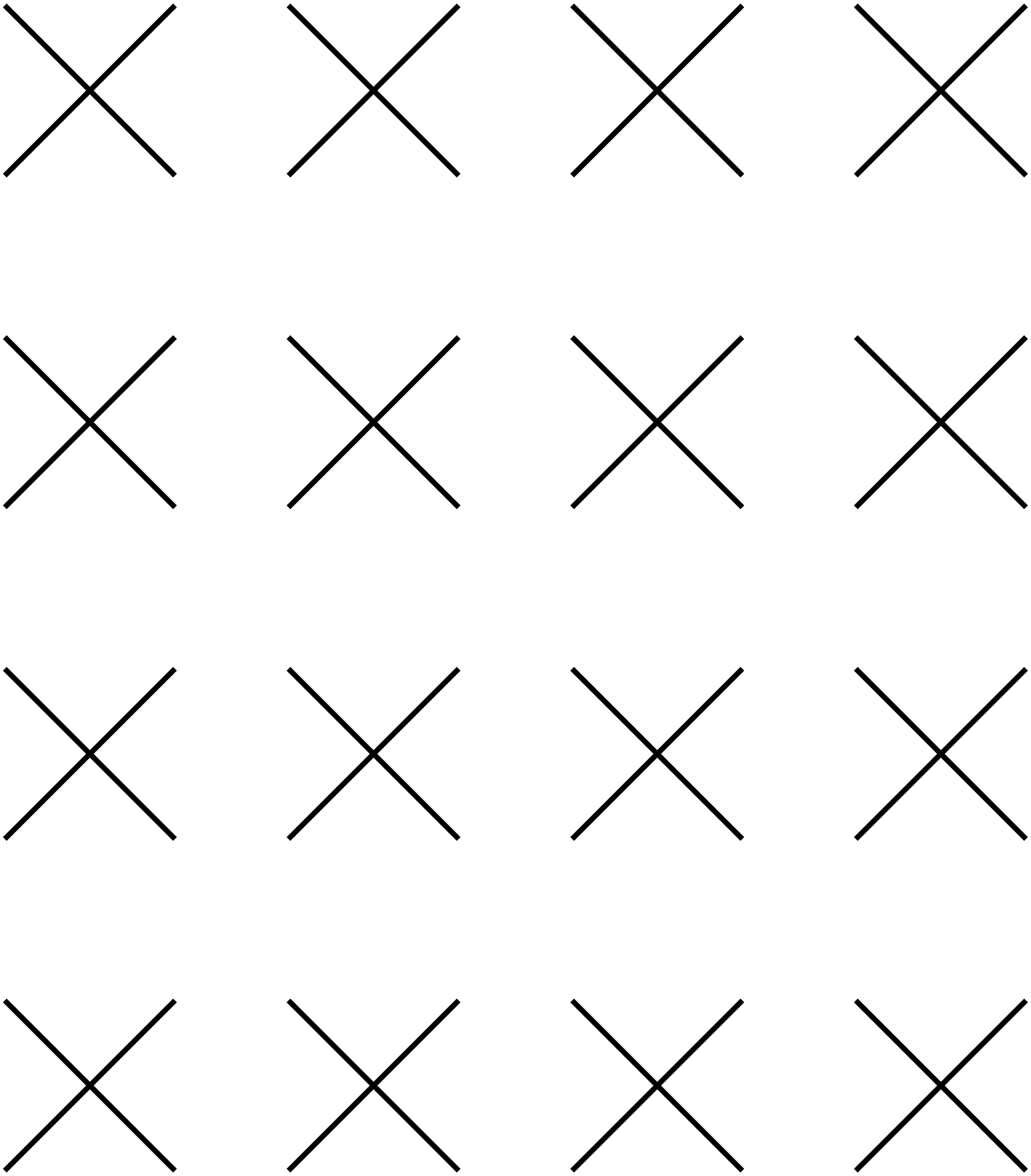
"X" Marks the Spot

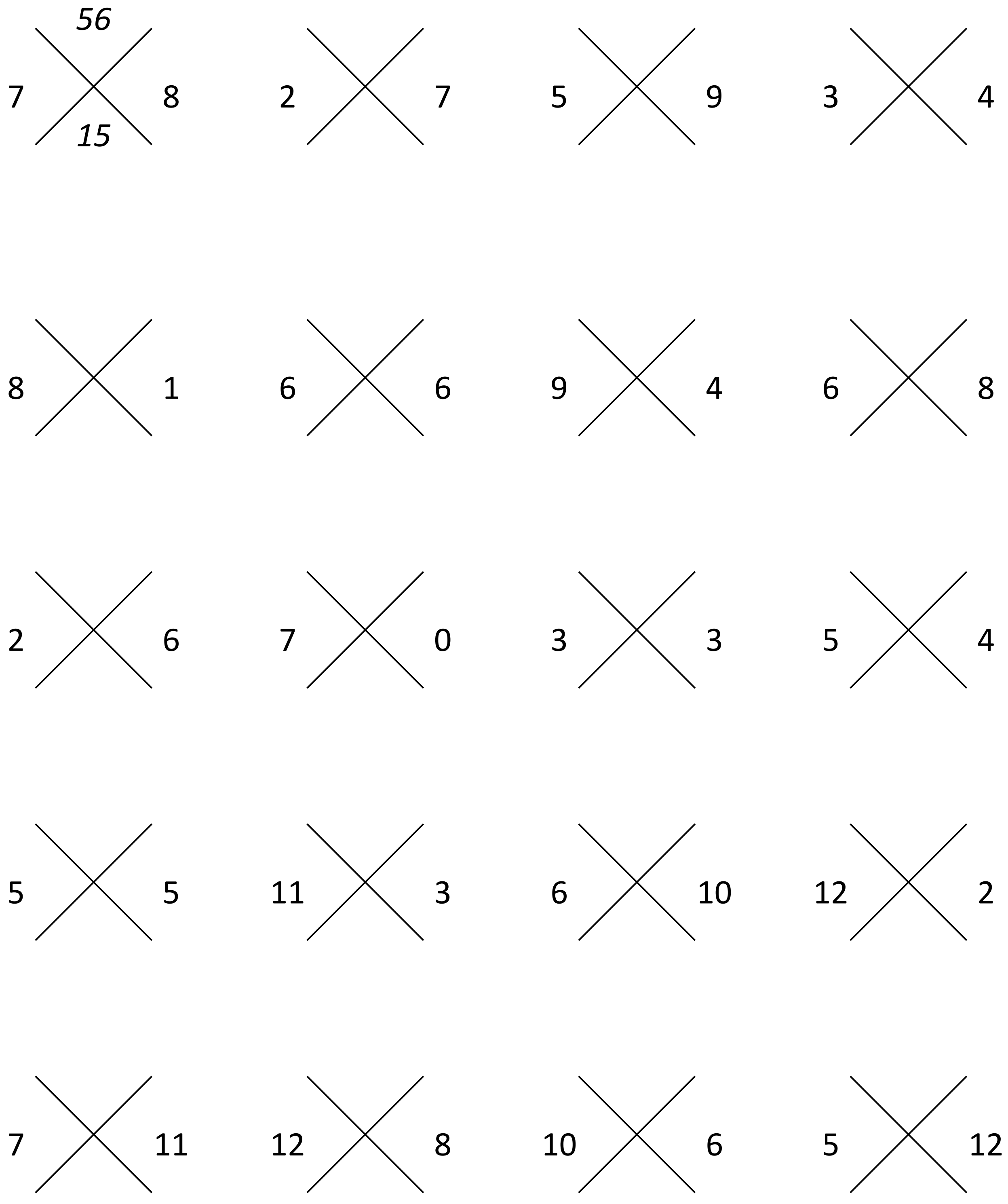
Multiply the two side numbers and put the product on the top. Add the two side numbers and put the sum on the bottom.



"X" Marks the Spot 1

Name______________________________

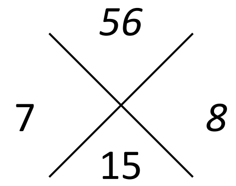
Multiply the two side numbers and put the product on the top. Add the two side numbers and put the sum on the bottom as shown.

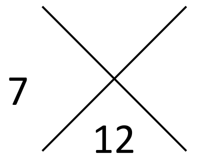


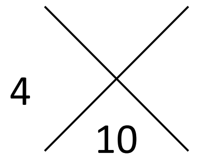
"X" Marks the Spot 2

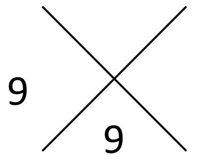
Name______________________________

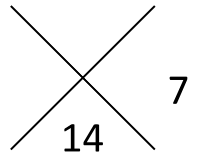
The number on the bottom is the sum of the two numbers on the sides. Find the missing side number. Then multiply the two side numbers and write the product on the top.

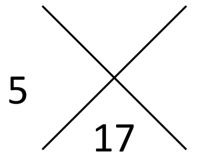


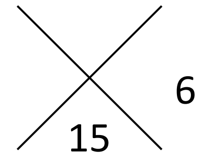


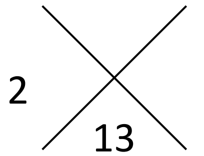


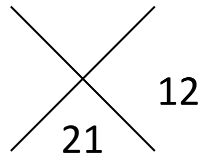


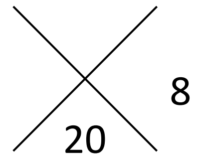


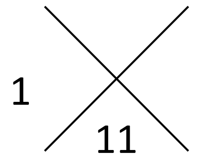


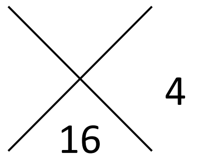


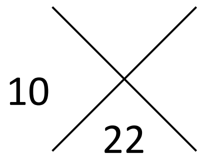


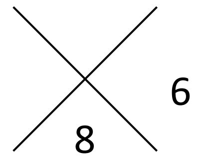


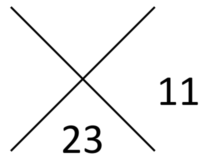


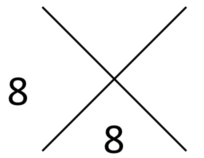


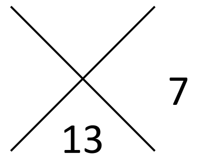


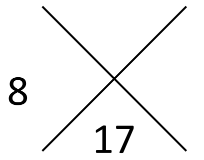


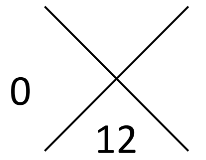


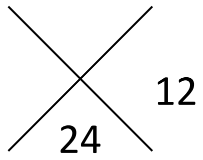








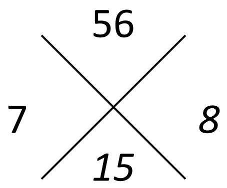


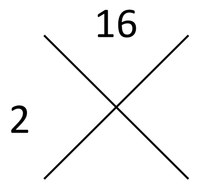


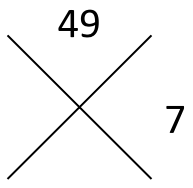
"X" Marks the Spot 3

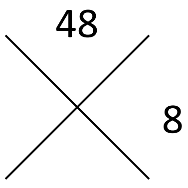
Name______________________________

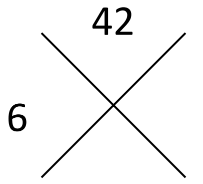
The top number is the product of the two numbers on the sides. Find the missing side number. Then add the two side numbers and write the sum on the bottom.

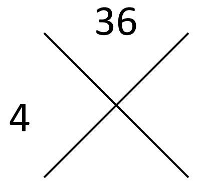


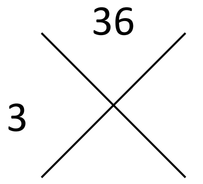


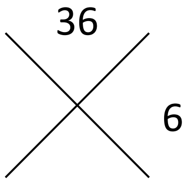


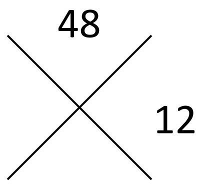


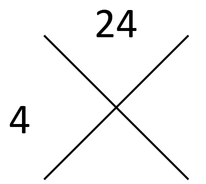


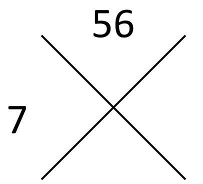


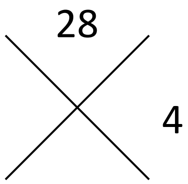


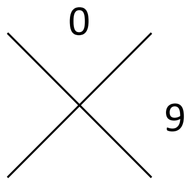


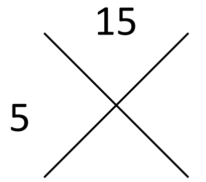


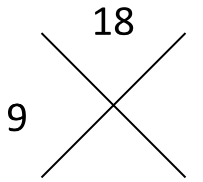


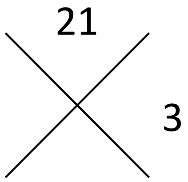


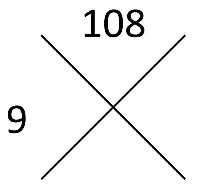


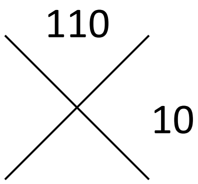


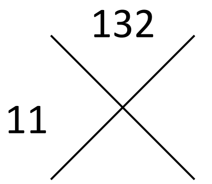


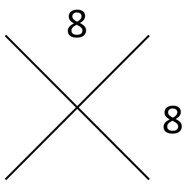












"X" Marks the Spot 4

Name______________________________

The top number is the product of the two missing side numbers. The bottom number is the sum of the two missing side numbers. Find the missing side numbers.

56 *7* *8* 15	28 11	32 12	36 15
48 14	36 12	0 12	36 13
63 16	64 16	60 16	60 17
88 19	120 22	132 23	84 19
144 24	72 17	99 20	81 18

"X" Marks the Spot 5

Name_____________________________

Multiply the two side numbers and put the product on the top. Add the two side numbers and put the sum on the bottom as shown.

