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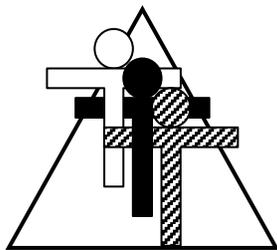
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FAST FACTS AND FRACTIONS TOO!

	1	4
2	<u>2</u>	<u>8</u>
3	<u>3</u>	12

$$2/3 + 1/4 = 11/12$$

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California League of Middle Schools
Educator of the Year, 2005
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Brad Fulton Educator of the Year

- ◆ Consultant
- ◆ Educator
- ◆ Author
- ◆ Keynote presenter
- ◆ Teacher trainer
- ◆ Conference speaker

Known throughout the country for motivating and engaging teachers and students, Brad has co-authored over a dozen books that provide easy-to-teach yet mathematically rich activities for busy teachers while teaching full time for over 30 years. In addition, he has co-authored over 40 teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

Seminar leader and trainer of mathematics teachers

- ◆ 2005 California League of Middle Schools Educator of the Year
- ◆ California Math Council and NCTM national featured presenter
- ◆ Lead trainer for summer teacher training institutes
- ◆ Trainer/consultant for district, county, regional, and national workshops

Author and co-author of mathematics curriculum

- ◆ Simply Great Math Activities series: six books covering all major strands
- ◆ Angle On Geometry Program: over 400 pages of research-based geometry instruction
- ◆ Math Discoveries series: bringing math alive for students in middle schools
- ◆ Teacher training seminar materials handbooks for elementary, middle, and secondary school

Available for workshops, keynote addresses, and conferences

All workshops provide participants with complete, ready-to-use activities that require minimal preparation and give clear and specific directions. Participants also receive journal prompts, homework suggestions, and ideas for extensions and assessment.

Brad's math activities are the best I've seen in 38 years of teaching!

Wayne Dequer, 7th grade math teacher, Arcadia, CA

"I can't begin to tell you how much you have inspired me!"

Sue Bonesteel, Math Dept. Chair, Phoenix, AZ

"Your entire audience was fully involved in math!! When they chatted, they chatted math. Real thinking!"

Brenda McGaffigan, principal, Santa Ana, CA

"Absolutely engaging. I can teach algebra to second graders!"

Lisa Fellers, teacher

References available upon request

With a Multiplication Table You Can:

- ✓ Simplify fractions
- ✓ Find equivalent fractions
- ✓ Add and subtract fractions of unlike denominators
- ✓ Multiply fractions
- ✓ Divide fractions
- ✓ Help students understand fraction procedures better
- ✓ Solve proportions
- ✓ Explore algebraic proofs
- ✓ Explore quadratic functions

(Oh yes, you can also multiply numbers!)

Fast Facts and Fractions

In classrooms across the United States, this program has proven successful at helping struggling students find success learning their multiplication facts and developing mastery with fraction operations. Students will also learn new strategies for putting a fraction in lowest terms and for solving proportions. Once initiated, this program requires about five minutes per day. I developed this program in 2010 to help my struggling 8th grade students, and they have shown impressive growth using these strategies.

Most of my students are daunted by the task of memorizing the 144 multiplication facts. Though you and I understand that the facts are interrelated – that 8×4 is the same as 4×8 – many struggling students fail to make these connections and see the chart as 144 unrelated facts. Here is how I implement the program. Feel free to adapt this program to suit the needs of your students.

I begin by explaining to my students that most adults don't have their multiplication facts memorized. I say this because first because it is true – most adults use calculators for math at virtually every opportunity and have thus forgotten the facts they once knew. I also say this because I don't want my students to think they aren't smart enough to learn their multiplication facts. In most cases, they simply have not invested the time necessary to memorize them.

I also explain to them that they could run a marathon with their shoelaces tied together, but they would be slow, they'd stumble often, and they'd probably give up running marathons. I conclude by saying that they can make it through middle and high school and perhaps college and into adulthood without memorizing their multiplication facts, but they would be slow, they would stumble often, and they'd likely give up.

I then give them the blank 12 by 12 multiplication grid and explain that I know some tricks that will help them get really fast. I use the patterns shown below to help them fill out the facts. Many students do not realize that patterns govern the multiplication chart. This is unfortunate because the brain is much better at recognizing and extending patterns than it is at memorizing information. I utilize this strength of the brain to get students "in the door" of the multiplication table. As the days progress, they begin to memorize the facts and rely less on patterning.

