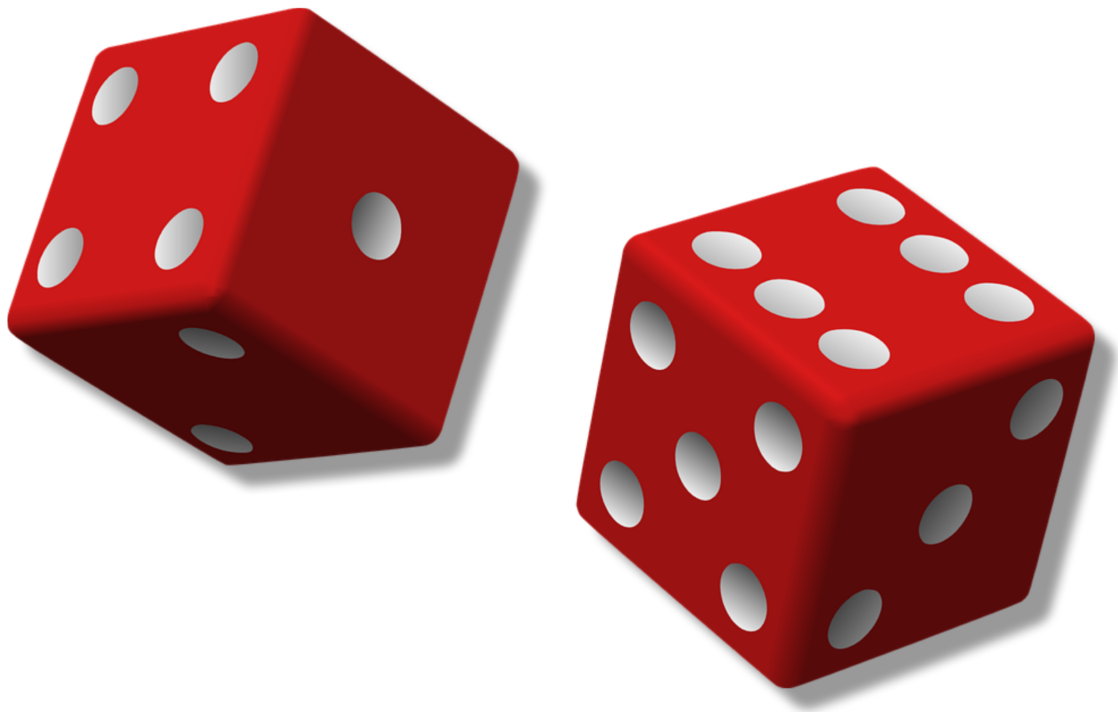
FAIR GAMES
An Exploration In Probability



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Brad Fulton and Bill Lombard

Teacher to Teacher Press

"Building Mathematical Skill on a Foundation of Understanding"

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- **Educators**
- **Authors**
- **Seminar leaders**
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Known throughout the country for motivating and engaging teachers and students, Brad and Bill have authored over ten books that provide easy-to-teach yet mathematically-rich activities for busy teachers. In addition, they have co-authored six teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

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Brad and Bill's math activities are the best I've seen in 30 years of teaching!
Wayne Dequer, 7th grade math teacher

"The high-energy, easy-to-follow handouts were clear. The instructors were great!"
DeLinda Van Dyke, middle school teacher

References available upon request.

ACTIVITY 1

Fair Games 1

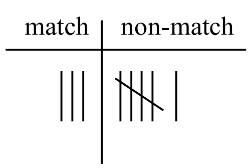
Overview: In this self-motivating investigation students discover the mathematical meaning of fair and unfair. As they work, they will also use reasoning skills, discover patterns, and use Pascal's triangle as a mathematical power tool.