5.6

7	.8	.2	7	.5	.9	.3	4

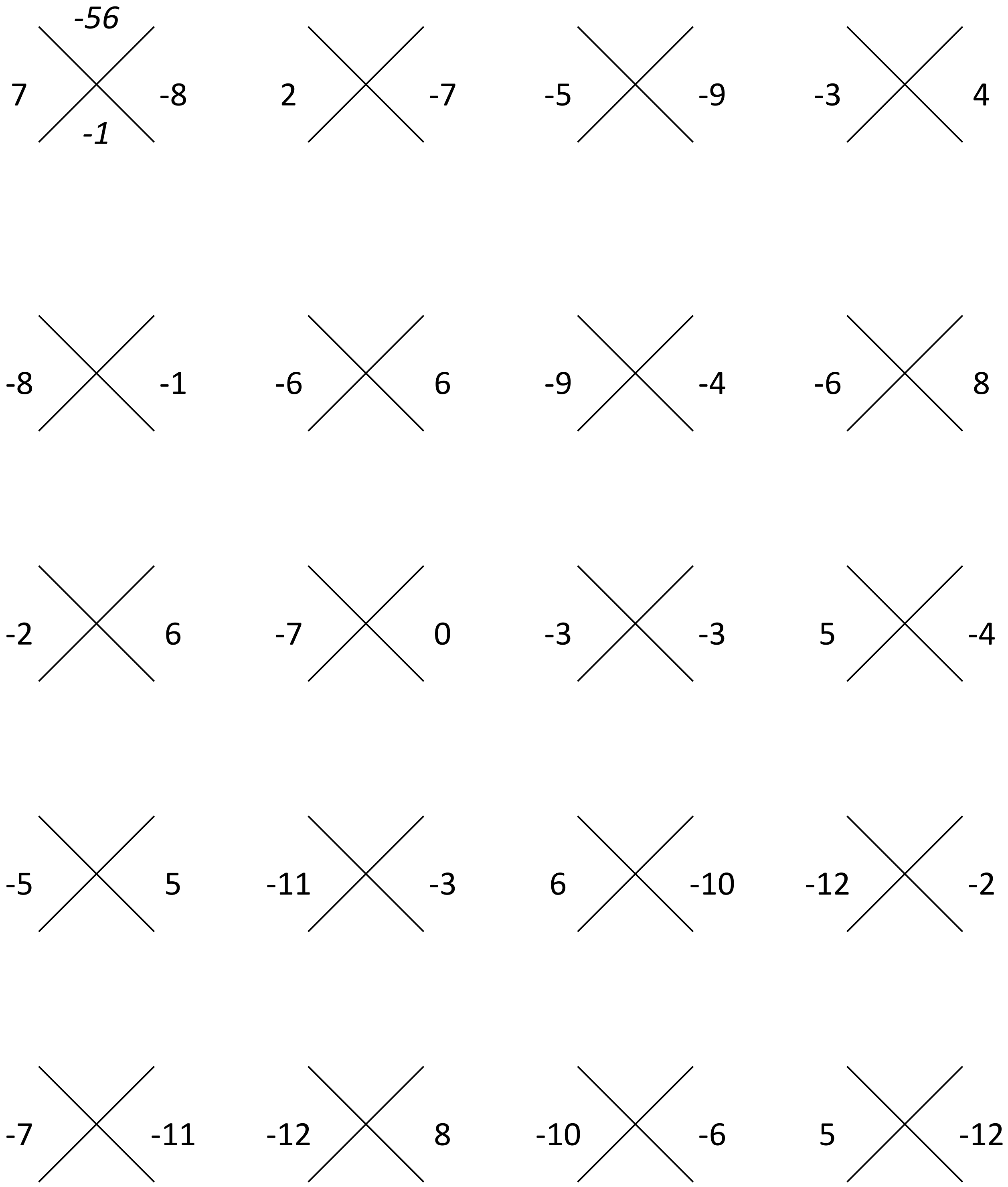
7.8

.08	.1	6	.6	.9	.4	.6	.08
.2	.6	.7	0	3	.3	.05	.04
.05	.5	1.1	.3	.6	10	.12	2
.7	.11	1.2	.08	10	.06	.5	12

"X" Marks the Spot 6

Name_______________________________

Multiply the two side numbers and put the product on the top. Add the two side numbers and put the sum on the bottom as shown.



"X" Marks the Spot 7

Name_____________________________

The top number is the product of the two missing side numbers. The bottom number is the sum of the two missing side numbers. Find the missing side numbers.

36	36	-36	-36
36 *1*			
37	-37	35	0

28	28	-28	-28
16	-16	-3	3

18	-18	18	18
-11	-3	-9	-19

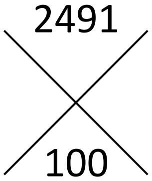
45	45	-45	-45
-14	-18	-12	44

48	-48	-48	-48
-14	2	-8	22

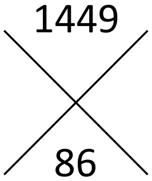
"X" Marks the Spot 8

Name______________________________

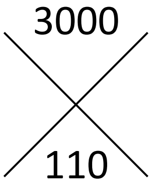
Use a guess and check table to find the missing side numbers. Always put your lower number in column a.



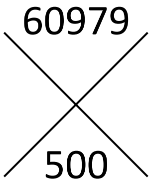
a	b	check



a	b	check



a	b	check



a	b	check

Leo's Pattern

Exploring Addition, Subtractions and More Using the Fibonacci Sequence

Overview:

Based on the famous Fibonacci sequence, this activity helps students develop numbers sense as they work with addition and subtraction. Decimals and integers are easily incorporated, and estimation skills are honed. It is a nice activity for the teacher since much of the work is self-correcting. Problem solving strategies can be taught, and students can even use algebra as a tool.

Required Materials:

☒ Paper

Optional Materials:

☐ Activity/Homework Master

☐ Calculators

Preface:

The Fibonacci sequence is a number pattern that has amazed mathematicians for centuries. It is based on the simple fact that 1 + 1 = 2. Leonardo Fibonacci is famous for his work with this pattern. **Each pair of adjacent numbers adds up to the subsequent number**:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597…

That is, not only does 1 + 1 = 2, but 1 + 2 = 3, 2 + 3 = 5, 3 + 5 = 8 and so on. The series goes on toward infinity.

The Fibonacci sequence has some unusual properties. The numbers are very common in the natural world. For example, the number 4 is missing, and this is a rare number in nature. (Four-leaf clovers are uncommon.) Starfish have 5 arms, an octopus has 8, and pinecones have 8 spirals in one direction and 13 in the other. Their needles come in groups of 1, 2, 3, or 5. A daisy has 21 petals, so "she loves you". The ratio of consecutive Fibonacci numbers approaches the golden ratio. For example, 1597 ÷ 987 = 1.61803445. The golden ratio by comparison is around 1.61803399.

A search on the Internet will yield a multitude of websites dedicated to the many fascinating properties of this sequence. To engage your students, you may wish to share some of these.

The following activity uses the structure of the Fibonacci sequence to explore addition and subtraction of different types of numbers. As the teacher, you will decide on which level to begin with your students, and on which level to end.

Procedure:

7. Display of Leo’s Pattern A,1. Explain the structure of the pattern, and tell students that each one of the patterns on the page follows the same rule: Add the first two terms to get the third term, add the second and third term to get the fourth term, and continue in this way. Two consecutive terms always add to get the following term. Patterns defined this way are called recursive patterns.

8. Show the class the rest of the patterns from set A. These patterns are examples of the recursive rule and show students that the rule can be used with many different sets of numbers.

9. Next ask the students to complete pattern B1. Set B patterns are missing the second term. Students should see that the second term is the *difference* between the third and first terms. Many students will be able to finish these patterns in their heads. You may create patterns as needed for more practice. Decimals, fractions, and negative integers make great entries for the terms.

10. After your students feel comfortable with the rule, patterns from set C may be introduced. Some of your students will feel challenged for the first time with set C patterns. Two numbers are missing between the numbers that are supplied in these patterns. This is a good time to introduce the problem solving technique of Guess and Check. With pattern C1, students may try to guess the number that comes after the 3, and then add the first two terms to get term three, and then add terms three and four to see if they sum to 17. If they guessed wrong the first time, they need to adjust their guess for term two, and try again.

    For pattern C1, this should not take too long, and students have a sense of accomplishment when they guess the right number.

11. Estimation skills and numbers sense will increase with the complexity of the problems and with the amount of practice. Your students will begin to notice patterns as they work. For example, if a student tries to put a four in the second blank of C1, they hit an 11 instead of a 17 in blank four. This tells the student that the guess was too low. If a 12 is tried, the student lands on 27, which is too high. Now the student knows that the correct answer is greater than four and less than 12. In fact, if the guess is decreased by one, and an 11 is tried, the target decreases by two to 25. This is double the guess. Since we want to decrease the target *eight* more to 17, we must decrease our guess *four* more to 7. This results in the correct answer.

    3, __, __, 17, __, __

    3, **4**, 7, 11 (low)
    3, **12**, 15, 27 (high)
    3, **11**, 14, 25 (high)
    3, **7**, 10, 17 ☺

12. More advanced students can use algebra instead of the Guess and Check strategy to solve these problems. Let’s use the following problem as an example:

    5, _____ , _____ , _____ , _____ , _____ , _____ , _____ , _____ , 241

    Since we do not know the value of the number in the second position, we will represent it with a variable:

$$5,\ \underline{x},\ \underline{\qquad},\ \underline{\qquad},\ \underline{\qquad},\ \underline{\qquad},\ \underline{\qquad},\ \underline{\qquad},\ \underline{\qquad},\ 241$$

Each subsequent number in the sequence is the sum of the previous two. This means that the value of the third position is the sum of 5 and x:

$$5,\ \underline{x},\ \underline{5 + x},\ \underline{\qquad},\ \underline{\qquad},\ \underline{\qquad},\ \underline{\qquad},\ \underline{\qquad},\ \underline{\qquad},\ 241$$

Similarly, the value of the fourth position is the sum of x and $5 + x$:

$$5,\ \underline{x},\ \underline{5 + x},\ \underline{5 + 2x},\ \underline{\qquad},\ \underline{\qquad},\ \underline{\qquad},\ \underline{\qquad},\ \underline{\qquad},\ 241$$

Continuing in this way yields the following series:

$$5,\ \underline{x},\ \underline{5 + x},\ \underline{5 + 2x},\ \underline{10 + 3x},\ \underline{15 + 5x},\ \underline{25 + 8x},\ \underline{40 + 13x},\ \underline{65 + 21x},\ 241$$

(Did you notice that the coefficients of the variables are Fibonacci numbers: $1x$, $1x$, $2x$, $3x$, $5x$, $8x$, $13x$, and $21x$?)

If we were to go one more step in this series, the final position would be the sum of $40 + 13x$ and $65 + 21x$ or $105 + 34x$. This means that:

$$105 + 34x = 241$$

Solving this equation yields:

$$105 - 105 + 34x = 241 - 105$$

$$34x = 136$$

$$34x/34 = 136/34$$

$$x = 4$$

We can now substitute 4 in place of x in the second position:

$$5,\ \underline{4},\ \underline{9},\ \underline{13},\ \underline{22},\ \underline{35},\ \underline{57},\ \underline{92},\ \underline{149},\ 241$$

* * *

Journal Prompts:

If the first term of Leo’s Pattern is 4 and the fourth term is 0, what must be true of the second term? Why?

If the first term of Leo’s Pattern is 2 and the fourth term is 5, how would you find the second term?

Describe to a student who was absent how to find the second and third terms of sequence C3 by the Guess and Check method.

Homework:

You can assign one of the activity/homework masters for homework or create ten new patterns for students to solve using the blank masters. Give terms 1 and 2 or 1 and 4 only. Or you may choose to give the last two terms in the series to teach subtraction concepts. You may wish to include decimals, fractions, or negative numbers as solutions.

You can also ask ten students create problems, write them on the board, and then assign the set for homework.

Taking a Closer Look:

There is a lot of algebra lurking in Leo's Patterns as shown in step 6 above. For a more complete treatment of the algebraic solutions, refer to our algebra book, *Simply Great Math Activities: Algebra Readiness, Volume 2*.

If you are teaching the pre-algebra concept of combining like terms, you can insert binomials into the series as shown here:

2x+7, 6x-1, _______ , _______ , _______ , _______ , _______ , ________

Assessment:

Use the answer key below or have students check each other's work.

Answer Key:

A1	5	8	13	21	34	55	89
A2	7	11	18	29	47	76	123
A3	12	19	31	50	81	131	212
A4	10	12	22	34	56	90	146
A5	6	10	16	26	42	68	110
A6	18	26	44	70	114	184	298
A7	1	21	22	43	65	108	173
A8	13	47	60	107	167	274	441
A9	7	49	56	105	161	266	427
A10	46	104	150	254	404	658	1062

B1	2	4	6	10	16	26
B2	3	6	9	15	24	39
B3	2	7	9	16	25	41
B4	1	5	6	11	17	28
B5	2	9	11	20	31	51
B6	8	1	9	10	19	29
B7	0	14	14	28	42	70
B8	19	0	19	19	38	57
B9	16	16	32	48	80	128
B10	74	37	111	148	259	407

C1	3	7	10	17	27	44
C2	4	8	12	20	32	52
C3	6	3	9	12	21	33
C4	5	2	7	9	16	25
C5	7	2	9	11	20	31
C6	8	11	19	30	49	79
C7	16	2	18	20	38	58
C8	0	24	24	48	72	120
C9	34	13	47	60	107	167
C10	43	3	46	49	95	144

D1	5	-2	3	1	4	5
D2	2	-5	-3	-8	-11	-19
D3	4	-6	-2	-8	-10	-18
D4	14	-10	4	-6	-2	-8
D5	-12	0	-12	-12	-24	-36
D6	13	-13	0	-13	-13	-26
D7	-23	8	-15	-7	-22	-29
D8	35	-21	14	-7	7	0
D9	31	-25	6	-19	-13	-32
D10	-31	25	-6	19	13	32

E1	9	-5	4	-1	3	2
E2	12	-12	0	-12	-12	-24
E3	-16	11	-5	6	1	7
E4	-14	9	-5	4	-1	3
E5	0	-6	-6	-12	-18	-30
E6	2	-3	-1	-4	-5	-9
E7	28	-25	3	-22	-19	-41
E8	-6	5	-1	4	3	7
E9	-17	10	-7	3	-4	-1
E10	-59	34	-25	9	-16	-7

F1	3	-6	-3	-9	-12	-21
F2	9	-11	-2	-13	-15	-28
F3	-6	4	-2	2	0	2
F4	-8	4	-4	0	-4	-4
F5	15	-7	8	1	9	10
F6	0	-8	-8	-16	-24	-40
F7	22	-14	8	-6	2	-4
F8	-11	0	-11	-11	-22	-33
F9	-26	16	-10	6	-4	2
F10	16	-15	1	-14	-13	-27
G1	3	3.8	6.8	10.6	17.4	28
G2	6.2	0.7	6.9	7.6	14.5	22.1
G3	0.23	0.09	0.32	0.41	0.73	1.14
G4	0.5	0.09	0.59	0.68	1.27	1.95
G5	6.4	1.7	8.1	9.8	17.9	27.7
G6	7.93	0.17	8.1	8.27	16.37	24.64
G7	3.2	0.6	3.8	4.4	8.2	12.6
G8	8.3	8.3	16.6	24.9	41.5	66.4
G9	0.17	0.38	0.55	0.93	1.48	2.41
G10	0.01	0.52	0.53	1.05	1.58	2.63
H1	3	4 ½	7 ½	12	19 ½	31 ½
H2	3 ½	2	5 ½	7 ½	13	20 ½
H3	6 ½	1 ½	8	9 ½	17 ½	27
H4	4 ¼	1 ¾	6	7 ¾	13 ¾	21 ½
H5	½	¼	¾	1	1 ¾	2 ¾
H6	¾	1 ½	2 ¼	3 ¾	6	9 ¾
H7	½	2	2 ½	4 ½	7	11 ½
H8	1 ¼	1 ½	2 ¾	4 ¼	7	11 ¼
H9	2 ½	1	3 ½	4 ½	8	12 ½
H10	0	3 ½	3 ½	7	10 ½	17 ½

The Common Core Connection

Fourth grade

Number and Operations in Base Ten

B4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Fifth grade

Number and Operations in Base Ten

B7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Sixth grade

The Number System

B3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Seventh grade

The Number System

1C: Understand subtraction of rational numbers as adding the additive inverse, p - q = p+ (-q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

1D: Apply properties of operations as strategies to add and subtract rational numbers.

High school algebra

Arithmetic with Polynomials and Rational Expressions

D7: …add, subtract, multiply, and divide rational expressions.