Patterns in the Multiplication Table:

1. Have students fill in the ones and twos column.
2. Likely their toughest column is the twelves. However, if you look at the first lines of the ones and twos, you'll see the products for the twelves.
3. Next have the students fill in the tens and fives.
4. Another easy column is the elevens. However, they may not know the elevens past 11 times 9. There is a trick here too.

x	1	2	12
1	1	2	12
2	2	4	24
3	3	6	36

To multiply a two-digit number, such as 12, times 11. Simply write the number 12 with a space in between. Then write the sum of the digits in the space.

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5. Now it's time to do the nines. Have the students fill in 1×9 , 10×9 , 11×9 , and 12×9 . For the missing products simply begin numbering from one through 8 starting at the top (2×9) and working toward the bottom (9×9). Then number from one through eight again starting from the bottom and working toward the top.
6. We now have seven of the 12 columns complete. If we complete the corresponding rows too, we have only 25 empty cells! Fill in the fours by doubling the twos. Doubling is a fairly rudimentary skill that students can master long before they have mastered other multiplication facts. You may find that some students who don't understand multiplication can still double small numbers.
7. Complete the eights column by doubling the fours.
8. We can complete the threes column by adding the products in the ones and twos column.
9. We can complete the sixes by doubling the threes or by adding the ones and fives products or the twos and fours products.
10. Similarly, we can complete the sevens columns by adding the ones and sixes products, the twos and fives products, or the threes and fours products.
11. Students should also look for the diagonal patterns that show up in the table. For example, beginning in the upper right corner where we have written the product of twelve times one and moving to the lower left corner where we have one times twelve reveals the palindromic pattern:
12, 22, 30, 36, 40, 42, 42, 40, 36, 30, 22, 12
12. Now allow the students to practice these skills as they are timed on completing the table.

Once this is done, and the students are energized and engaged by the patterns that govern these elusive facts, I ask them to put away their chart. I then give them a new blank one and ask them to see how many they can complete in ten minutes. At the end of the time, I ask them to count how many they have and log their result on the accompanying **Fast Facts** graph. Thus if they filled in 93 facts, they would write the date on the first line of the graph's horizontal axis and put a dot above it at 93. This concludes the first day of instruction.

On the second day, I ask them what patterns they recall from the previous day. If they have forgotten any, we review them. Sometimes students will suggest patterns they discovered or learned in a previous year. Then they are given a blank chart and the timer begins. After ten minutes, they log their progress. Invariably they do better than the day before. I ask them to connect the points to form a line graph, and they are impressed by their growth in only one day.

Eventually students will be able to fill in all 144 facts in ten minutes. At that point, they turn their graph over and log their progress on the second graph where the

vertical axis represents time. Now the process takes less than ten minutes per day. Eventually my students get their times incredibly low. My classroom record belongs to a student who could complete the chart in 1:52. The class median is around three minutes. Remember, these are struggling students! Eventually we need only about five minutes of time at the start of class to practice our facts, and then we move on to our daily instruction. I have found this to be a much more practical use of warm up time.

I reward students who can complete the chart in less than three minutes; they only need to take the test once every three weeks. They are quite proud to be able to enter the class and work on their homework while the rest of the class does the multiplication warm up. Students who reach this goal turn in their timed test so that I can verify their accuracy. This is the only time I monitor their results.

I also give my students a randomized multiplication test once every three weeks. These tests have 36 problems that are not in order like the chart. This ensures and verifies that students are actually memorizing the facts.

Once students have gained proficiency with their multiplication facts, I begin to show them how to use the chart as a tool for solving fraction operations. I only introduce one skill per week. They learn how to add fractions one week, and I give them time to gain mastery with that skill. Then I show them how to use the chart to subtract fractions the next week.

Eventually they learn how to use the ***Fantastic Fraction Grid***. This allows students who have mastered their multiplication facts to solve fraction problems without writing out the full 12 by 12 chart.

Some of my students now begin their standardized tests by making a multiplication chart on their scratch paper. I have seen students who struggled for years with low scores on these tests make significant improvements. As their skill increases, so does their confidence. They often begin to see that they are much better at math than they thought, all because they took the time to untangle their shoelaces.