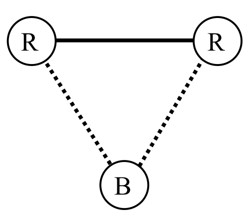
Vocabulary: fair, unfair, theoretical probability, experimental probability

Materials:
- ☒ Paper
- ☒ colored chips
- ☐ colored pencils

PROCEDURE

Skills:
- Finding probability
- Using probability notation
- Understanding probability
- Finding patterns
- Simplifying fractions





1. Explain to the class that you are going to play a game. In this game two red chips and one blue chip will be placed in a can. Two will be drawn at random. If they match, the students get one point; if they don't match, the teacher gets a point. Ask them who they think will win? Also ask them to explain their reasoning. (Some younger students often reason that since there are two matching red chips but only one blue chip the students will win twice as many games.)
2. Play 20 rounds of the game keeping track of matches and non-matches on a tally chart as shown. The students will be surprised to discover that the non-match occurs about twice as many times as the match. Ask them to try to explain why this occurs. How can the game be made fair? Suggest that they consider putting two red chips and two blue chips in the can.
3. Play 20 rounds of this new game to see if it appears to be fair. Again the teacher will likely win by a considerable margin. Why does this happen?
4. Display activity master A or sketch it on the board. Have them make a sketch in their notes. Draw connecting lines between chips to record matches and non-matches. It is best visually to use two different colors. Alternatively, you can use solid lines for matches and dotted lines for non-matches. The result is shown in the margin. This shows that there are two ways to achieve a non-match and only one way to make a match. It is much easier to see why the teacher will probably win this game. Of the three ways that these chips

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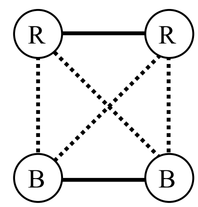
can be drawn, one of the three results in a match, and two of the three result in a non-match. This means that a player of this game will *probably* get a match one out of every three games played and a non-match two out of every three games played. Mathematicians have developed a very convenient shorthand way to write this:

$$P(\text{match}) = {}^{1}/_{3} \qquad P(\text{non-match}) = {}^{2}/_{3}$$

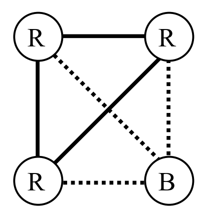
When writing probability, the denominator indicates the total possible outcomes. The numerator indicates the total number of outcomes that result in the desired condition (a match or a non-match).

5. Now display activity master B and have the students sketch this problem in their notes. Have them try to find the probability of a match and non-match. The result is shown in the margin. They will likely be surprised to discover that the outcome is the same as problem A.



$$P(\text{match}) = {}^{2}/_{6} = {}^{1}/_{3} \quad P(\text{non-match}) = {}^{4}/_{6} = {}^{2}/_{3}$$

6. The probability should be written on the transparency master exactly as shown above. It should be noted that probability is written as a common fraction in simplest terms. It is usually not written as a decimal or percent.
7. Ask the students to suggest a game that might be fair. Transparency masters are provided for most situations, but you may have to sketch some that the students suggest. Let's explore problem C. Most students will think the class has an advantage in this game since there are three red chips and only one blue chip. When they play the game, the score will *probably* be much closer than they expect. Once they analyze the game, they will see why that this is a fair game.



$$P(\text{match}) = {}^{3}/_{6} = {}^{1}/_{2} \quad P(\text{non-match}) = {}^{3}/_{6} = {}^{1}/_{2}$$

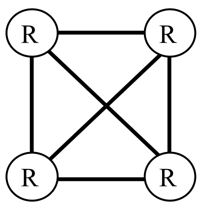
8. Some students will think this game is fair *if they win* and unfair if the teacher wins. This is a good time to introduce the mathematical definition of fair. A game is "fair" if each team has an *equal probability of winning*. With two teams that translates into a probability of one half. This does not mean that the game will always result in a tie. It simply means that if enough games are played, there will *probably* be a nearly equal number of both possible outcomes.

9. This leads into the difference between theoretical probability and experimental probability. Let's assume that in game C, there were 12 matches and 8 non-matches in 20 attempts. The *experimental probability* is shown here:

   P(match) = $^{12}/_{20} = {}^{3}/_{5}$ P(non-match) = $^{8}/_{20} = {}^{2}/_{5}$

   However, the *theoretical probability* is still one half for each outcome. Experimental probability is what actually occurs when the experiment in conducted. Theoretical probability is what is expected to occur in the long run given a reasonable amount of experiments.