Activity/Homework Master

Name__________________________________

Date____________________ Class_________

Leo’s Pattern A

These patterns all follow the same rule: add two successive terms to get the next term. The original Fibonacci sequence is given, along with additional examples. Find the missing terms.

A1 1, 1, 2, 3, 5, 8, ___, ___, ___, ___, ___

A2 1, 3, 4, 7, 11, ___, ___, ___, ___, ___

A3 2, 5, 7, 12, 19, ___, ___, ___, ___, ___

A4 8, 2, 10, 12, ___, ___, ___, ___, ___

A5 2, 4, 6, 10, ___, ___, ___, ___, ___

A6 8, 18, 26, ___, ___, ___, ___, ___

A7 20, 1, 21, ___, ___, ___, ___, ___

A8 34, 13, 47, ___, ___, ___, ___, ___

A9 42, 7, 49, ___, ___, ___, ___, ___

A10 58, 46, 104, ___, ___, ___, ___, ___

Activity/Homework Master

Name_________________________________

Date___________________ Class_________

Leo’s Pattern B

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

B1 2, ___, 6, ___, ___, ___

B2 3, ___, 9, ___, ___, ___

B3 ___, ___, ___, ___, 25, 41

B4 ___, ___, ___, ___, 17, 28

B5 ___, ___, ___, 20, ___, 51

B6 ___, ___, ___, ___, 19, 29

B7 ___, ___, ___, 28, ___, 70

B8 ___, ___, 19, 19, ___, ___

B9 ___, ___, 32, ___, 80, ___

B10 ___, ___, ___, 148, ___, 407

Name________________________________

Date___________________ Class_________

Leo’s Pattern C

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

C1 3, ___, ___, 17, ___, ___

C2 4, ___, ___, 20, ___, ___

C3 6, ___, ___, 12, ___, ___

C4 5, ___, ___, ___, 16, ___

C5 7, ___, ___, ___, ___, 31

C6 ___, ___, 19, ___, ___, 79

C7 ___, 2, ___, ___, ___, 58

C8 ___, 24, ___, ___, 72, ___

C9 ___, ___, 47, ___, ___, 167

C10 43, ___, ___, ___, ___, 144

Name__________________________________

Date____________________ Class_________

Leo’s Pattern D

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

D1 5, -2, 3, ____, ____, ____

D2 2, -5, -3, ____, ____, ____

D3 4, -6, ____, ____, ____, ____

D4 14, -10, ____, ____, ____, ____

D5 -12, 0, ____, ____, ____, ____

D6 13, -13, ____, ____, ____, ____

D7 -23, 8, ____, ____, ____, ____

D8 35, -21, ____, ____, ____, ____

D9 31, -25, ____, ____, ____, ____

D10 -31, 25, ____, ____, ____, ____

Name___________________________________

Date____________________ Class_________

Leo’s Pattern E

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

E1 ___, -5, 4, ___, ___ , ___

E2 ___, ___, ___, -12, -12, ___

E3 ___, ___, ___, ___, 1, 7

E4 ___, ___, ___, ___, -1, 3

E5 ___, ___, ___, ___, -18, -30

E6 ___, ___, ___, ___, -5, -9

E7 ___, ___, ___, ___, -19, -41

E8 ___, ___, ___, ___, 3, 7

E9 ___, ___, ___, ___, -4, -1

E10 ___, ___, ___, ___, -16, -7

Activity/Homework Master

Name________________________________

Date___________________ Class_________

Leo's Pattern F

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

F1 ___, -6, ___, -9, ___, ___

F2 ___, ___, -2, ___, -15, ___

F3 ___, ___, -2, ___, 0, ___

F4 ___, 4, ___, ___, -4, ___

F5 15, ___, ___, 1, ___, ___

F6 ___, ___, -8, ___, ___, -40

F7 22, ___, ___, -6, ___, ___

F8 -11, ___, ___, -11, ___, ___

F9 -26, ___, ___, ___, ___, 2

F10 16, ___, ___, ___, ___, -27

Activity/Homework Master

Name___________________________________

Date____________________ Class_________

Leo's Pattern G

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

G1 3, 3.8, ___, ___, ___, ___

G2 6.2, .7, ___, ___, ___, ___

G3 .23, .09, ___, ___, ___, ___

G4 .5, .09, ___, ___, ___, ___

G5 ___, 1.7, 8.1, ___, ___, ___

G6 ___, .17, 8.1, ___, ___, ___

G7 ___, ___, ___, ___, 8.2, 12.6

G8 ___, ___, 16.6, ___, 41.5, ___

G9 ___, .38, ___, .93, ___, ___

G10 ___, .52, ___, ___, 1.58, ___

Activity/Homework Master

Name___________________________________

Date____________________ Class_________

Leo’s Pattern H

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

H1 3, 4 ½, ___, ___, ___, ___

H2 3 ½, 2, ___, ___, ___, ___

H3 6 ½, 1 ½, ___, ___, ___, ___

H4 4 ¼, 1 ¾, ___, ___, ___, ___

H5 ½, ¼, ___, ___, ___, ___

H6 ¾, 1 ½, ___, ___, ___, ___

H7 ___, ___, ___, 4 ½, 7, ___

H8 ___, ___, ___, 4 ¼, 7, ___

H9 ___, ___, 3 ½, ___, 8, ___

H10 ___, ___, 3 ½, ___, 10 ½, ___

Activity/Homework Master

Name___________________________________

Date____________________ Class_________

Leo’s Pattern

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

1 ____, ____, ____, ____, ____, ____, ____, ____

2 ____, ____, ____, ____, ____, ____, ____, ____

3 ____, ____, ____, ____, ____, ____, ____, ____

4 ____, ____, ____, ____, ____, ____, ____, ____

5 ____, ____, ____, ____, ____, ____, ____, ____

6 ____, ____, ____, ____, ____, ____, ____, ____

7 ____, ____, ____, ____, ____, ____, ____, ____

8 ____, ____, ____, ____, ____, ____, ____, ____

9 ____, ____, ____, ____, ____, ____, ____, ____

10 ____, ____, ____, ____, ____, ____, ____, ____

Fast Facts and Fractions

In classrooms across the United States, this program has proven successful at helping struggling students find success learning their multiplication facts and developing mastery with fraction operations. Students will also learn new strategies for putting a fraction in lowest terms and for solving proportions. Once initiated, this program requires about five minutes per day. I developed this program in 2010 to help my struggling 8th grade students, and they have shown impressive growth using these strategies.

Most of my students are daunted by the task of memorizing the 144 multiplication facts. Though you and I understand that the facts are interrelated – that 8 x 4 is the same as 4 x 8 – many struggling students fail to make these connections and see the chart as 144 unrelated facts. Here is how I implement the program. Feel free to adapt this program to suit the needs of your students.

I begin by explaining to my students that most adults don’t have their multiplication facts memorized. I say this because first because it is true – most adults use calculators for math at virtually every opportunity and have thus forgotten the facts they once knew. I also say this because I don’t want my students to think they aren’t smart enough to learn their multiplication facts. In most cases, they simply have not invested the time necessary to memorize them.

I also explain to them that they could run a marathon with their shoelaces tied together, but they would be slow, they’d stumble often, and they’d probably give up running marathons. I conclude by saying that they can make it through middle and high school and perhaps college and into adulthood without memorizing their multiplication facts, but they would be slow, they would stumble often, and they’d likely give up.

I then give them the blank 12 by 12 multiplication grid and explain that I know some tricks that will help them get really fast. I use the patterns shown below to help them fill out the facts. Many students do not realize that patterns govern the multiplication chart. This is unfortunate because the brain is much better at recognizing and extending patterns than it is at memorizing information. I utilize this strength of the brain to get students “in the door” of the multiplication table. As the days progress, they begin to memorize the facts and rely less on patterning.

Patterns in the Multiplication Table:

1. Have students fill in the ones and twos column.
2. Likely their toughest column is the twelves. However, if you look at the first lines of the ones and twos, you’ll see the products for the twelves.
3. Next have the students fill in the tens and fives.
4. Another easy column is the elevens. However, they may not know the elevens past 11 times 9. There is a trick here too. To multiply a two-digit number, such as 12, times 11. Simply write the number 12 with a space in between. Then write the sum of the digits in the space.

x	1	2
1	**1**	**2**
2	**2**	**4**
3	**3**	**6**

12
12
24
36

5. Now it's time to do the nines. Have the students fill in 1 x 9, 10 x 9, 11 x 9, and 12 x 9. For the missing products simply begin numbering from one through 8 starting at the top (2 x 9) and working toward the bottom (9 x 9). Then number from one through eight again starting from the bottom and working toward the top.
6. We now have seven of the 12 columns complete. If we complete the corresponding rows too, we have only 25 empty cells! Fill in the fours by doubling the twos. Doubling is a fairly rudimentary skill that students can master long before they have mastered other multiplication facts. You may find that some students who don't understand multiplication can still double small numbers.
7. Complete the eights column by doubling the fours.
8. We can complete the threes column by adding the products in the ones and twos column.
9. We can complete the sixes by doubling the threes or by adding the ones and fives products or the twos and fours products.
10. Similarly, we can complete the sevens columns by adding the ones and sixes products, the twos and fives products, or the threes and fours products.
11. Students should also look for the diagonal patterns that show up in the table. For example, beginning in the upper right corner where we have written the product of twelve times one and moving to the lower left corner where we have one times twelve reveals the palindromic pattern:

12, 22, 30, 36, 40, 42, 42, 40, 36, 30, 22, 12

12. Now allow the students to practice these skills as they are timed on completing the table.

Once this is done, and the students are energized and engaged by the patterns that govern these elusive facts, I ask them to put away their chart. I then give them a new blank one and ask them to see how many they can complete in ten minutes. At the end of the time, I ask them to count how many they have and log their result on the accompanying ***Fast Facts*** graph. Thus, if they filled in 93 facts, they would write the date on the first line of the graph's horizontal axis and put a dot above it at 93. This concludes the first day of instruction.

On the second day, I ask them what patterns they recall from the previous day. If they have forgotten any, we review them. Sometimes students will suggest patterns they discovered or learned in a previous year. Then they are given a blank chart and the timer begins. After ten minutes, they log their progress. Invariably they do better than the day before. I ask them to connect the points to form a line graph, and they are impressed by their growth in only one day.

Eventually students will be able to fill in all 144 facts in ten minutes. At that point, they turn their graph over and log their progress on the second graph where the vertical axis represents time. Now the process takes less than ten minutes per day. Eventually my students get their times incredibly low. My classroom record belongs to a student who could complete the chart in 1:52. The class median is around three minutes. Remember, these are struggling students! Eventually we need only about five minutes of time at the start of class to practice our facts, and then we move on to our daily instruction. I have found this to be a much more practical use of warm up time.

I reward students who can complete the chart in less than three minutes; they only need to take the test once every three weeks. They are quite proud to be able to enter the class and work on their homework while the rest of the class does the multiplication warm up. Students who reach this goal turn in their timed test so that I can verify their accuracy. This is the only time I monitor their results.

I also give my students a randomized multiplication test once every three weeks. These tests have 36 problems that are not in order like the chart. This ensures and verifies that students are actually memorizing the facts.

Once students have gained proficiency with their multiplication facts, I begin to show them how to use the chart as a tool for solving fraction operations. I only introduce one skill per week. They learn how to add fractions one week, and I give them time to gain mastery with that skill. Then I show them how to use the chart to subtract fractions the next week.

Eventually they learn how to use the ***Fantastic Fraction Grid***. This allows students who have mastered their multiplication facts to solve fraction problems without writing out the full 12 by 12 chart.

Some of my students now begin their standardized tests by making a multiplication chart on their scratch paper. I have seen students who struggled for years with low scores on these tests make significant improvements. As their skill increases, so does their confidence. They often begin to see that they are much better at math than they thought, all because they took the time to untangle their shoelaces.