Multiplication Race

Name _____

Date _____ Period _____

X	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

Multiplication Race

Name _____

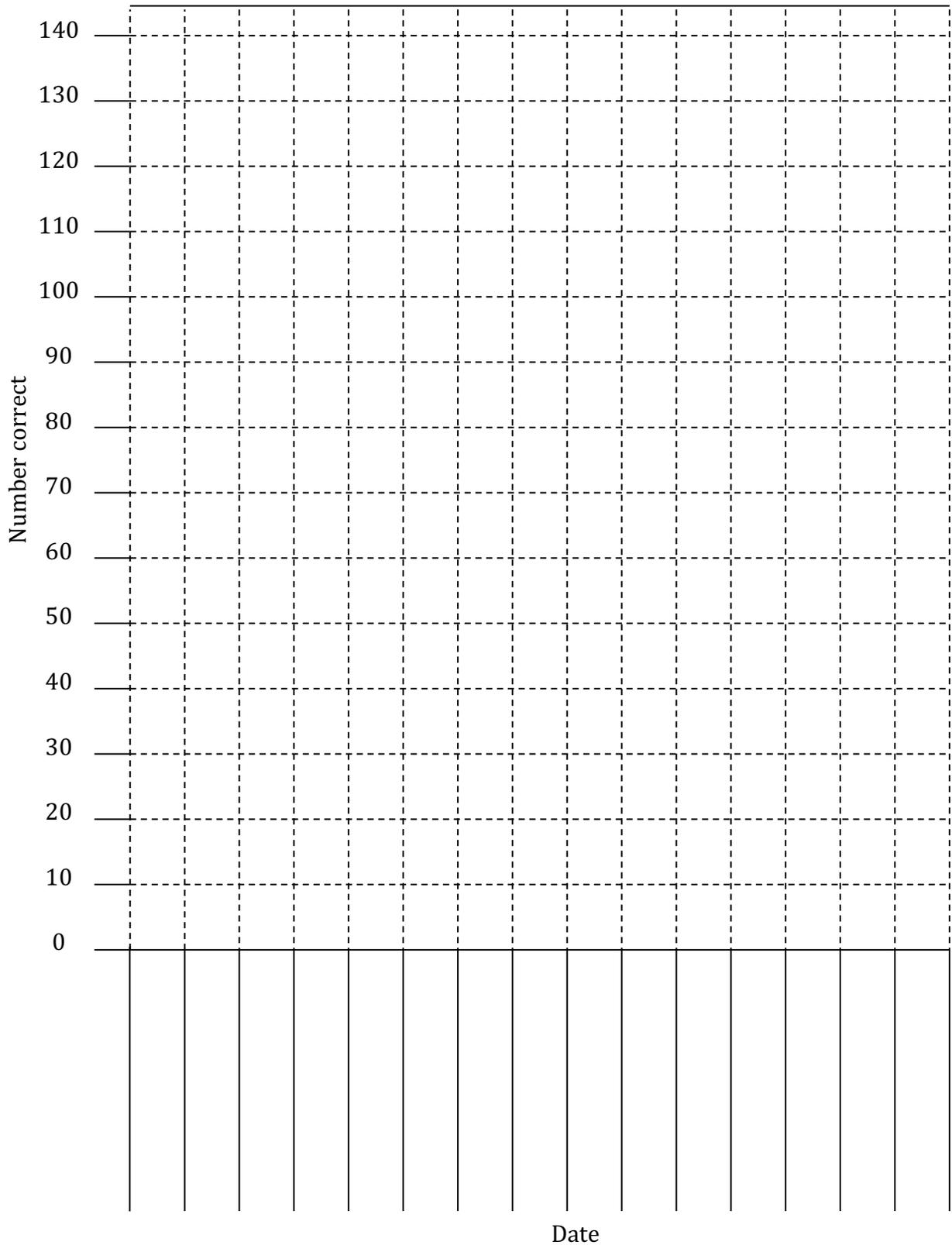
Date _____ Period _____

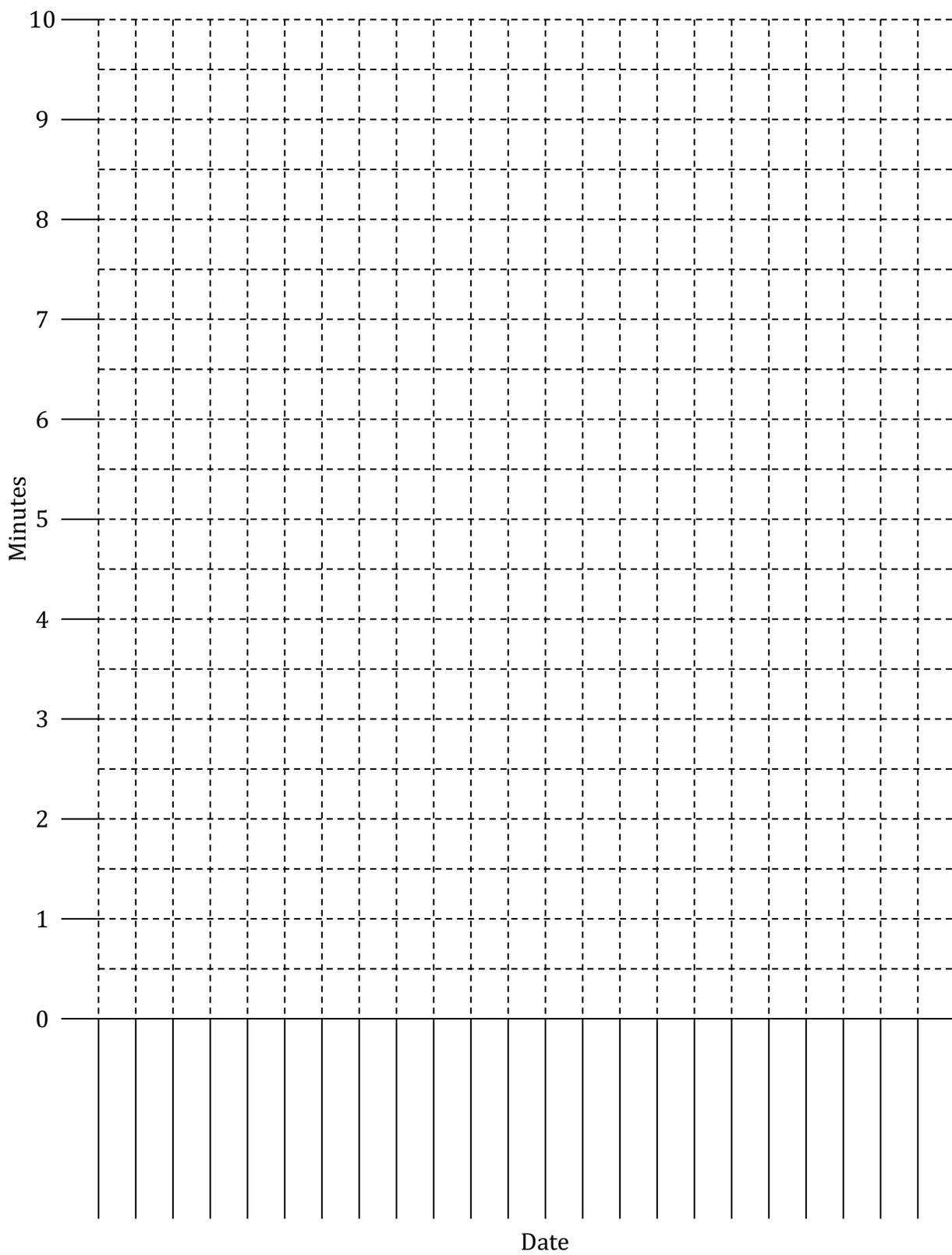
X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

FAST FACTS

Name _____

Period _____





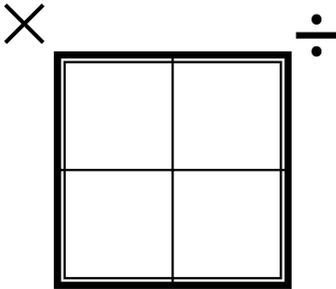
**FANTASTIC
FRACTION
GRID**

Name _____

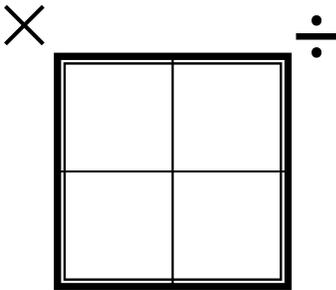
Date _____ Period _____

Find the sum, difference, product, and quotient of each pair of fractions.

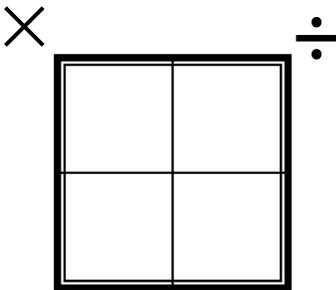
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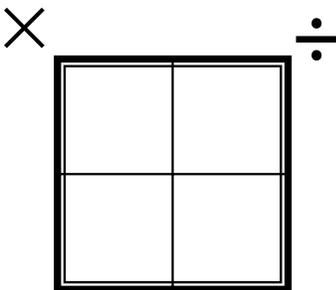
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3



4



Simplifying fractions:

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Example: Simplify $\frac{12}{21}$

Find your fraction vertically in the multiplication table.