10. To make a point, have the students analyze problem D. Although it is obvious that every game is a match, this situation illustrates two important points.



    P(match) = $^{6}/_{6} = 1$ P(non-match) = $^{0}/_{6} = 0$

    A probability of one is an absolute certainty. A probability of zero is an absolute impossibility. That is why a probability is always written as a fraction greater than or equal to zero and less than or equal to one.

11. Have students explore other games with the goal of finding another fair game. A homework master is provided.

Journal Prompts:

What does fair mean? How has your definition of that term changed during this activity?

How has your definition of the word "fair" changed during this activity? What does "fair" mean to a mathematician?

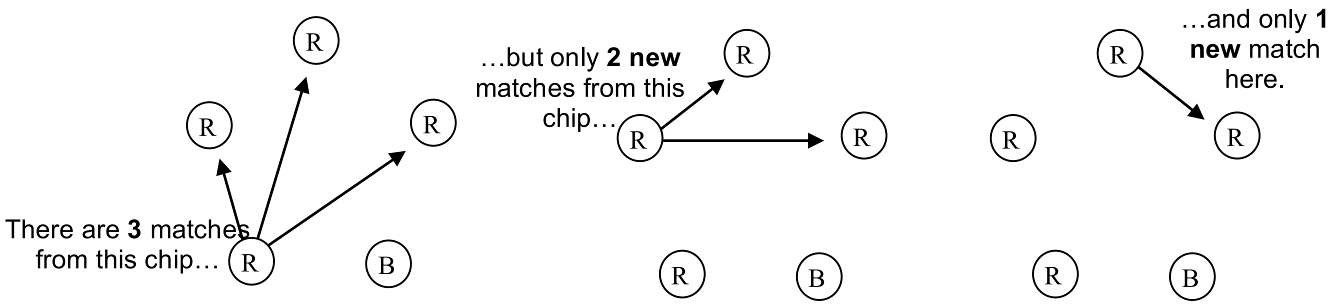
Homework:

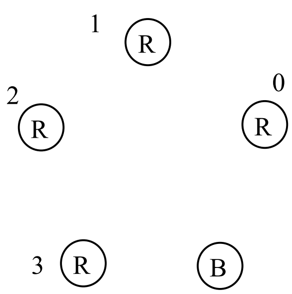
A homework master of sample problems is provided.

Taking a Closer Look: B

Many extensions are explored in the next chapter.

Ask students to describe patterns they see. This will help them analyze games with lots of chips. Some students may notice that the number of possible non-matches is the product of the quantities of the two colors. For example, in game E, there are four red chips and one blue chip. Therefore, there will be 4 x 1 possible non-matches. There is also a trick to finding the matches without drawing all the lines. This is shown below.





The diagram shows that there are six total matches (3 + 2 + 1 + 0). Since we found out that there are four non-matches (4 red times 1 blue), there must be ten total outcomes. Thus we can write:

P(match) = $^6/_{10}$ = $^3/_5$ P(non-match) = $^4/_{10}$ = $^2/_5$

This method allows us to analyze larger games without drawing and counting so many lines.

Assessment: ☑

Check to see that students are writing the correct notation for probability. Avoid the temptation to write, "P(m)." This means, "The probability of getting an m." Also see if they are simplifying fractions. A student may have noticed that the probability of the match and the probability of the non-match will always add up to one.

Invite students to the board or overhead projector to solve problems. This allows you to assess their work and allows the class to see any shortcuts they are using.