Multiplication Race

Name______________________________

Date____________________ Period______

X	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

Multiplication Race

Name______________________________

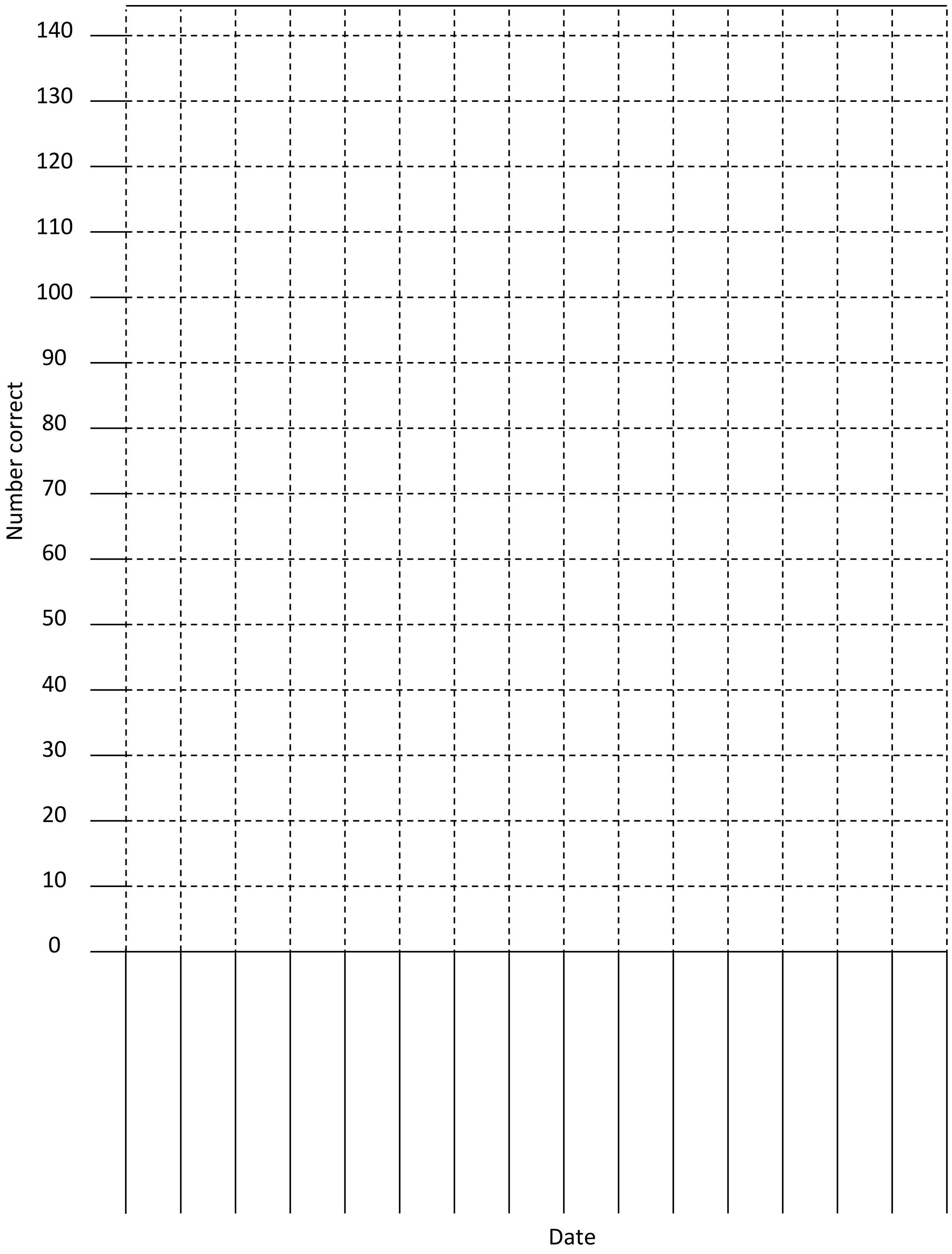
Date____________________ Period______

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

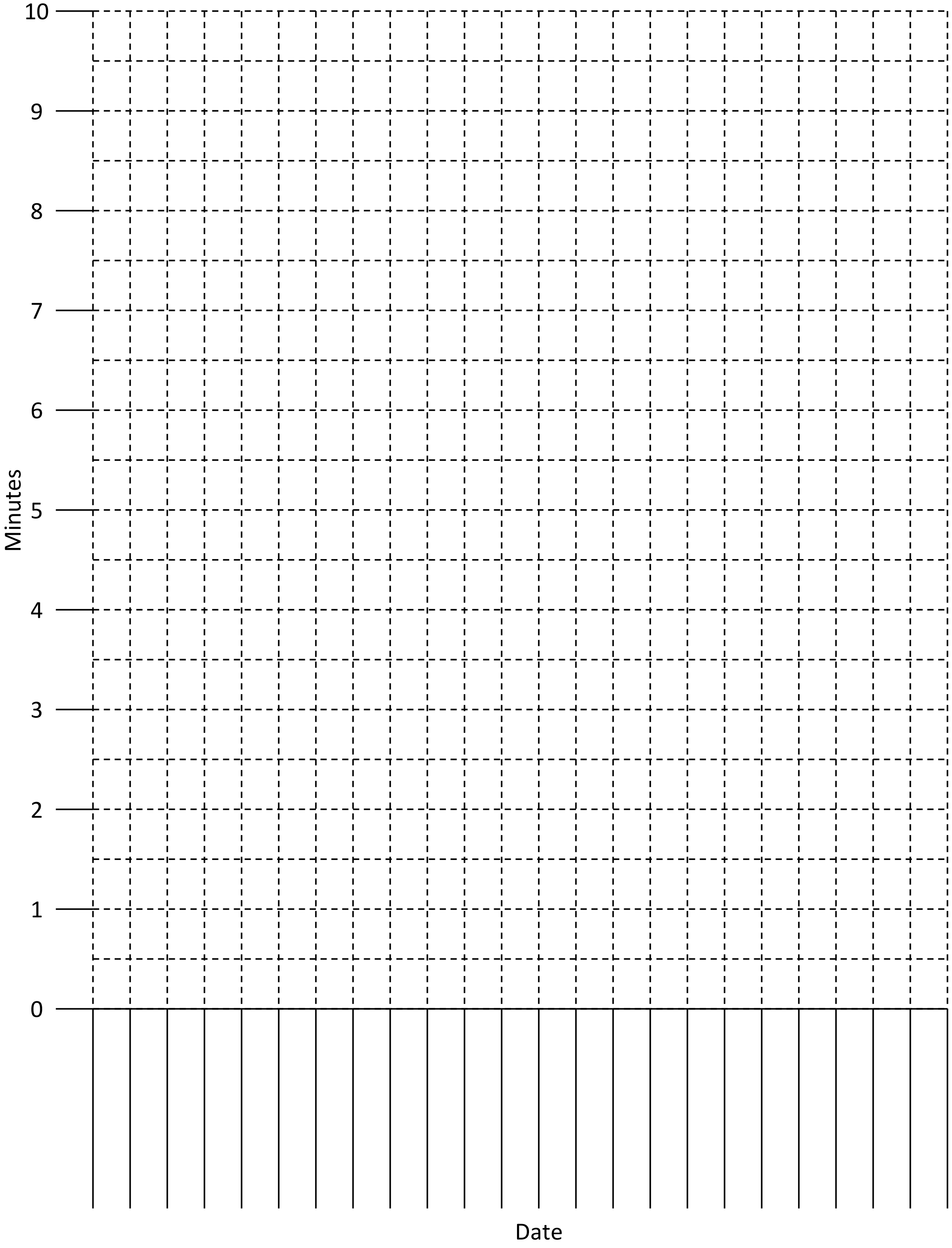
FAST FACTS

Name______________________________

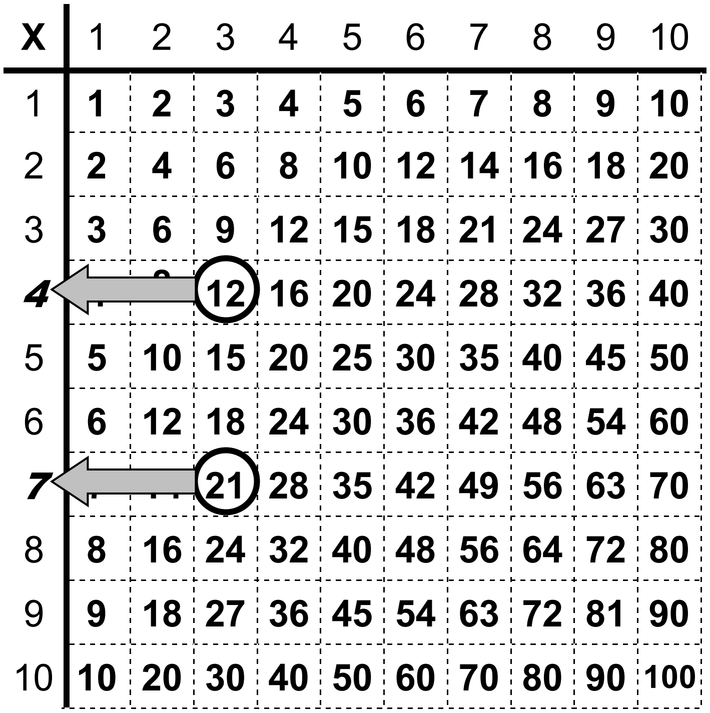
Period ________



FAST FACTS



Simplifying fractions:



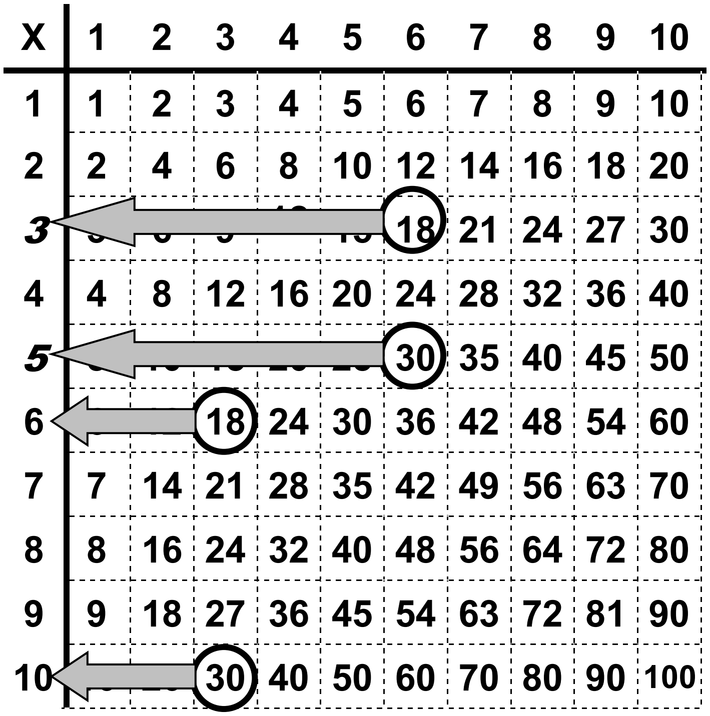
X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	[illegible]	[illegible]	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	[illegible]	[illegible]	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Example: Simplify $^{12}/_{21}$

Find your fraction vertically in the multiplication table.

Read its simplified value from the left-hand column.

Answer: $^{4}/_{7}$



X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	30	35	40	45	50
6	[illegible]	[illegible]	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	[illegible]	[illegible]	30	40	50	60	70	80	90	100

When looking for your fraction in the table, you may find it more than once. Use the uppermost location to find the simplest form.

Example: Simplify $^{18}/_{30}$

Answer: $^{3}/_{5}$

Simplifying Fractions
Practice Page

Name___________________________

Date _______________Period ______

Simplify the following fractions using your multiplication table.

1) $\frac{3}{6}$ =_____

2) $\frac{5}{15}$ =_____

3) $\frac{3}{9}$ =_____

4) $\frac{10}{25}$ =_____

5) $\frac{14}{21}$ =_____

6) $\frac{60}{70}$ =_____

7) $\frac{10}{12}$ =_____

8) $\frac{3}{9}$ =_____

9) $\frac{28}{35}$ =_____

10) $\frac{4}{16}$ =_____

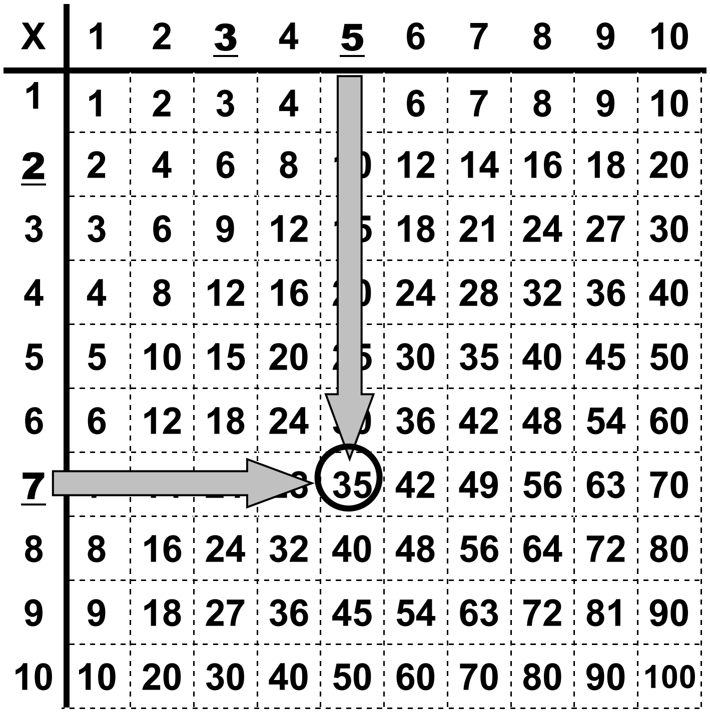
11) $\frac{12}{18}$ =_____

12) $\frac{20}{24}$ =_____

13) $\frac{24}{30}$ =_____

14) $\frac{18}{36}$ =_____

Adding and Subtracting
Of Unlike Denominators:



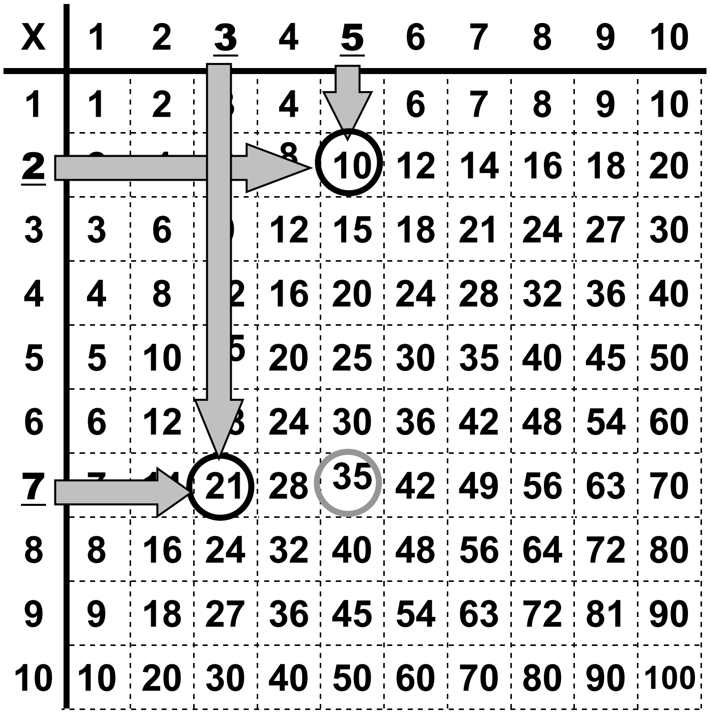
X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	[illegible]	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	[illegible]	[illegible]	[illegible]	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Example: Add ${}^{2}/_{7} + {}^{3}/_{5}$

Find the first fraction on the left of the table.

Find the second fraction at the top of the table.

Multiply the denominators as shown. This is the denominator of your answer.

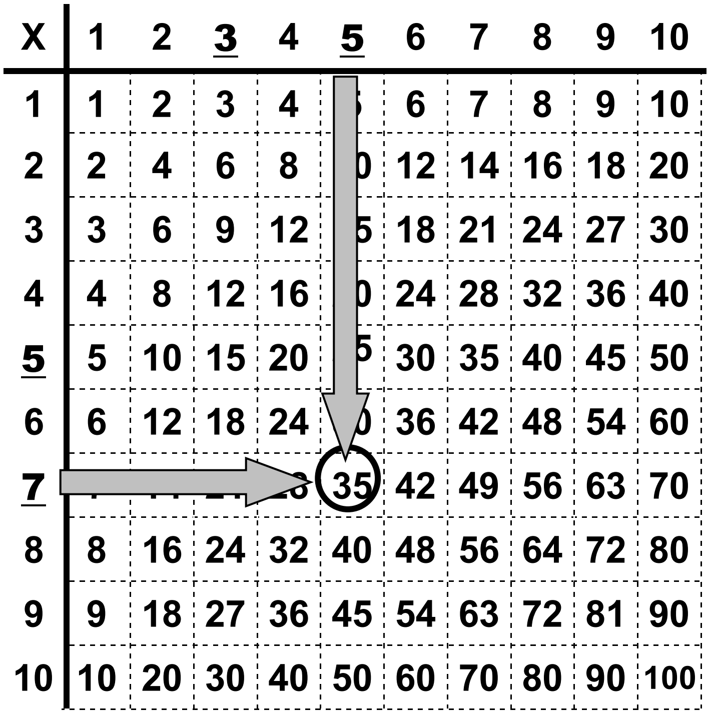


X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	[illegible]	6	7	8	9	10
2	2	4	[illegible]	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Multiply as shown and add the products.
This is the numerator of your answer.

Answer: ${}^{31}/_{35}$

Simplify if necessary.



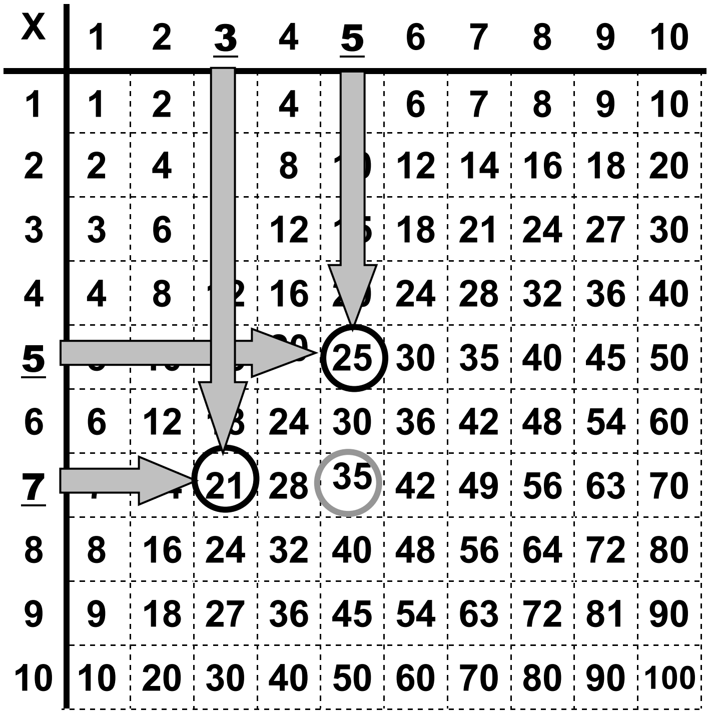
X	1	2	<u>3</u>	4	<u>5</u>	6	7	8	9	10
1	1	2	3	4	[illegible]	6	7	8	9	10
2	2	4	6	8	[illegible]	12	14	16	18	20
3	3	6	9	12	[illegible]	18	21	24	27	30
4	4	8	12	16	[illegible]	24	28	32	36	40
<u>5</u>	5	10	15	20	[illegible]	30	35	40	45	50
6	6	12	18	24	[illegible]	36	42	48	54	60
<u>7</u>	[illegible]	[illegible]	[illegible]	[illegible]	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Example: Subtract $^5/_7 - ^3/_5$

Find the first fraction on the left of the table.