Read its simplified value from the left-hand column.

Answer: $\frac{4}{7}$

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

When looking for your fraction in the table, you may find it more than once. Use the uppermost location to find the simplest form.

Example: Simplify $\frac{18}{30}$

Answer: $\frac{3}{5}$

Simplifying Fractions
Practice Page

Name _____

Date _____ Period _____

Simplify the following fractions using your multiplication table.

1) $\frac{3}{6} = \text{-----}$

2) $\frac{5}{15} = \text{-----}$

3) $\frac{3}{9} = \text{-----}$

4) $\frac{10}{25} = \text{-----}$

5) $\frac{14}{21} = \text{-----}$

6) $\frac{60}{70} = \text{-----}$

7) $\frac{10}{12} = \text{-----}$

8) $\frac{3}{9} = \text{-----}$

9) $\frac{28}{35} = \text{-----}$

10) $\frac{4}{16} = \text{-----}$

11) $\frac{12}{18} = \text{-----}$

12) $\frac{20}{24} = \text{-----}$

13) $\frac{24}{30} = \text{-----}$

14) $\frac{18}{36} = \text{-----}$

Adding and Subtracting Of Unlike Denominators:

X	1	2	<u>3</u>	4	<u>5</u>	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
<u>2</u>	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
<u>7</u>	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Example: Add $\frac{2}{7} + \frac{3}{5}$

Find the first fraction on the left of the table.

Find the second fraction at the top of the table.

Multiply the denominators as shown. This is the denominator of your answer.

X	1	2	<u>3</u>	4	<u>5</u>	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
<u>2</u>	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
<u>7</u>	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Multiply as shown and add the products. This is the numerator of your answer.

Answer: $\frac{31}{35}$

Simplify if necessary.

X	1	2	<u>3</u>	4	<u>5</u>	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
<u>5</u>	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
<u>7</u>	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Example: Subtract $\frac{5}{7} - \frac{3}{5}$

Find the first fraction on the left of the table.

Find the second fraction at the top of the table.

Multiply the denominators as shown. This is the denominator of your answer.

X	1	2	<u>3</u>	4	<u>5</u>	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
<u>5</u>	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
<u>7</u>	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Multiply as shown and subtract the products. This is the numerator of your answer.

Answer: $\frac{4}{35}$

Simplify if necessary.

Adding Fractions
Practice Page

Name _____

Date _____ Period _____

Add the following fractions using your multiplication table. Simplify as necessary.

1) $\frac{1}{2} + \frac{1}{3} =$ _____

2) $\frac{1}{4} + \frac{1}{5} =$ _____

3) $\frac{1}{3} + \frac{1}{4} =$ _____

4) $\frac{2}{3} + \frac{1}{5} =$ _____

5) $\frac{3}{7} + \frac{1}{2} =$ _____

6) $\frac{2}{5} + \frac{1}{3} =$ _____

7) $\frac{1}{2} + \frac{1}{4} =$ _____

8) $\frac{1}{6} + \frac{1}{4} =$ _____

9) $\frac{1}{2} + \frac{2}{3} =$ _____

10) $\frac{3}{5} + \frac{2}{3} =$ _____

11) $\frac{3}{8} + \frac{1}{2} =$ _____

12) $\frac{7}{8} + \frac{1}{4} =$ _____

Subtracting Fractions
Practice Page

Name _____

Date _____ Period _____

Subtract the following fractions using your multiplication table.
Simplify as necessary.

1) $\frac{1}{2} - \frac{1}{3} = \text{-----}$

2) $\frac{1}{4} - \frac{1}{5} = \text{-----}$

3) $\frac{1}{3} - \frac{1}{4} = \text{-----}$

4) $\frac{2}{3} - \frac{1}{5} = \text{-----}$

5) $\frac{5}{7} - \frac{1}{2} = \text{-----}$

6) $\frac{2}{5} - \frac{1}{3} = \text{-----}$

7) $\frac{1}{2} - \frac{1}{4} = \text{-----}$

8) $\frac{5}{6} - \frac{1}{4} = \text{-----}$

9) $\frac{1}{2} - \frac{2}{9} = \text{-----}$

10) $\frac{3}{4} - \frac{2}{3} = \text{-----}$

11) $\frac{1}{2} - \frac{3}{8} = \text{-----}$

12) $\frac{7}{8} - \frac{1}{4} = \text{-----}$

Multiplying Fractions:

X	1	2	<u>3</u>	<u>4</u>	5	6	7	8	9	10
1	1	2			5	6	7	8	9	10
2	2	4			10	12	14	16	18	20
3	3	6			15	18	21	24	27	30
<u>4</u>			12		20	24	28	32	36	40
<u>5</u>				20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Example: Multiply $\frac{4}{5} \times \frac{3}{4}$

Find the first fraction on the left of the table.