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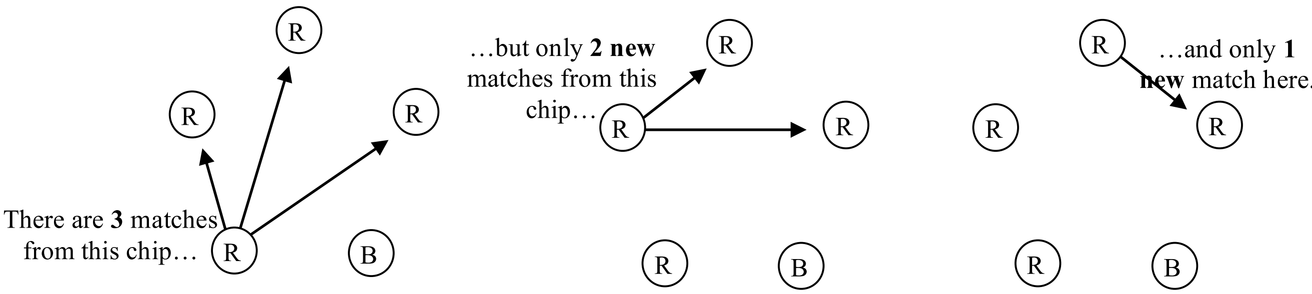
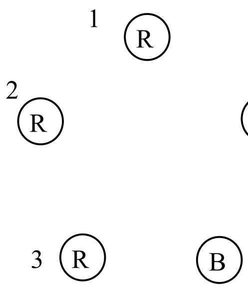




The diagram to the left shows that there are six total matches (3 + 2 + 1 + 0). Since we found out that there are four non-matches (4 red times 1 blue), there must be ten total outcomes. Thus we can write:

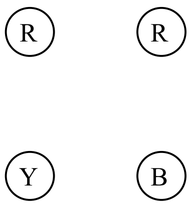
$P(\text{match}) = {}^{6}/_{10} = {}^{3}/_{5}$ $\quad P(\text{non-match}) = {}^{4}/_{10} = {}^{2}/_{5}$

This method allows us to analyze larger games without drawing and counting so many lines.

Homework: Fair Games 1 Name________________________

Analyze each game shown to determine the probability of a match and the probability of a non-match. Use different colors to show matches and non-matches. Remember to simplify all fractions.

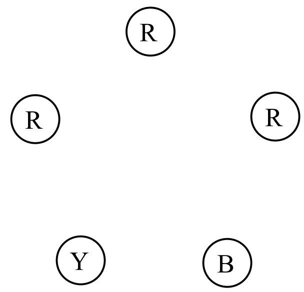
1



P(match) = _____

P(non-match) = _____

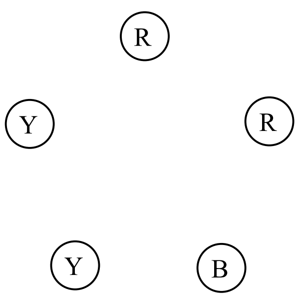
2



P(match) = _____

P(non-match) = _____

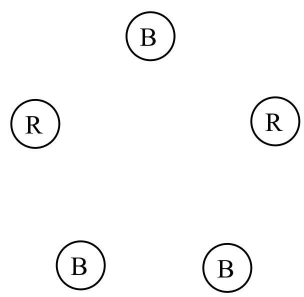
3



P(match) = _____

P(non-match) = _____

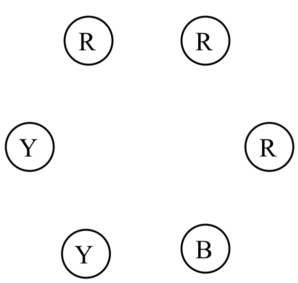
4



P(match) = _____

P(non-match) = _____

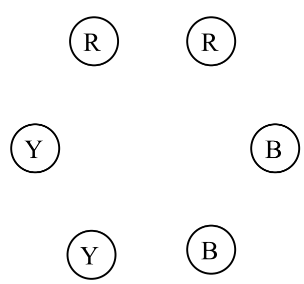
5



P(match) = _____

P(non-match) = _____

6



P(match) = _____

P(non-match) = _____

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Homework: Fair Games 2 Name________________________

Analyze each game shown to determine the probability of a match and the probability of a non-match. Use different colors to show matches and non-matches. Remember to simplify all fractions.

1

R
R R
R R
R B

P(match) = _____

P(non-match) = _____

2

R
R R
Y B
Y B

P(match) = _____

P(non-match) = _____

3

R
R R
R B
Y G

P(match) = _____

P(non-match) = _____

4

R R
R R
R R
R B

P(match) = _____

P(non-match) = _____

5

R R
R R
Y B
Y B

P(match) = _____

P(non-match) = _____

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The following chips are to be placed in a can. Two will be drawn at random. What is the probability of a match? What is the probability of a non-match?