Find the second fraction at the top of the table.

Multiply the denominators as shown. This is the denominator of your answer.



X	1	2	<u>3</u>	4	<u>5</u>	6	7	8	9	10
1	1	2	[illegible]	4	[illegible]	6	7	8	9	10
2	2	4	[illegible]	8	[illegible]	12	14	16	18	20
3	3	6	[illegible]	12	[illegible]	18	21	24	27	30
4	4	8	[illegible]	16	[illegible]	24	28	32	36	40
<u>5</u>	[illegible]	[illegible]	[illegible]	[illegible]	25	30	35	40	45	50
6	6	12	[illegible]	24	30	36	42	48	54	60
<u>7</u>	[illegible]	[illegible]	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Multiply as shown and subtract the products.
This is the numerator of your answer.

Answer: $^4/_{35}$

Simplify if necessary.

Adding Fractions
Practice Page

Name___________________________

Date _______________Period ______

Add the following fractions using your multiplication table. Simplify as necessary.

1) $\frac{1}{2}+\frac{1}{3}=$_____

2) $\frac{1}{4}+\frac{1}{5}=$_____

3) $\frac{1}{3}+\frac{1}{4}=$_____

4) $\frac{2}{3}+\frac{1}{5}=$_____

5) $\frac{3}{7}+\frac{1}{2}=$_____

6) $\frac{2}{5}+\frac{1}{3}=$_____

7) $\frac{1}{2}+\frac{1}{4}=$_____

8) $\frac{1}{6}+\frac{1}{4}=$_____

9) $\frac{1}{2}+\frac{2}{3}=$_____

10) $\frac{3}{5}+\frac{2}{3}=$_____

11) $\frac{3}{8}+\frac{1}{2}=$_____

12) $\frac{7}{8}+\frac{1}{4}=$_____

Subtracting Fractions

Practice Page

Name__________________________

Date _______________Period ______

Subtract the following fractions using your multiplication table. Simplify as necessary.

1) $\frac{1}{2} - \frac{1}{3} =$_____

2) $\frac{1}{4} - \frac{1}{5} =$_____

3) $\frac{1}{3} - \frac{1}{4} =$_____

4) $\frac{2}{3} - \frac{1}{5} =$_____

5) $\frac{5}{7} - \frac{1}{2} =$_____

6) $\frac{2}{5} - \frac{1}{3} =$_____

7) $\frac{1}{2} - \frac{1}{4} =$_____

8) $\frac{5}{6} - \frac{1}{4} =$_____

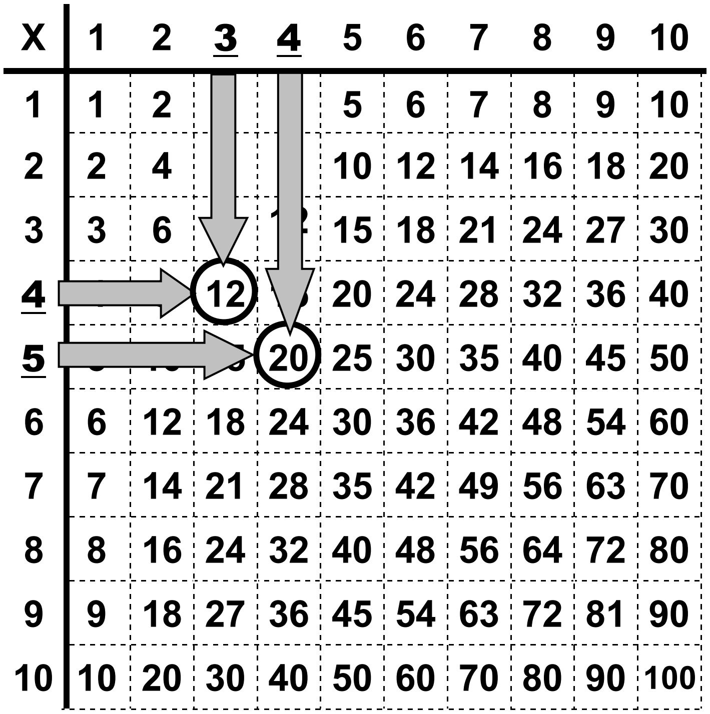
9) $\frac{1}{2} - \frac{2}{9} =$_____

10) $\frac{3}{4} - \frac{2}{3} =$_____

11) $\frac{1}{2} - \frac{3}{8} =$_____

12) $\frac{7}{8} - \frac{1}{4} =$_____

Multiplying Fractions:



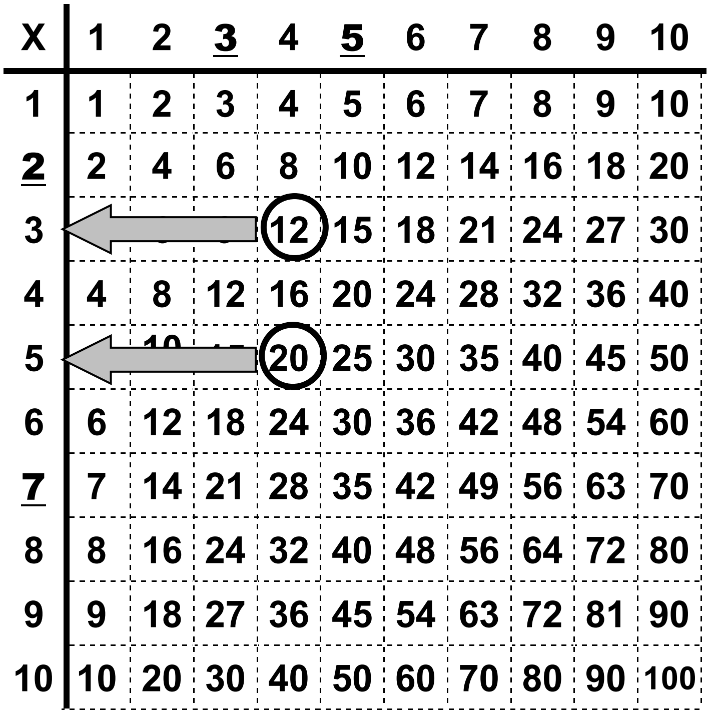
X	1	2	3	4	5	6	7	8	9	10
1	1	2			5	6	7	8	9	10
2	2	4			10	12	14	16	18	20
3	3	6		12	15	18	21	24	27	30
4			12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Example: Multiply $^4/_5$ x $^3/_4$

Find the first fraction on the left of the table.

Find the second fraction at the top of the table.

Multiply as shown. The upper number is the numerator of your answer. The lower number is the denominator.



X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3				12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5		10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Simplify if necessary.

Answer: $^3/_5$

Multiplying Fractions

Practice Page

Name__________________________

Date _______________Period ______

Multiply the following fractions using your multiplication table. Simplify as necessary.

1) $\frac{1}{2} \times \frac{1}{3} =$______

2) $\frac{1}{4} \times \frac{1}{5} =$______

3) $\frac{1}{3} \times \frac{1}{4} =$______

4) $\frac{2}{3} \times \frac{1}{5} =$______

5) $\frac{5}{7} \times \frac{1}{2} =$______

6) $\frac{2}{5} \times \frac{1}{3} =$______

7) $\frac{2}{3} \times \frac{1}{4} =$______

8) $\frac{5}{6} \times \frac{3}{4} =$______

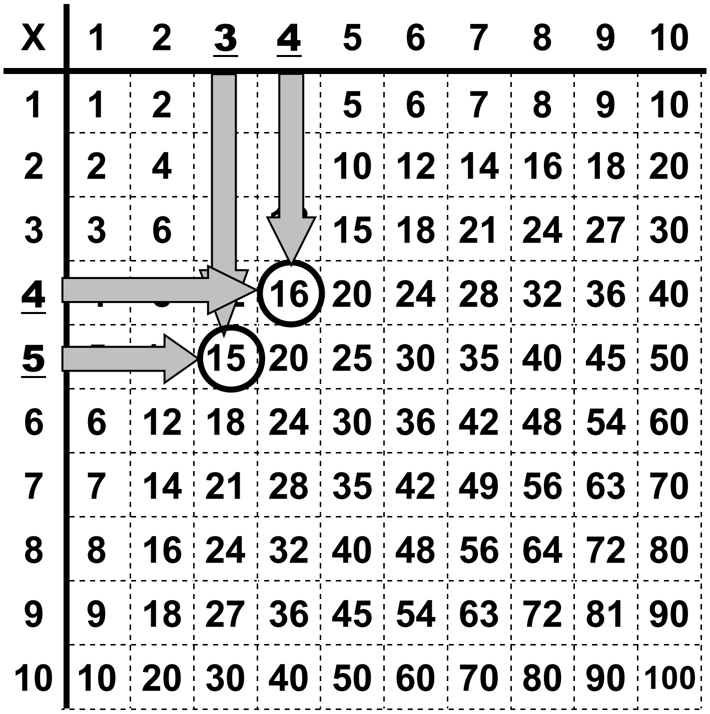
9) $\frac{1}{2} \times \frac{2}{9} =$______

10) $\frac{3}{4} \times \frac{2}{3} =$______

11) $\frac{1}{6} \times \frac{3}{8} =$______

12) $\frac{2}{9} \times \frac{3}{4} =$______

Dividing Fractions:



X	1	2	3	4	5	6	7	8	9	10
1	1	2			5	6	7	8	9	10
2	2	4			10	12	14	16	18	20
3	3	6			15	18	21	24	27	30
4				16	20	24	28	32	36	40
5			15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Example: Divide $^4/_5 \div {}^3/_4$

Find the first fraction on the left of the table.

Find the second fraction at the top of the table.

Multiply as shown. The upper number is the numerator of your answer. The lower number is the denominator.

Result: $^{16}/_{15}$

Simplify if necessary.

Answer: $1^1/_{15}$

Dividing Fractions
Practice Page

Name___________________________

Date _______________Period ______

Divide the following fractions using your multiplication table. Simplify as necessary.

1) $\frac{1}{3} \div \frac{1}{2} =$_____

2) $\frac{1}{5} \div \frac{1}{3} =$_____

3) $\frac{1}{3} \div \frac{3}{4} =$_____

4) $\frac{1}{7} \div \frac{2}{5} =$_____

5) $\frac{2}{5} \div \frac{3}{5} =$_____

6) $\frac{2}{5} \div \frac{2}{5} =$_____

7) $\frac{2}{3} \div \frac{1}{3} =$_____

8) $\frac{1}{6} \div \frac{5}{6} =$_____

9) $\frac{5}{8} \div \frac{3}{8} =$_____

10) $\frac{3}{4} \div \frac{2}{3} =$_____

11) $\frac{1}{6} \div \frac{3}{8} =$_____

12) $\frac{2}{9} \div \frac{3}{4} =$_____

Fraction Operations
Without a Multiplication Table:

Often students are not allowed to use a multiplication table during testing. Here is a simple way to show them to create a do-it-yourself template for solving all four operations:

Add $^{2}/_{3} + {}^{1}/_{4} =$

Subtract $^{2}/_{3} - {}^{1}/_{4} =$

Multiply $^{2}/_{3} \text{ x } {}^{1}/_{4} =$

Divide $^{2}/_{3} \div {}^{1}/_{4} =$

Draw a two by two grid:

	1	4
2		
3		

Write the first fraction on the left of the grid.

Write the second fraction on the top of the grid.

Multiply the digits to complete the grid.

	1	4
2	2	8
3	3	12

The sum is found by adding the 8 + 3 and writing the answer over the 12.
$^{2}/_{3} + {}^{1}/_{4} = {}^{11}/_{12}$

The difference is found by subtracting the 8 – 3 and writing the answer over the 12.
$^{2}/_{3} - {}^{1}/_{4} = {}^{5}/_{12}$

	1	4
2	2	8
3	3	12

The product is found in one of the diagonals as shown.
$^{2}/_{3} \text{ x } {}^{1}/_{4} = {}^{2}/_{12}$
Simplify as necessary. $^{2}/_{12} = {}^{1}/_{6}$

	1	4
2	2	8
3	3	12

The quotient is found in the other diagonal as shown.
$^{2}/_{3} \div {}^{1}/_{4} = {}^{8}/_{3}$
Simplify as necessary. $^{8}/_{3} = 2\ {}^{2}/_{3}$

	1	4
2	2	8
3	3	12

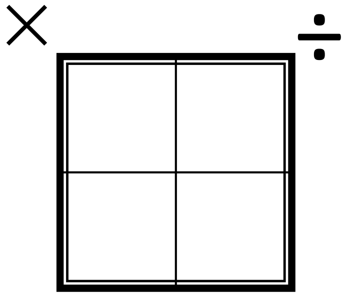
Fantastic
Fraction Grid

Name________________________

Date____________ Period _____

Find the sum, difference, product, and quotient of each pair of fractions.

1



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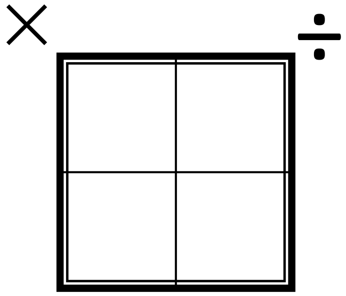

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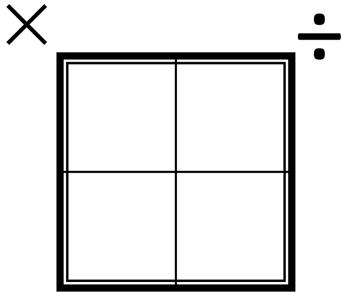

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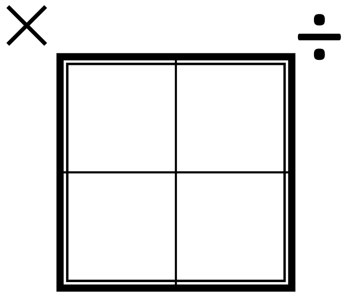


Solving Proportions:

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Example: Solve $\frac{12}{30} = \frac{16}{x}$

Find the three known numbers as vertices of a rectangle.

The missing vertex is the solution to the proportion.

x = 40

Why this works:

Notice that the 12 is the product of 6 x 2.

The 16 is the product of 8 x 2.

The 30 is the product of the 6 x 5.