Find the second fraction at the top of the table.

Multiply as shown. The upper number is the numerator of your answer. The lower number is the denominator.

X	1	2	<u>3</u>	<u>4</u>	<u>5</u>	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
<u>2</u>	2	4	6	8	10	12	14	16	18	20
3				12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
<u>5</u>					20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54	60
<u>7</u>	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Simplify if necessary.

Answer: $\frac{3}{5}$

Multiplying Fractions
Practice Page

Name _____

Date _____ Period _____

Multiply the following fractions using your multiplication table.
Simplify as necessary.

1) $\frac{1}{2} \times \frac{1}{3} =$ _____

2) $\frac{1}{4} \times \frac{1}{5} =$ _____

3) $\frac{1}{3} \times \frac{1}{4} =$ _____

4) $\frac{2}{3} \times \frac{1}{5} =$ _____

5) $\frac{5}{7} \times \frac{1}{2} =$ _____

6) $\frac{2}{5} \times \frac{1}{3} =$ _____

7) $\frac{2}{3} \times \frac{1}{4} =$ _____

8) $\frac{5}{6} \times \frac{3}{4} =$ _____

9) $\frac{1}{2} \times \frac{2}{9} =$ _____

10) $\frac{3}{4} \times \frac{2}{3} =$ _____

11) $\frac{1}{6} \times \frac{3}{8} =$ _____

12) $\frac{2}{9} \times \frac{3}{4} =$ _____

Dividing Fractions:

X	1	2	3	4	5	6	7	8	9	10
1	1	2			5	6	7	8	9	10
2	2	4			10	12	14	16	18	20
3	3	6			15	18	21	24	27	30
4					20	24	28	32	36	40
5					25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Example: Divide $\frac{4}{5} \div \frac{3}{4}$

Find the first fraction on the left of the table.

Find the second fraction at the top of the table.

Multiply as shown. The upper number is the numerator of your answer. The lower number is the denominator.

Result: $\frac{16}{15}$

Simplify if necessary.

Answer: $1\frac{1}{15}$

Dividing Fractions
Practice Page

Name _____

Date _____ Period _____

Divide the following fractions using your multiplication table. Simplify as necessary.

1) $\frac{1}{3} \div \frac{1}{2} =$ _____

2) $\frac{1}{5} \div \frac{1}{3} =$ _____

3) $\frac{1}{3} \div \frac{3}{4} =$ _____

4) $\frac{1}{7} \div \frac{2}{5} =$ _____

5) $\frac{2}{5} \div \frac{3}{5} =$ _____

6) $\frac{2}{5} \div \frac{2}{5} =$ _____

7) $\frac{2}{3} \div \frac{1}{3} =$ _____

8) $\frac{1}{6} \div \frac{5}{6} =$ _____

9) $\frac{5}{8} \div \frac{3}{8} =$ _____

10) $\frac{3}{4} \div \frac{2}{3} =$ _____

11) $\frac{1}{6} \div \frac{3}{8} =$ _____

12) $\frac{2}{9} \div \frac{3}{4} =$ _____

Fraction Operations Without a Multiplication Table:

Often students are not allowed to use a multiplication table during testing. Here is a simple way to show them to create a do-it-yourself template for solving all four operations:

Add $\frac{2}{3} + \frac{1}{4} =$

Subtract $\frac{2}{3} - \frac{1}{4} =$

Multiply $\frac{2}{3} \times \frac{1}{4} =$

Divide $\frac{2}{3} \div \frac{1}{4} =$

Draw a two by two grid:

Write the first fraction on the left of the grid.

Write the second fraction on the top of the grid.

Multiply the digits to complete the grid.