R R

B

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The following chips are to be placed in a can. Two will be drawn at random. What is the probability of a match? What is the probability of a non-match?

R R

B B

The following chips are to be placed in a can. Two will be drawn at random. What is the probability of a match? What is the probability of a non-match?

R R

R B

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The following chips are to be placed in a can. Two will be drawn at random. What is the probability of a match? What is the probability of a non-match?

R R

R R

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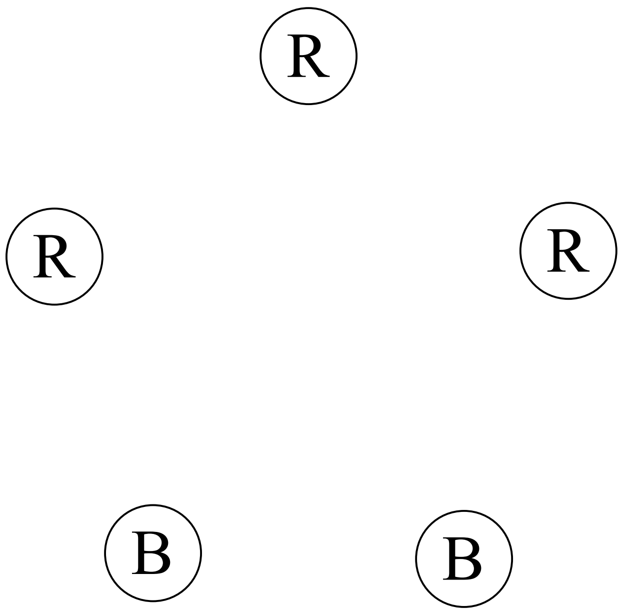
The following chips are to be placed in a can. Two will be drawn at random. What is the probability of a match? What is the probability of a non-match?



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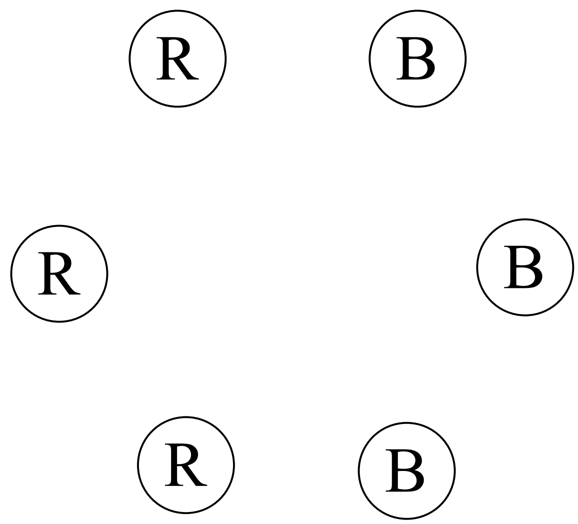

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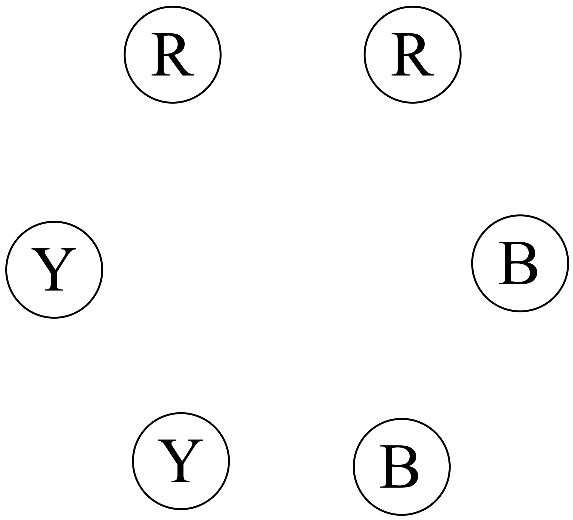