The proportion could be written:

$$\frac{(6 \times 2)}{(6 \times 5)} = \frac{(8 \times 2)}{x}$$

Using the cross products rule gives us:

$$(6 \text{ x } 2)(x) = (6 \text{ x } 5)(8 \text{ x } 2)$$

The associative property gives us:

$$(6 \text{ x } 2)(x) = (6 \text{ x } 2)(8 \text{ x } 5)$$

Canceling the common factors leaves:

$$x = (8 \text{ x } 5) = 40$$

Solving Proportions Name___________________________

Practice Page

Date _______________Period ______

Solve the proportions using your multiplication table.

1) $\frac{9}{12} = \frac{15}{x}$

2) $\frac{20}{28} = \frac{35}{x}$

3) $\frac{8}{24} = \frac{x}{36}$

4) $\frac{10}{12} = \frac{x}{30}$

5) $\frac{9}{x} = \frac{1}{3}$

6) $\frac{12}{x} = \frac{10}{15}$

7) $\frac{x}{24} = \frac{15}{18}$

8) $\frac{x}{14} = \frac{35}{49}$

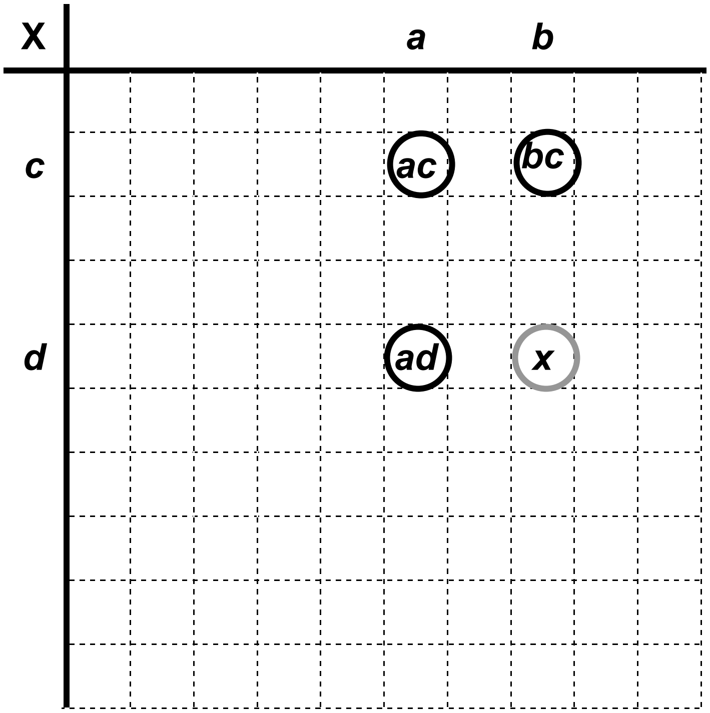
9) $\frac{15}{36} = \frac{x}{24}$

10) $\frac{9}{6} = \frac{21}{x}$

11) $\frac{12}{20} = \frac{3}{x}$

12) $\frac{15}{20} = \frac{x}{8}$

Solving Proportions,
An Algebraic Proof:



Given that three numbers in a proportion can be located on a multiplication table as the vertices of a rectangle, prove that the fourth vertex is the solution to the proportion.

If x is the solution to the proportion, then,

$$\frac{ac}{ad} = \frac{bc}{x}$$

$x = bd$	Given
$\frac{ac}{ad} = \frac{bc}{bd}$	Substitution
$(ac)(bd) = (ad)(bc)$	Cross products
$(abcd) = (abcd)$	Associative property

Exploring Quadratic Functions:

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

x	y
1	1
2	4
3	9
4	16
5	25
6	36

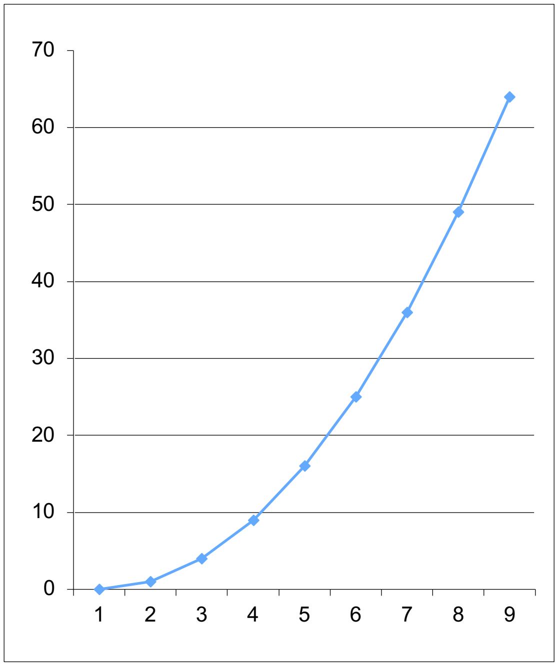
Looking at numbers in the diagonals of the multiplication table allows us to explore quadratic functions. Here we see the square numbers which are generated of course by multiplying a factor in the left column by its matching factor in the top row.

These functions are of the form:

$$y = ax^2 + bx + c$$

We can view the circled numbers and the numbers to their far left as the columns of a t-table.

When graphed they always form parabolas as shown. But what about other diagonals in the multiplication table?



Exploring Quadratic Functions:

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Here we look at the next diagonal to the right. These are not square numbers, but they still represent a quadratic function.

Notice that the numbers can be represented as their products as shown in the t-table. We see that each number is the product of the step (x) and one more than the step ($x + 1$):

Thus, we can represent the function as:

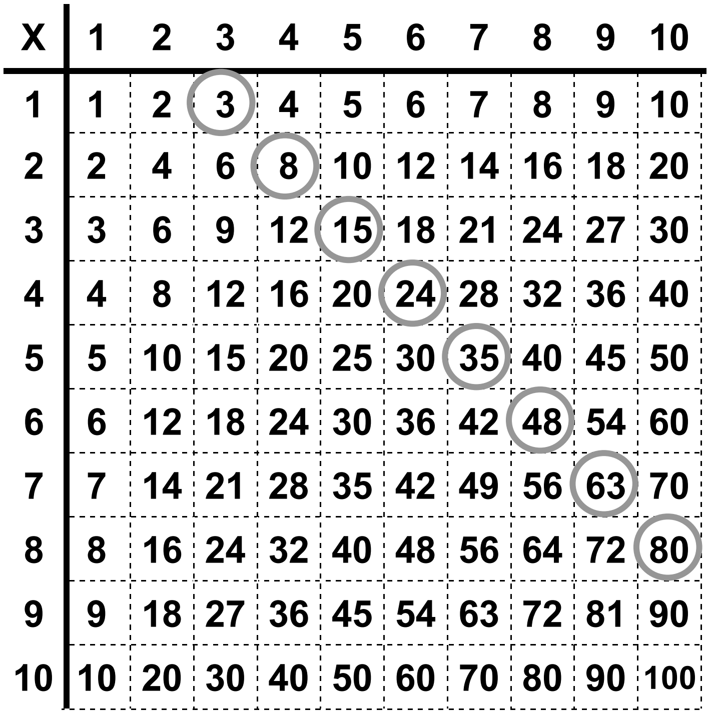
$$y = x(x + 1)$$

or

$$y = x^2 + x$$

x	y	
1	2	= 1 x 2
2	6	= 2 x 3
3	12	= 3 x 4
4	20	= 4 x 5
5	30	= 5 x 6
6	42	= 6 x 7

Exploring Quadratic Functions:



X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

x	y	
1	3	= 1 x 3
2	8	= 2 x 4
3	15	= 3 x 5
4	24	= 4 x 6
5	35	= 5 x 7
6	48	= 6 x 8

Here is the next diagonal in the chart. Again the numbers can be represented as their products in the t-table.

Now each number is the product of the step (x) and *two* more than the step ($x + 2$):

Thus, we can represent the function as:

$$y = x(x + 2)$$

or

$$y = x^2 + 2x$$

This pattern continues throughout the chart creating the following sequence of functions:

$$y = x^2 + 3x$$

$$y = x^2 + 4x$$

$$y = x^2 + 5x$$

If the diagonal is moved to the left of the square numbers we get functions of this nature:

$$y = x^2 - x$$

$$y = x^2 - 2x$$

$$y = x^2 - 3x$$

Exploring Quadratic Functions:

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

What if we flip the diagonal? Now we get a new arrangement of numbers.

Now each number is the product of the step (x) and eleven minus the step ($11 - x$):

Thus, we can represent the function as:

$$y = x(11 - x)$$

or

$$y = 11x - x^2$$

Put in standard form, the equation is:

$$y = -x^2 + 11x$$

Students can now explore quadratic functions that can be found in other diagonals of the multiplication table.

x	y
1	10 = 1 x (11 – 1)
2	18 = 2 x (11 – 2)
3	24 = 3 x (11 – 3)
4	28 = 4 x (11 – 4)
5	30 = 5 x (11 – 5)
6	30 = 6 x (11 – 6)

Knowing the Standards

For each of the standards below, write the grade level at which it is addressed in your state standards.

- Working with circle graphs (pie charts).. ______
- Solving proportions using cross products.. ______
- Making box plots.. ______

We have a lot of standards to teach…and never enough time to teach them. As we seek to help students who need to catch up to grade level standards, we need to focus on the areas of greatest concern. We need to major on the majors. What are those areas at your grade level?

> *“**Students should spend the large majority of their time on the major work of their grade.** Supporting work, and where appropriate, additional work can engage students in the major work of their grade.”*
>
> Achieve the Core

____

____

____

A good way to find out is to look at the areas of major work as outlined in your state math standards. Here are the areas for grades K-8 in the Common Core Standards.

K-2	Addition and subtraction – concepts, skills, and problem solving; place value
3-5	Multiplication and division of whole numbers and fractions – concepts, skills, and problem solving
6	Ratios and proportional relationships: early expressions and equations
7	Ratios and proportional relationships; arithmetic of rational numbers
8	Linear algebra and linear functions

Here is a good resource to find the major clusters at your grade level if you are a Common Core state: http://bit.ly/MainTopics

For example, in 8th grade these are the major, supporting, and additional clusters:

♦ MAJOR △ SUPPORTING ☐ ADDTIONAL

8.NS.A	△	Know that there are numbers that are not rational, and approximate them.
8.EE.A	♦	Work with radical and integer exponents.
8.EE.B	♦	Understand the connections between proportions, lines, and linear equations.
8.EE.C	♦	Analyze and solve linear equations and pairs of simultaneous equations.
8.F.A	♦	Define, evaluate, and compare functions.
8.F.B	♦	Use functions to model relationships between quantities.
8.G.A	♦	Understand congruence & similarity using models, transparencies, and software.
8.G.B	♦	Understand and apply the Pythagorean Theorem.
8.G.C	☐	Solve real-world problems involving volume of cylinders, cones, and spheres.
8.SP.A	△	Investigate patters of association in bivariate data.

By taking the time to familiarize yourself with the content and major focus of your state standards, you will save yourself a lot of work and maximize your minutes with your students.

Tangram Math

Integrating Fractions, Decimals, Percent, Geometry, and Algebra

Overview:

In this powerfully engaging activity students of all skill levels will study standard and nonstandard tangrams to determine the values of the pieces. Students will compare the pieces to see how they relate to one another as fractions, decimals, percent, and areas. They will also develop geometric vocabulary and form an understanding of congruence and similarity. The best part is that the entire process will lead them seamlessly into algebraic thinking as they navigate among the pieces and their representations. The activity can also be used to help students learn probability, solve proportions and equations and understand algebraic properties!

Required Materials:

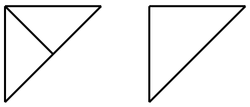
- ☒ Student copies of the tangrams

Optional Materials:

- ☐ Rulers
- ☐ Scissors
- ☐ Calculators
- ☐ Document camera or projection device

Procedure:

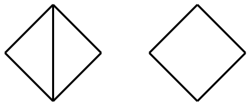
1 This activity works best when students work in groups of two to four. This fosters important dialogue that facilitates understanding. Display a copy of tangram 1. Students should have individual copies. You may wish to distribute scissors for this activity as some students find it helpful to cut the pieces for comparison.



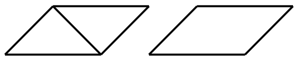
$2c = b$

2 Ask the students, "If the entire square tile has a value of 1, what is the value of the region *a*?" They can see that it is $^{1}/_{4}$ since four of the large triangles can fit in the square.

3 Next ask them to evaluate the medium-sized triangle, region b. Since two b's will fit into one a, b is $^{1}/_{2}$ of a, or $^{1}/_{8}$ of the whole. Another way to see this is by showing that the entire tile can be cut into eight b's.



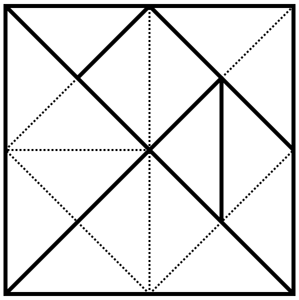
$2c = d$



$2c = e$

4 Ask the students to find the values of the other regions. When they find answers, ask them to justify them. They may do this verbally, by rearranging pieces on their desks or on your projection device. They will

see that the small triangle c is$^{1}/_{16}$ of the tile since two of them can fit into b. Since two small triangles also fit into the square, d is equal to b. Two c's also fit into the parallelogram e, so b, d, and e are all $^{1}/_{8}$. This is shown in the margin on the previous page.



5 Another way to show the relationships among the polygons is by drawing lines to subdivide the tangram into the smallest unit (in this case, triangle c) as shown here in the margin. It is then easy to see that c is $^{1}/_{16}$ of the tile. Regions b, d, and e are each $^{2}/_{16}$ or $^{1}/_{8}$ of the tile, and region a is $^{4}/_{16}$ or $^{1}/_{4}$ of the tile. This cut-up method will not work on all of the other tangram patterns however.

6 After the students have found the fractional values of each piece, they can add them together to check their work. Remember that there are two of shape a and two of shape c. Adding all of these pieces gives a total of $^{16}/_{16}$ or 1 (whole tangram).

7 If students are familiar with decimals, you can ask them to find the decimal values of each piece. They will find that a = 0.25. Ask them to explain their reasoning. They may say that in money a quarter is \$.25.

8 When they try to find the value of b, students may give different answers. Beginning students fail to see that c, which is $^{1}/_{8}$, has a decimal representation of 0.125. They may think that 0.125 is greater than 0.25 since it has more places. I have seen students suggest that b = 0.12½. This may confuse some as it incorporates both decimal and common fractions, but essentially it is correct. We see these sorts of representations on gasoline prices: $\$2.99^{9}/_{10}$. By annexing a zero and writing the value of a as 0.250 instead of 0.25, many students are able to halve the 0.250 and get 0.125.

9 I have also seen students suggest that c has a value of 0.625. They annex a zero on b to get 0.1250 and then cut it in half without regard to the place value. These discrepancies will be discovered when students check their answers by adding all the decimals (keeping in mind that there are two a's and two c's). They should get a total value of 1.0000.

10 Next you can ask students to write these as percent representations. This should be much easier than it typically is since students are beginning to see the connections among the shapes and their other representations. In fact, they will see that half of 25% (a) is 12½% (b). Similarly, if they had written that c had a value of 0.625, they will think it has a percent value of 62.5%.

This is a contradiction since they can see that it is not over half the total shape. However, by writing the correct answer of 0.0625, they can then convert that to 6¼%. Adding the percent values of the pieces yields 100%.