The sum is found by adding the $8 + 3$ and writing the answer over the 12.
 $\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$

The difference is found by subtracting the $8 - 3$ and writing the answer over the 12.
 $\frac{2}{3} - \frac{1}{4} = \frac{5}{12}$

The product is found in one of the diagonals as shown.
 $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12}$
Simplify as necessary. $\frac{2}{12} = \frac{1}{6}$

The quotient is found in the other diagonal as shown.
 $\frac{2}{3} \div \frac{1}{4} = \frac{8}{3}$
Simplify as necessary. $\frac{8}{3} = 2 \frac{2}{3}$

	1	4
2		
3		

	1	4
2	2	8
3	3	12

	1	4
2	2	<u>8</u>
3	<u>3</u>	12

	1	4
2	<u>2</u>	8
3	3	<u>12</u>

	1	4
2	2	<u>8</u>
3	<u>3</u>	12

Solving Proportions:

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Example: Solve $\frac{12}{30} = \frac{16}{x}$

Find the three known numbers as vertices of a rectangle.

The missing vertex is the solution to the proportion.

$$x = 40$$

Why this works:

Notice that the 12 is the product of 6 x 2.

The 16 is the product of 8 x 2.

The 30 is the product of the 6 x 5.

The proportion could be written:

$$\frac{(6 \times 2)}{(6 \times 5)} = \frac{(8 \times 2)}{x}$$

Using the cross products rule gives us:

$$(6 \times 2)(x) = (6 \times 5)(8 \times 2)$$

The associative property gives us:

$$(6 \times 2)(x) = (6 \times 2)(8 \times 5)$$

Canceling the common factors leaves:

$$x = (8 \times 5) = 40$$

Solving Proportions
Practice Page

Name _____

Date _____ Period _____

Solve the proportions using your multiplication table.

1) $\frac{9}{12} = \frac{15}{x}$

2) $\frac{20}{28} = \frac{35}{x}$

3) $\frac{8}{24} = \frac{x}{36}$

4) $\frac{10}{12} = \frac{x}{30}$

5) $\frac{9}{x} = \frac{1}{3}$

6) $\frac{12}{x} = \frac{10}{15}$

7) $\frac{x}{24} = \frac{15}{18}$

8) $\frac{x}{14} = \frac{35}{49}$

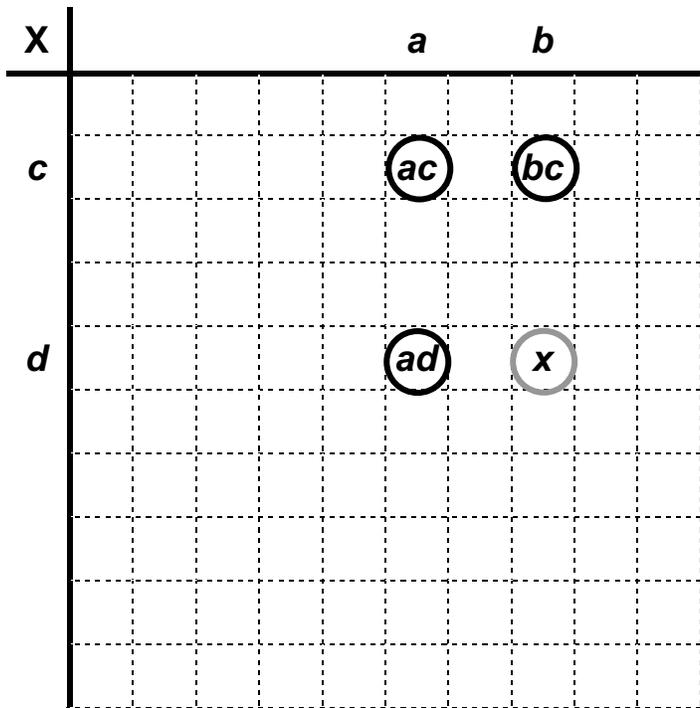
9) $\frac{15}{36} = \frac{x}{24}$

10) $\frac{9}{6} = \frac{21}{x}$

11) $\frac{12}{20} = \frac{3}{x}$

12) $\frac{15}{20} = \frac{x}{8}$

Solving Proportions, An Algebraic Proof:



Given that three numbers in a proportion can be located on a multiplication table as the vertices of a rectangle, prove that the fourth vertex is the solution to the proportion.

If x is the solution to the proportion, then, $\frac{ac}{ad} = \frac{bc}{x}$

$$x = bd \quad \text{Given}$$

$$\frac{ac}{ad} = \frac{bc}{bd} \quad \text{Substitution}$$

$$(ac)(bd) = (ad)(bc) \quad \text{Cross products}$$

$$(abcd) = (abcd) \quad \text{Associative property}$$

Exploring Quadratic Functions:

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Looking at numbers in the diagonals of the multiplication table allows us to explore quadratic functions. Here we see the square numbers which are generated of course by multiplying a factor in the left column by its matching factor in the top row.