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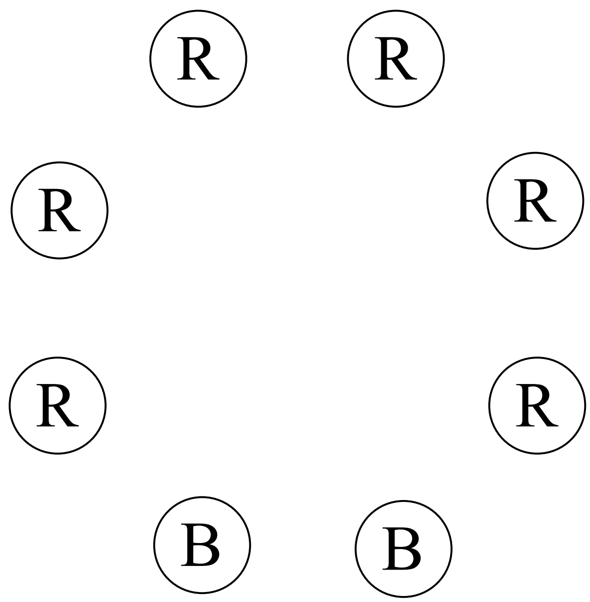

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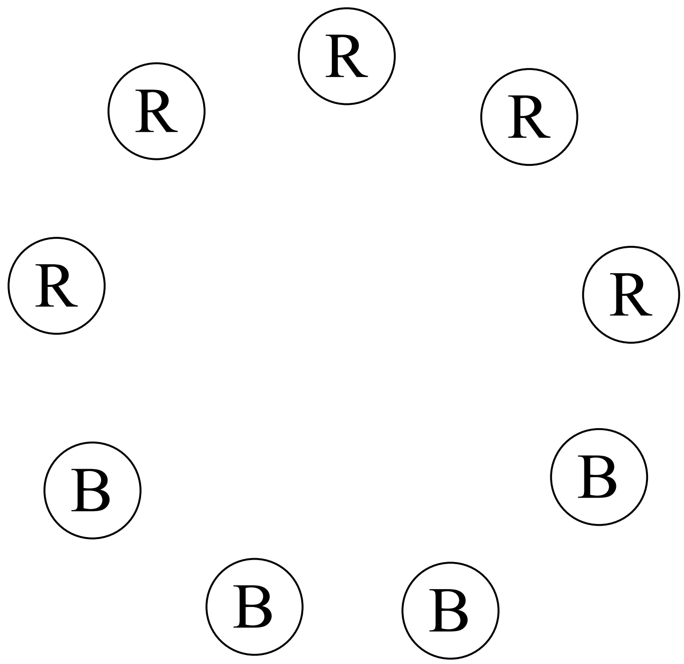

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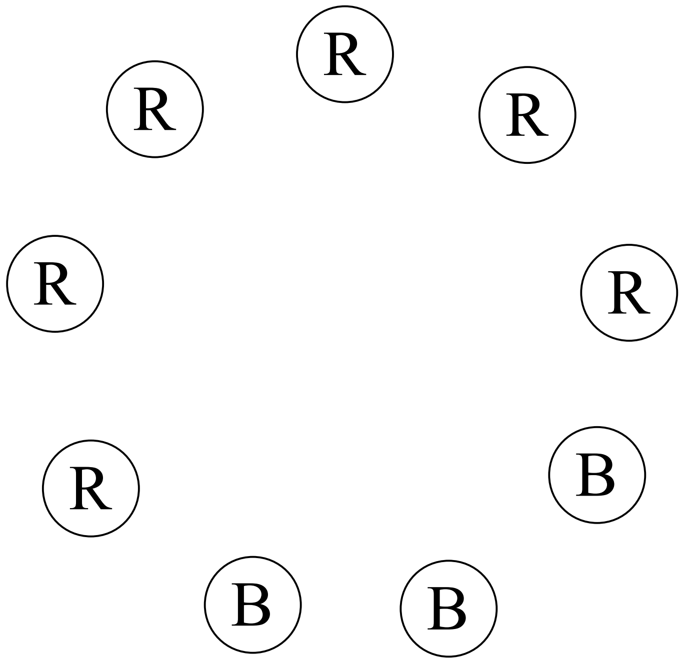

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