11 Students can also calculate the areas of the pieces. Have the students measure the base and height of large triangle a. They will see that the base is four inches long and the height is equal to two inches. Using the area formula for a triangle leads to:

$$A = \frac{4 \bullet 2}{2} = 4\ sq\ in.$$

The students may not need to use these formulas if they make connections with the values of the pieces. For example, since a has an area of 4 in^2, and b = ½ of a, it must have an area of 2 in^2.

12 Ask them to find the areas of the other triangular regions using the same formula. This will show that medium-sized triangle b has a base and height of two inches and an area of two square inches.

$$A = \frac{2 \bullet 2}{2} = 2\ sq.\ in.$$

The small triangle has a base of two inches, a height of one inch, and an area of one square inch.

$$A = \frac{2 \bullet 1}{2} = 1\ sq.\ in.$$

13 The students should now find the area of parallelogram e. They can see that it is composed of two small triangles (c), so its area must be two square inches. Measuring its base and height shows they are equal to two inches and one inch respectively. Multiplying these gives us the area:

$$A = 2 \bullet 1 = 2\ sq.\ in.$$

14 The square will be more difficult to solve. Measuring the side shows it to be approximately $1^3/_8$ inches. Squaring this shows that the area is $1^{57}/_{64}$ square inches. This is almost two square inches. The Pythagorean theorem is more accurate in this case. The sides of the square will represent legs a and b of a right triangle. Since the sides are equal, we will use only the letter a. The diagonal of the square is the hypotenuse, c. Since measuring the hypotenuse shows that it is two inches long, the Pythagorean Theorem is written:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + a^2 &= 2^2 \\ 2a^2 &= 2^2 \\ 2a^2 &= 4 \\ a^2 &= 2 \text{ (\textit{the area of the square})} \\ a = \sqrt{2} &\approx 1.414 \textit{ in.} \end{aligned}$$

15 Students should also write the names of each shape. Younger students may want to label shape a with the name triangle. I have my middle school students use the full name: isosceles right triangle. Similarly, some students will say that shape d is a diamond. Others may opt for the more formal term, rhombus. However, the most specific name is square. The fact that it has been rotated from a familiar orientation causes many students to assume it is no longer a square. This is because most squares they have seen have a base parallel to the edges of their paper. When students change the name of a shape when it is rotated, it shows that they are not functioning at a high level of geometric thinking. They assume that shapes are defined by orientation instead of by their properties. Since the formal definition of a square is a quadrilateral with four congruent sides and four congruent angles, d is a square.

Good Tip 👍

This activity even can be used as a formative assessment tool! When students talk about their thinking, their vocabulary will show the level of sophistication in their thinking as explained here.

⇦

Remind them that triangles are always described by their angles and their sides. There are three classifications of each:

<u>angles</u>
acute – all three angles are less than 90º
right – one angle is exactly 90º
obtuse – one angle is more than 90º
<u>sides</u>
equilateral – all three sides are equal
isosceles – two sides are equal
scalene – no sides are equal

16 Notice that as students discuss their thinking in their group, they will be talking algebraically! You will hear statements such as, “Two c’s is a d,” and, “D is equal to b.” These can then be written as algebraic equations: 2c = d and d = b. I require my students to write an equation for

each shape. The equation must be correct and must contain the letter for that region. For example, these five equations could be used for Tangram 1:

Region	Equation
a	$4c = \mathbf{a}$
b	$\mathbf{b} = 2d$
c	$^{a}/_{4} = \mathbf{c}$
d	$\mathbf{d} = e$
e	$2a = b + 2c + d + \mathbf{e}$

17 If you want to explore proportions, assign a new value to the tangram. For example, if the entire tangram has a value of \$3.00, what is the value of a? Since a = ¼ of the total, it has a value of \$.75.
If b has a value of ½, what is the value of the entire tangram? Since 8 b's can fit in the tangram, the total value of the tangram is 8 • ½ = 4.

18 Advanced students can calculate the perimeters of the pieces. However, this will require the use of the Pythagorean theorem or very accurate measurement. Each tangram has a side length of 4 inches. Therefore the legs of triangle b are each 2" long. The Pythagorean theorem gives the length of the hypotenuse as:

$$a^2 + b^2 = c^2$$

$$2^2 + 2^2 = c^2$$

$$4 + 4 = c^2$$

$$8 = c^2$$

$$2.83 \approx c$$

$$\text{Perimeter} = a + b + c \approx 2 + 2 + 2.83 = 6.83$$

19 Have students explore the other tangram patterns. The first six involve fractions that are based on halves such as $^{1}/_{4}$, $^{1}/_{8,}$ and $^{1}/_{16}$. Halves are easier to work with conceptually than thirds or fifths. For this reason, only the last two tangrams involve thirds or fifths. Also, in the first three tangrams, all areas use unit fractions; that is, their numerators are one. This changes in tangrams four, five, and six where we start to see fractions such as $^{3}/_{8}$. Again this represents another conceptual step in part/whole thinking.

* * *

Journal Prompts:

Explain how polygons b, d, and e are alike and how they are different.

Why does the cut-up method only work on some tiles but not on others?

Homework:

You may wish to assign an unfinished tangram as homework.

Another option is to have students make a tile of their own and write fractions for each region. It should be made of at least seven sections using at least five different shapes. This should be drawn on a 4” or a 6" square. You may wish to pass out the included grid paper for this task.

Taking a Closer Look:

Assign a cost or value to various regions. If the medium-sized triangle of tangram one sells for $1.37, what is the cost of each region and the whole tile? If tangram two costs $5.64, what would each piece cost?

Explore probability. What is the probability of a dart randomly landing on c?

$P(c) = {}^{1}/_{16}$

What is the probability of a dart randomly hitting a quadrilateral?

$P(\text{quadrilateral}) = d + e = {}^{1}/_{8} + {}^{1}/_{8} =$ ¼

Assessment:

By allowing students to work in groups and by asking them to rationalize their answers, you will be able to assess their levels of understanding. Listen to their discussions as mentioned previously. Are they using sophisticated reasoning and accurate vocabulary?

Homework can be assessed after collecting it using the enclosed answer keys, or you may wish to have students trade papers and solve each other’s puzzles.

Answer Key

Tangram 1

Region	Name	Frac.	Dec.	%	Area
a	Isosceles right triangle	1/4	0.25	25	4 in^2
b	Isosceles right triangle	1/8	0.125	12.5	2 in^2
c	Isosceles right triangle	1/16	0.0625	6.25	1 in^2
d	Square	1/8	0.125	12.5	2 in^2
e	Parallelogram	1/8	0.125	12.5	2 in^2

Tangram 2

Region	Name	Frac.	Dec.	%	Area
a	Isosceles right triangle	1/4	0.25	25	4 in^2
b	Isosceles right triangle	1/16	0.0625	6.25	1 in^2
c	Isosceles right triangle	1/8	0.125	12.5	2 in^2
d	Rectangle	1/4	0.25	25	4 in^2
e	Scalene right triangle	1/16	0.0625	6.25	1 in^2
f	Obtuse isosceles triangle	1/8	0.125	12.5	2 in^2

Tangram 3

Region	Name	Frac.	Dec.	%	Area
a	Rectangle	1/8	0.125	12.5	2 in^2
b	Scalene right triangle	1/16	0.0625	6.25	1 in^2
c	Square	1/16	0.0625	6.25	1 in^2
d	Rectangle	1/32	0.03125	3.125	.5 in^2
e	Acute isosceles triangle	1/8	0.125	12.5	2 in^2
f	Rhombus	1/4	0.25	25	4 in^2
g	Obtuse isosceles triangle	1/32	0.03125	3.125	.5 in^2
h	Acute isosceles triangle	1/32	0.03125	3.125	.5 in^2

Tangram 4

Region	Name	Frac.	Dec.	%	Area
a	Isosceles right triangle	1/4	0.25	25	4 in^2
b	Isosceles right triangle	1/8	0.125	12.5	2 in^2
c	Right trapezoid	1/8	0.125	12.5	2 in^2
d	Isosceles right triangle	1/16	0.0625	6.25	1 in^2
e	Isosceles trapezoid	3/16	0.1875	18.75	3 in^2

Tangram 5

Region	Name	Frac.	Dec.	%	Area
a	Rectangle	1/8	0.125	12.5	2 in^{2}
b	Scalene right triangle	1/16	0.0625	6.25	1 in^{2}
c	Isosceles right triangle	1/16	0.0625	6.25	1 in^{2}
d	Isosceles right triangle	1/32	0.03125	3.125	.5 in^{2}
e	Isosceles trapezoid	3/32	0.09375	9.375	1.5 in^{2}
f	Right trapezoid	3/8	0.375	37.5	6 in^{2}

Tangram 6

Region	Name	Frac.	Dec.	%	Area
a	Rectangle	1/8	0.125	12.5	2 in^{2}
b	Acute isosceles triangle	1/16	0.0625	6.25	1 in^{2}
c	Acute right triangle	1/32	0.03125	3.125	.5 in^{2}
d	Rectangle	1/8	0.125	12.5	2 in^{2}
e	Isosceles trapezoid	3/16	0.1875	18.75	3 in^{2}
f	Scalene right triangle	1/16	0.0625	6.25	1 in^{2}
g	Acute isosceles triangle	1/8	0.125	12.5	2 in^{2}
h	Parallelogram	3/16	0.1875	18.75	3 in^{2}

Tangram 7

Region	Name	Frac.	Dec.	%	Area
a	Rectangle	1/3	$0.\overline{3}$	$33^{1}/_{3}$	$5^{1}/_{3}$ in^{2}
b	Obtuse isosceles triangle	1/6	$0.1\overline{6}$	$16^{2}/_{3}$	$2^{2}/_{3}$ in^{2}
c	Scalene right triangle	1/12	$0.08\overline{3}$	$8^{1}/_{3}$	$1^{1}/_{3}$ in^{2}
d	Rectangle	1/6	$0.1\overline{6}$	$16^{2}/_{3}$	$2^{2}/_{3}$ in^{2}
e	Rectangle	1/12	$0.08\overline{3}$	$8^{1}/_{3}$	$1^{1}/_{3}$ in^{2}
f	Scalene right triangle	1/24	$0.041\overline{6}$	$4^{1}/_{6}$	$^{2}/_{3}$ in^{2}

Tangram 8

Region	Name	Frac.	Dec.	%	Area
a	Rectangle	1/5	0.2	20	3.2 in^{2}
b	Scalene right triangle	1/10	0.1	10	1.6 in^{2}
c	Obtuse isosceles triangle	1/20	0.05	5	0.8 in^{2}
d	Acute isosceles triangle	1/20	0.05	5	0.8 in^{2}
e	Scalene right triangle	1/20	0.05	5	0.8 in^{2}
f	Parallelogram	1/10	0.1	10	1.6 in^{2}
g	Rectangle	1/20	0.05	5	0.8 in^{2}
h	Acute isosceles triangle	1/10	0.1	10	1.6 in^{2}
i	Concave pentagon	1/5	0.2	20	3.2 in^{2}

Look at all the Common Core Standards you can teach with tangrams!

4th grade:

CCSS.MATH.CONTENT.4.NF.A.1
Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

CCSS.MATH.CONTENT.4.NF.A.2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

CCSS.MATH.CONTENT.4.NF.B.3
Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

CCSS.MATH.CONTENT.4.NF.B.3.A
Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

CCSS.MATH.CONTENT.4.NF.B.3.B
Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2\ 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$.

CCSS.MATH.CONTENT.4.G.A.2
Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

5th grade:

CCSS.MATH.CONTENT.5.NBT.A.1
Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

CCSS.MATH.CONTENT.5.NBT.A.3
Read, write, and compare decimals to thousandths.

CCSS.MATH.CONTENT.5.NBT.A.3.A
Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.

CCSS.MATH.CONTENT.5.NBT.A.3.B
Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

CCSS.MATH.CONTENT.5.NF.A.1
Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

CCSS.MATH.CONTENT.5.NF.A.2
Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

CCSS.MATH.CONTENT.5.G.B.3
Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

6th grade:

CCSS.MATH.CONTENT.6.RP.A.1
Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
CCSS.MATH.CONTENT.6.RP.A.3
Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

CCSS.MATH.CONTENT.6.RP.A.3.C
Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

CCSS.MATH.CONTENT.6.RP.A.3.D
Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

CCSS.MATH.CONTENT.6.NS.C.6
Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

CCSS.MATH.CONTENT.6.EE.A.3
Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

CCSS.MATH.CONTENT.6.EE.A.4
Identify when two expressions are equivalent (i.e., when the two expressions name the same

number regardless of which value is substituted into them). *For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.*

CCSS.MATH.CONTENT.6.EE.B.6
Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

CCSS.MATH.CONTENT.6.G.A.1
Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

7th grade:

CCSS.MATH.CONTENT.7.RP.A.2
Recognize and represent proportional relationships between quantities.

CCSS.MATH.CONTENT.7.RP.A.2.C
Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.*

CCSS.MATH.CONTENT.7.NS.A.2

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

CCSS.MATH.CONTENT.7.NS.A.2.D
Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

CCSS.MATH.CONTENT.7.EE.B.4
Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

CCSS.MATH.CONTENT.7.G.B.6
Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

8th grade:

CCSS.MATH.CONTENT.8.NS.A.2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,

π^2). *For example, by truncating the decimal expansion of* $\sqrt{2}$*, show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

CCSS.MATH.CONTENT.8.NS.A.1
Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually

CCSS.MATH.CONTENT.8.EE.A.2
Use square root and cube root symbols to represent solutions to equations of the form$x^2 = p$ and $x^3 = \mathrm{p}$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

CCSS.MATH.CONTENT.8.G.A.2
Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

CCSS.MATH.CONTENT.8.G.A.4
Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

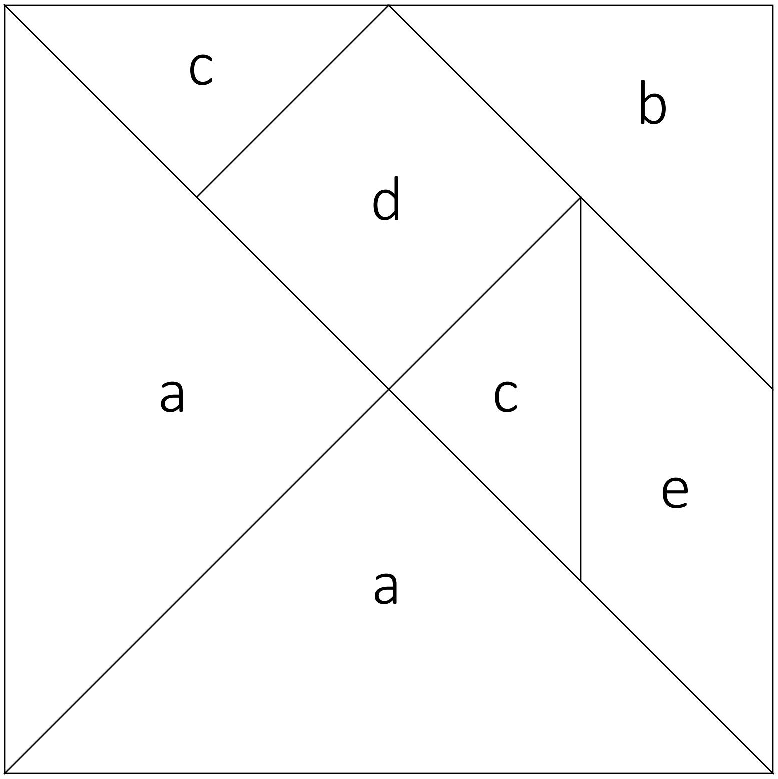
CCSS.MATH.CONTENT.8.G.A.5
Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