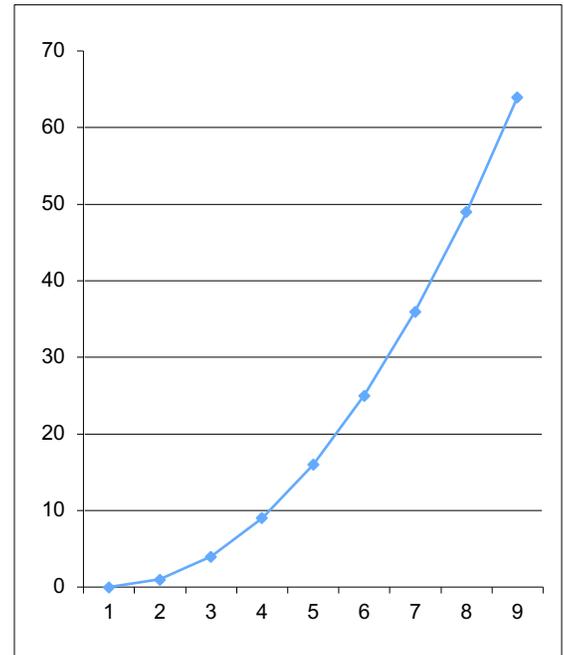
These functions are of the form:

$$y = ax^2 + bx + c$$

We can view the circled numbers and the numbers to their far left as the columns of a t-table.

When graphed they always form parabolas as shown. But what about other diagonals in the multiplication table?

x	y
1	1
2	4
3	9
4	16
5	25
6	36



Exploring Quadratic Functions:

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Here we look at the next diagonal to the right. These are not square numbers, but they still represent a quadratic function.

Notice that the numbers can be represented as their products as shown in the t-table. We see that each number is the product of the step (x) and one more than the step ($x + 1$):

Thus we can represent the function as:

$$y = x(x + 1)$$

or

$$y = x^2 + x$$

x	y
1	2 = 1 x 2
2	6 = 2 x 3
3	12 = 3 x 4
4	20 = 4 x 5
5	30 = 5 x 6
6	42 = 6 x 7

Exploring Quadratic Functions:

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Here is the next diagonal in the chart. Again the numbers can be represented as their products in the t-table.

Now each number is the product of the step (x) and *two* more than the step ($x + 2$):

Thus we can represent the function as:

$$y = x(x + 2)$$

or

$$y = x^2 + 2x$$

This pattern continues throughout the chart creating the following sequence of functions:

$$y = x^2 + 3x$$

$$y = x^2 + 4x$$

$$y = x^2 + 5x$$

If the diagonal is moved to the left of the square numbers we get functions of this nature:

$$y = x^2 - x$$

$$y = x^2 - 2x$$

$$y = x^2 - 3x$$

x	y
1	3 = 1 x 3
2	8 = 2 x 4
3	15 = 3 x 5
4	24 = 4 x 6
5	35 = 5 x 7
6	48 = 6 x 8

Exploring Quadratic Functions:

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

What if we flip the diagonal?

Now we get a new arrangement of numbers.

Now each number is the product of the step (x) and eleven minus the step ($11 - x$):

Thus we can represent the function as:

$$y = x(11 - x)$$

or

$$y = 11x - x^2$$

Put in standard form, the equation is:

$$y = -x^2 + 11x$$

Students can now explore quadratic functions that can be found in other diagonals of the multiplication table.

x	y
1	10 = 1 x (11 - 1)
2	18 = 2 x (11 - 2)
3	24 = 3 x (11 - 3)
4	28 = 4 x (11 - 4)
5	30 = 5 x (11 - 5)
6	30 = 6 x (11 - 6)

And Finally:

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Let's not forget that the multiplication table is pretty handy for multiplying numbers too!

While we want to help students to be independent from relying on a multiplication table, allowing them time to explore the mathematics hidden in this simple chart will not only make them more familiar with its members and its properties, it will help them enjoy the intricate beauty of the mathematics it reveals.

And who knows what jewels your students may discover buried here!

“The real voyage of discovery consists not in seeking new lands but seeing with new eyes.”

Marcel Proust

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Feel free to contact me if you have questions or comments or would like to discuss a staff development training or keynote address at your site.

Happy teaching,

Brad