CCSS.MATH.CONTENT.8.G.B.7
Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Tangram 1

Name_______________________________

Date_______________________ Class_____

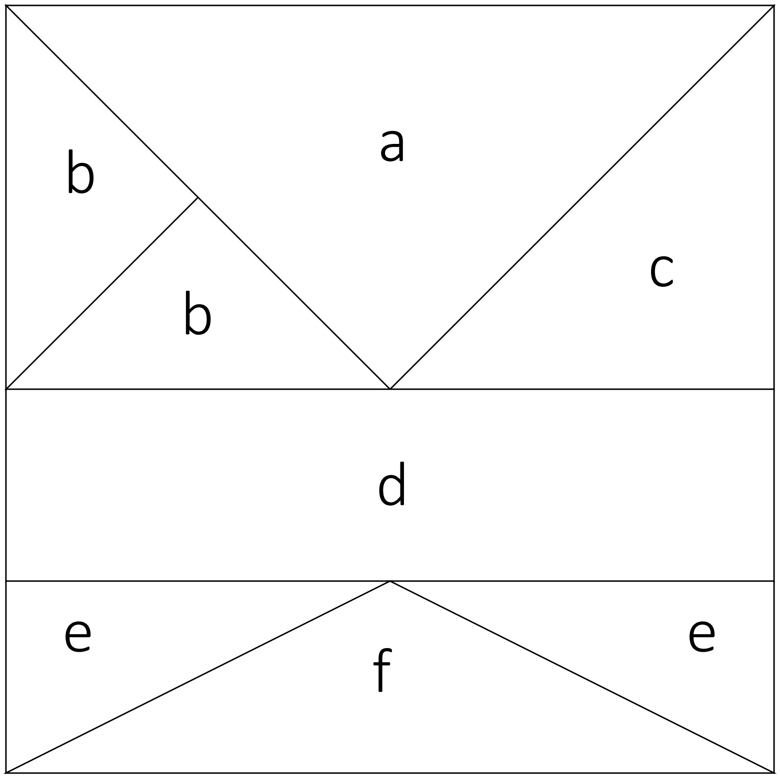


Region	Name	Fraction	Decimal	Percent	Area	Formula
a						
b						
c						
d						
e						

Tangram 2

Name________________________________

Date______________________ Class_____

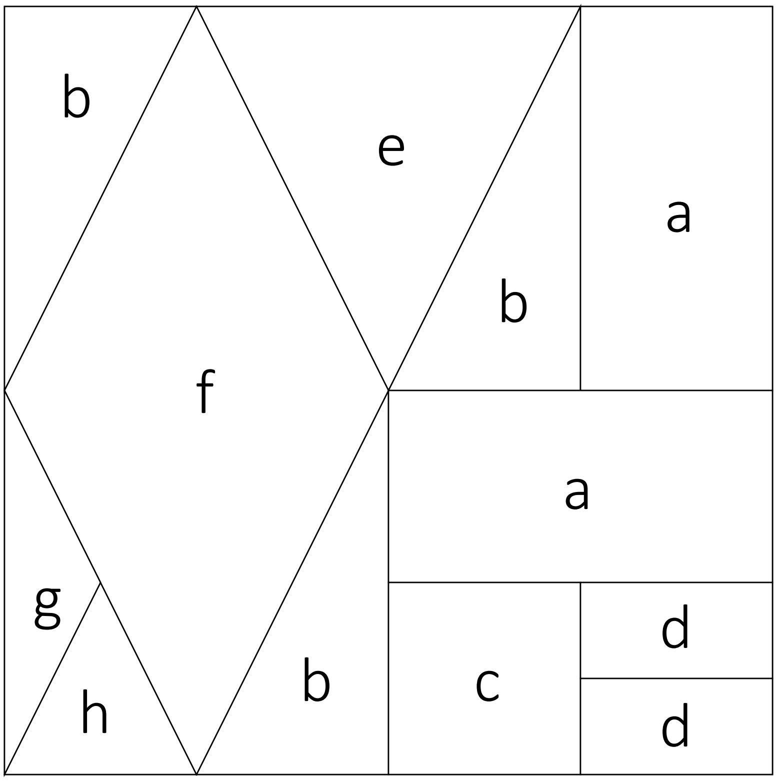


Region	Name	Fraction	Decimal	Percent	Area	Formula
a						
b						
c						
d						
e						
f						

Tangram 3

Name________________________________

Date______________________ Class______

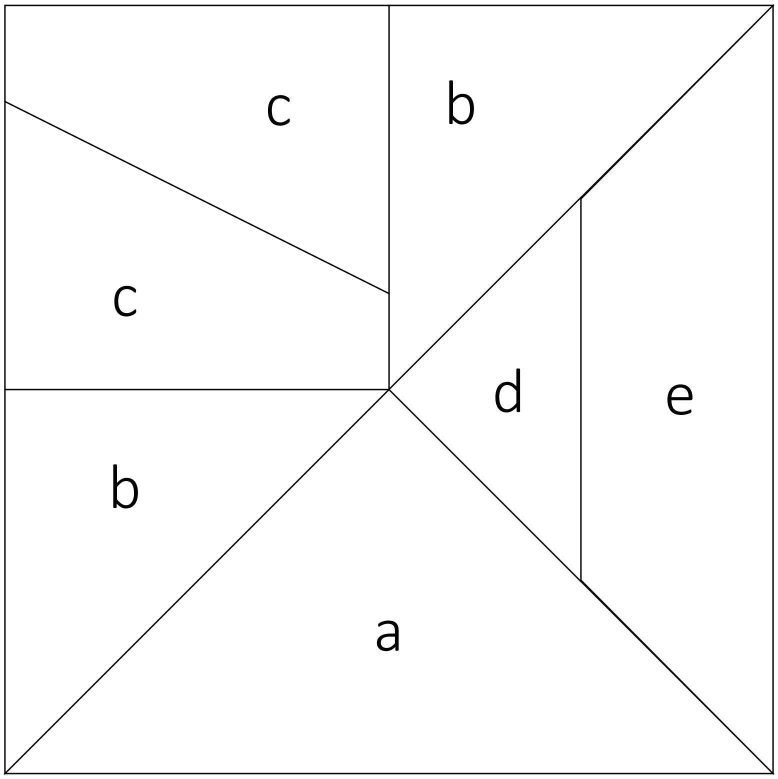


Region	Name	Fraction	Decimal	Percent	Area	Formula
a						
b						
c						
d						
e						
f						
g						
h						

Tangram 4

Name______________________________

Date____________________ Class_____

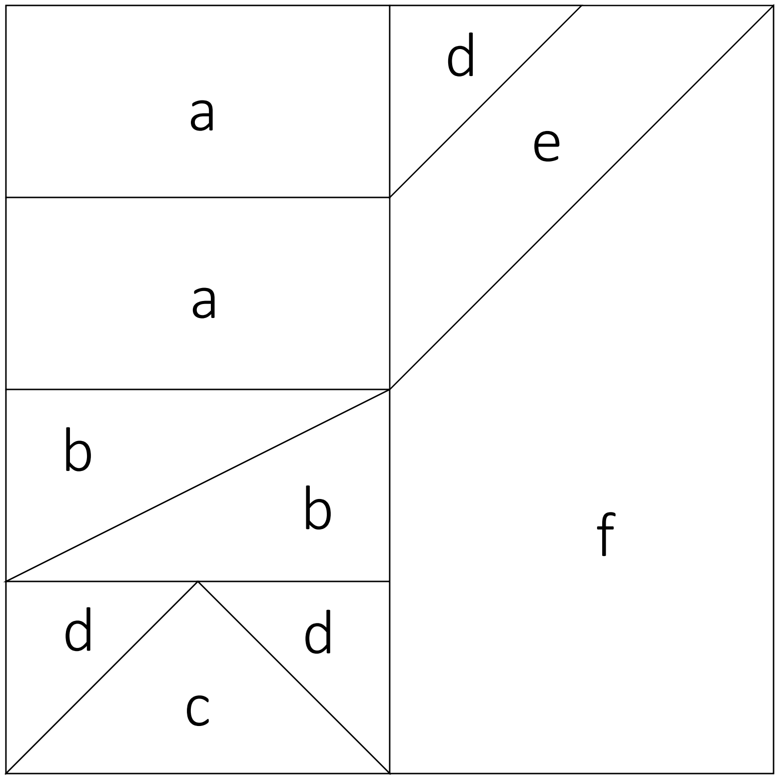


Region	Name	Fraction	Decimal	Percent	Area	Formula
a						
b						
c						
d						
e						

Tangram 5

Name________________________________

Date_____________________ Class_____

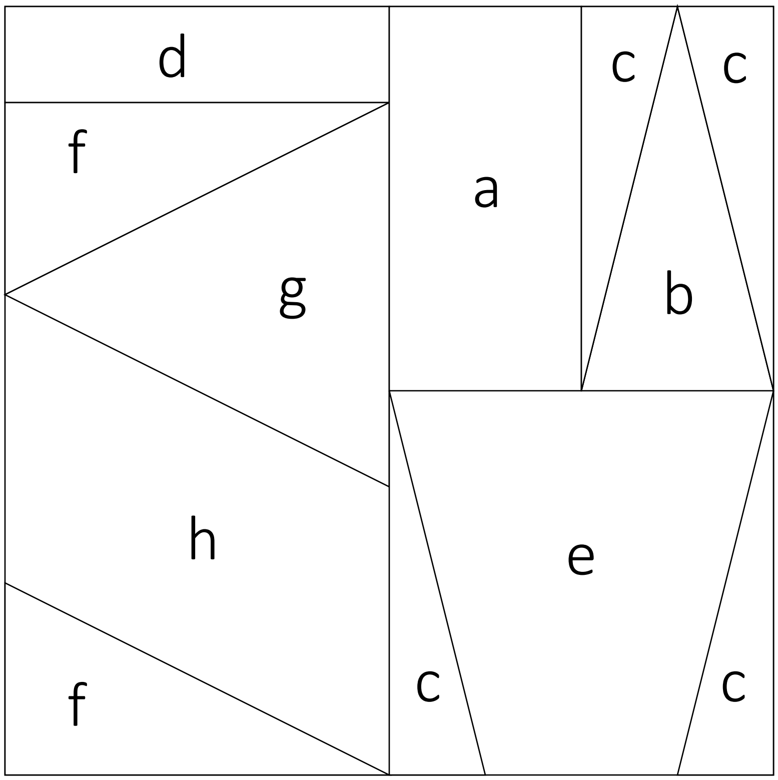


Region	Name	Fraction	Decimal	Percent	Area	Formula
a						
b						
c						
d						
e						
f						

Tangram 6

Name_______________________________

Date______________________ Class_____

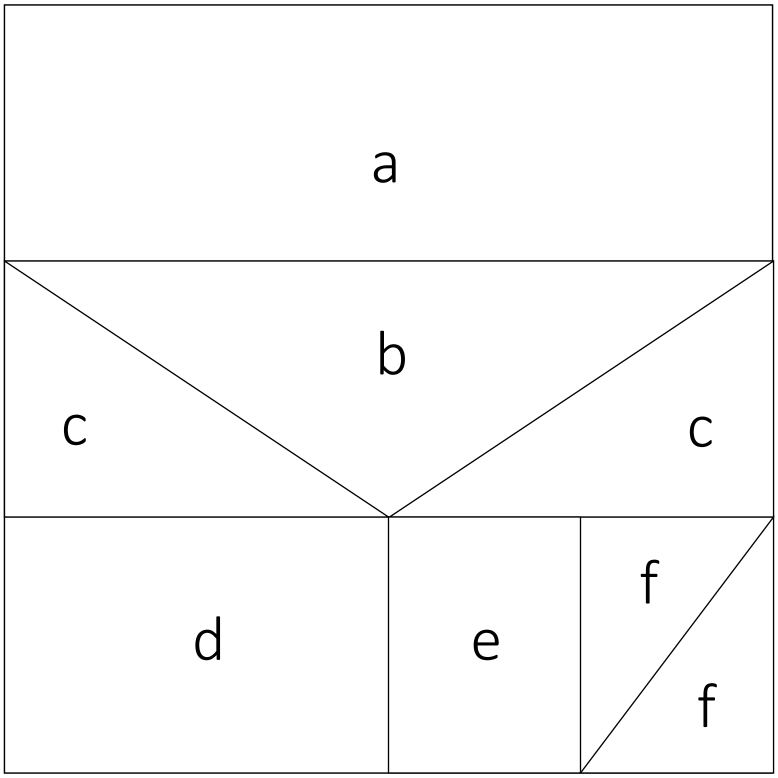


Region	Name	Fraction	Decimal	Percent	Area	Formula
a						
b						
c						
d						
e						
f						
g						
h						

Tangram 7

Name________________________________

Date______________________ Class_____

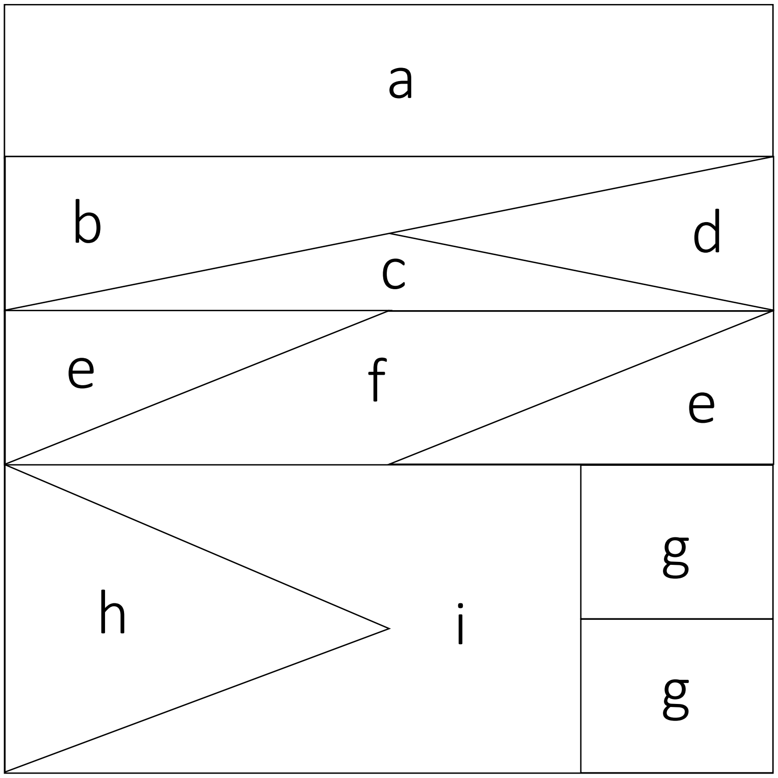


Region	Name	Fraction	Decimal	Percent	Area	Formula
a						
b						
c						
d						
e						
f						

Tangram 8

Name______________________________

Date____________________ Class_____



Region	Name	Fraction	Decimal	Percent	Area	Formula
a						
b						
c						
d						
e						
f						
g						
h						
i						

My Personal Tangram

Name______________________________________

Date___________________________ Class______

Create a tangram of your own design using at least five different shapes and seven different pieces as in Tangram 1. Use your creativity. Assign a value of 1 to your tangram, and then find the values of each piece.

Region	Name	Fraction	Decimal	Percent	Area	Formula
____	____________________________	________	________	________	________	____________
____	____________________________	________	________	________	________	____________
____	____________________________	________	________	________	________	____________
____	____________________________	________	________	________	________	____________
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Notes: