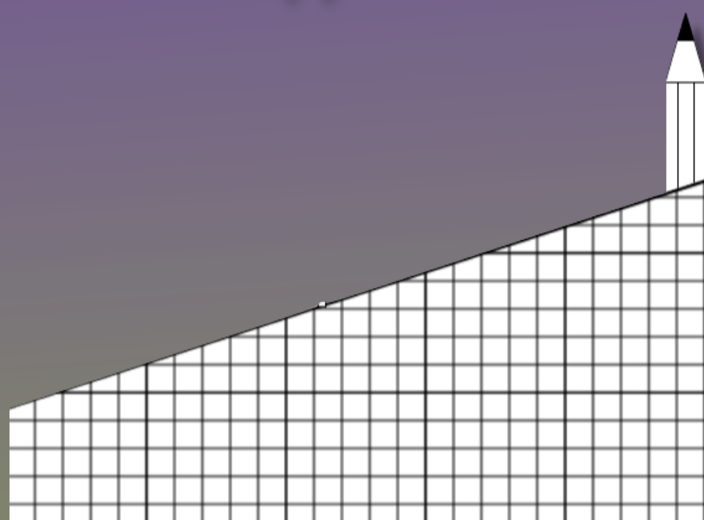
Function Fun

Unit 3: Applications of Linear Functions



By Brad Fulton

Educator of the Year, 2005

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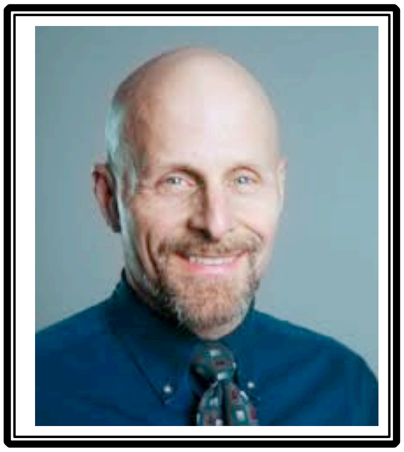

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Educator of the Year

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Known throughout the country for motivating and engaging teachers and students, Brad has co-authored over a dozen books that provide easy-to-teach yet mathematically rich activities for busy teachers while teaching full time for over 30 years. In addition, he has co-authored over 40 teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

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Function Fun

Part 3: Applications of Linear Functions

Overview:

This activity is part 3 in a five-part series on developing an understanding of and proficiency with functions. Part 1 offered an introduction helping students to understand slope and y-intercept when integrated across a multi-representational approach that included visual, graphical, tabular, linguistic, and equation models.

In part 2, the student developed even greater proficiency as negative and fractional slopes and y-intercepts are encountered.

Part 3 allows students to apply their understanding of functions in tackling three realistic problems: "Banking on Functions", "The Great Yo-yo Festival", and "A Nutty Function."

Part 4, *Quadratic Functions*, incorporates functions of the form $ax^2 + bx + c$ and is geared toward an algebra 1 course.

Part 5, *The King's Pathway*, provides an engaging project your students can do to create their own beautiful functions based on their work with the previous sections.

Procedure:

Section 1: Banking on Functions.. 1 class period

Required Materials:

☒ Banking on Functions Activity Master

☒ Banking on Functions Student Worksheet

☒ Banking on Functions homework

Optional Materials:

☐ Calculators

1. Students may work individually or in small groups on this activity. Display a copy of the Savings Account Problems showing only the first two sentences of problem 1. Students will likely volunteer that they need more information to figure out the problem. Acting confused, ask them what information they need. When they tell that they need to know how many nonths are involved, answer that you don't know—Randy didn't tell you. Then ask students if they could solve the problem if they knew that Randy would make only one monthly deposit. Students may tell you the answer is \$40 + \$70 = \$110. If they do, ask them what would happen if he made two monthly deposits. When students see that the equation for answering this qeustions is 2(\$40) + \$70 = \$150, ask them how how much Randy would have if he made *zero* monthly deposits. The answer is \$70, the y-intercept. Then ask students how much Randy's balance goes up each month. They will see it boes up by \$40. This is the slope. Ask them if they could make a t-table showing the infomration. Most students will see that they can.

2. Pass out ¼ inch grid paper to each student. (The enclosed master includes wide margins that can be used for recording the t-table and equation.) Students will need to create axes and fgure out the best way to scale them so that all of their data will fit. You may wish to tell them this or let them discover on their own. Counting by $10 or $20 per line on the vertical (y) axis will work nicely. Counting by twos on the horizontal (x) axis will also work; the answers to subsequent questions will not fit on the graph if the scale on the x-axis is one unit. Have students construct the t-table for Randy's account, graph the function, and write the formula: $y = 40x + 70$. Ask them what the variables x and y represent (number of months and total dollars in the account respectively).

> ***Teacher Tip!***
> If you would prefer students become accustomed to function notation, you could have them write the formula:
> $$f(m) = 40m + 70$$
> In this case, the variable m represents the number of months. It is read, "The function of m is 40 times m plus 70."

3. Now give some values for x (months). What if Randy makes 17 monthly deposits? What if he makes 37? What if he makes deposits for 4 years (48 months)? Students can see that even when they don't have enough information to solve a problem, they can find a formula that will solve all related problems. Reaveal the remainder of problem 1 and have students calculate answers. Alternatively students can extend their graphs to answer each query. Respectively, the answers are $270, $470, and $790.

4. Reveal problem 2 in its entirety. Most students will construct a second t-table and extend it until a y-value is found that matches the former t-table. You may also show sstudents that graphing the two functions shows their intersection as the solution. Lastly, you may wish to show students an algebraic solution:

$$\begin{aligned} 40x + 70 &= 35x + 310 \\ -35x \quad & \quad -35x \\ 5x + 70 &= 310 \\ -70 \quad & \quad -70 \\ \frac{5x}{5} &= \frac{240}{5} \\ x &= 48 \end{aligned}$$

By substituting 48 months into either equation, students can find that Randy and Angela will each have $1,990.

5. In problems 3 and 4, students are given the value of the savings account (y) and are asked to find the period of time (x). Many students may rely on guess-and-check procedures. Others may extend their t-tables or graphs until they get matching values. Here is the algebraic solution:

$$40x + 70 = 1{,}000$$
$$-70 \qquad -70$$
$$\frac{40x}{40} = \frac{930}{40}$$
$$x = 23.25$$

Since Randy makes deposits monthly, and since 23 months yields only \$990, we see that Randy must save for 24 months to have at least \$1,000.

For problem 4, the algebraic solution is:

$$35x + 310 = 1{,}220$$
$$-310 \qquad -310$$
$$\frac{35x}{35} = \frac{910}{35}$$
$$x = 26$$

Angela will have exactly \$1,220 after 26 months.

6. Ask students to translate this sentence into a formula. "Tomeka had \$29 in the bank. She added \$22 more each month." They should be able to get $y = 22x + 29$. Next ask them to write a sentence for this formula: $y = 24x + 112$. One possible answer is, "Juan had \$112 in the bank. He added \$24 per month."
7. Assign Homework 6. The third page provides more challenging problems. The last problem illustrates negative slope.

> ***Teacher Tip!***
> Most of the graphs provided at this point in the unit no longer have numbered or labeled axes. Encourage your students to label the axes *x* and *y* and choose an appropriate scale.

Answer Key

1. $y = 25x + 75$
2. $y = 14x + 306$
3. After 65 months, Derek will have exactly \$1,700.
4. After 21 months
5. Both Rosa and Derek will have \$600
6. $y = 27x + 72$; after 9 months, San will have enough to buy the bike.
7. $y = 16x + 115$; no, Keisha cannot buy her bike before San; Keisha would need 12 months to have enough money.
8. $y = 32x + 39$; $y = -10x + 435$; after 10 months, Chris can afford the bike.

Banking on Functions

1. Randy already has \$70 in his savings account. He will add \$40 per month. How much money will he have?

   How much money will have after 5 months?

   How much money will have after 10 months?

   How much money will he have after 18 months?

2. Angela already has \$310 in her savings account. She will add \$35 per month. How many months will it take before she and Randy have the same amount of money in their accounts? How much will each of them have?

3. Randy wants to know when he will have at least \$1,000. Find a way to determine this for him.

4. Angela finds she has exactly \$1,220. How many months has she been saving?

Student Worksheet

Banking on Functions

Name_______________________________

Date____________________ Class_____

Homework

Banking on Functions

Name____________________________________

Date________________________ Class______

1. Derek has $75 in his savings account. He will add $25 per month. Complete the t-table, write a formula, and graph his problem.

month	principal
0	
1	
2	
3	
4	
5	
x	

2. Rosa has $306 in her account. She will add $14 each month. Complete the t-table, write a formula, and graph his problem.

month	principal
0	
1	
2	
3	
4	
5	
x	

3. When will Derek have $1,700?

____

4. When will Rosa and Derek have the same amount of money?

____

5. How much money will they each have then?

____

Banking on Functions

6. San want to buy a bicycle for $295. He currently has $72 in his account. If he adds $27 to it each month, when will he have enough money for the bike?

____

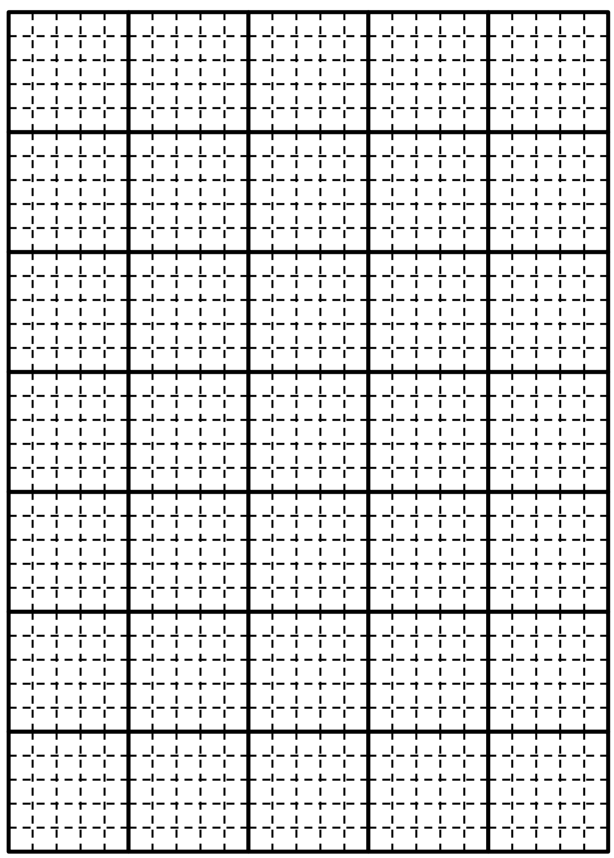
month	principal
0	
1	
2	
3	
4	
5	
x	

7. Keisha wants the same bike that San wants. If she already has $115 in her account, and she adds $16 per month, can she buy the bike before San?

____

month	principal
0	
1	
2	
3	
4	
5	
x	

Graph each function from problems 6 and 7 here.



Banking on Functions

8. Chris has $39 in the bank. He adds $32 to it each month. He wants to buy a bike that costs $435. Every month the bike's price is reduced by $10. How many months will it take for Chris to afford the bike?

____

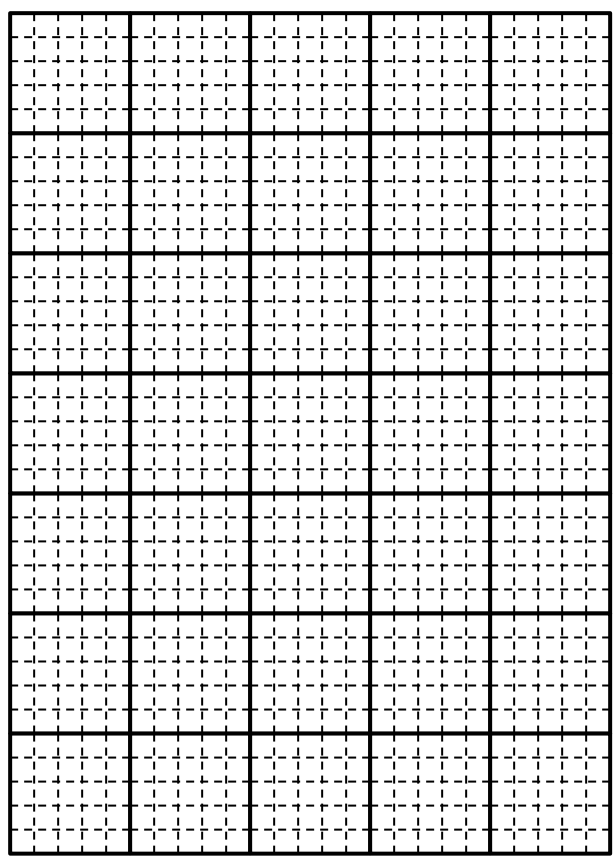
Chris's Account

month	principal
0	
1	
2	
3	
4	
5	
x	

Bike Sale

month	principal
0	
1	
2	
3	
4	
5	
x	

Graph both functions for problem 8 here. Chose a scale that allows the data to fit. Label each function on the graph with its formula.



Section 2: The Great Yo-yo Festival .... 1 class period

I still remember the confusion we faced as students when my high school algebra teacher introduced the $y = mx + b$ form of a linear function. He quickly stated that "m represented slope, the change in y compared to the change in x or $(y_2 – y_1) \div (x_2 – x_1)$ or $\Delta y \div \Delta x$. He went on to say that b represented the y-intercept of the function or the y value when $x = 0$, $(0, y)$." It was all perfectly clear…to him. Later when I was an algebra teacher myself, I decide there had to be a better way. Here is my approach.

Procedure:

Required Materials:

- ☒ Yo-yo Festival Activity Master
- ☒ Yo-yo Festival Student Worksheet
- ☒ ¼ inch grid paper
- ☒ Homework 7

Optional Materials:

- ☐ Calculators

1. Students may work individually or in small groups on this activity. Make sure the students have paper on which to work. Display a copy of the Yo-yo Festival Activity Master showing only the first problem. Read the problem together and ask the students to write an equation that represents the problem and solve it.
2. Ask the students what equations they wrote and what answer they got. While some equations may differ, they essentially will communicate the same idea. Write this form of the equation on the board:

$$y = 5 \bullet 27 + 34 = 169$$

3. Ask them what the y represents in this equaiton. The will probably say it stands for yo-yos. Ask them to elaborate; there are three sets of yo-yos in this problem: the ones brought by the experts, the ones in the box, and the total yo-yos at the tournament. In this case, the variable represents the total yo-yos.
4. Now display the second problem and ask them to write and solve an equation. Again, though their equations may differ somewhat, they are similar in what they communicate. Write this version on the board:

$$154 = 6 \bullet 22 + b$$

In this case $b = 22$. Some students may solve this by a guess-and-check method. If they have had experience solving equations, they should be encouraged to solve it algebraically.

$$\begin{aligned} 154 &= 6 \bullet 22 + b \\ 154 &= 132 + b \\ -132 &\quad -132 \\ 22 &= b \end{aligned}$$

5. Ask them what they suppose the *b* represents. They will likely surmise that it also represents yo-yos, but in this case they are the yo-yos in the *box*.
6. Display the third problem and ask them to repeat the task. In this case, focus them on this representation of the problem:

$$\begin{array}{c} 182 = m\bullet 31 + 58 \\ -58 \qquad\qquad -58 \\ \underline{124} = \underline{m\bullet 31} \\ 31 \qquad 31 \\ 4 = m \end{array}$$

7. Ask them what the m represents. Some will see that it represents the *main* yo-yos brought by each expert.
8. Next ask the students to solve the fourth problem. By now, students may be trying to second guess the teacher and write an equation that uses the variable *e* for *experts*. However, write it this way on the board:

$$\begin{array}{c} 214 = 7x + 46 \\ -46 \qquad\quad -46 \\ \underline{168} = \underline{7x} \\ 7 \qquad 7 \\ 24 = x \end{array}$$

9. Ask them what the *x* represents and they will realize it stands for the number of *experts*. If they ask why you didn't use *e*, you can jokingly reply that you have to have an *x* somewhere on the page or you're not doing algebra!
10. Now show them the fifth problem. Most will say they cannot answer it because there is not enough information. Ask them how many total yo-yos there will be if one expert comes. (40) What if two experts show up? (48). Continue in this manner until the see the pattern.
11. Pass out the Student Worksheet. Ask them to complete a t-table and graph for the problem.
12. When they finish ask them to find the formula. What do the variables *y* and *x* represent in the formula? (total yo-yos and the number of experts). Then write this equation on the board and ask them to write a sentence translating it into words:

    $$y = \mathrm{mx} + \mathrm{b}$$

    "The total number of yo-yos is equal to the main yo-yos times the number of experts plus the yo-yos in the box."

    Students now have a context for y = mx $+ b$.
13. Assign the homework if you wish.

Answer Key

Questions 1–7 are based on the formula $y = 4x + 12$

1. 40
2. 52
3. 72
4. 120
5. 8
6. 1
7. 18

Questions 8–16 are based on the formula $y = 9x + 20$

8. 119
9. 92
10. 164
11. 1,316
12. 2
13. 0
14. 10
15. 2,234
16. about 109

Activity Master

The Great Yo-yo Festival

At the annual yo-yo festival, all the yo-yo experts bring their favorite models. The festival also provides a box of spare yo-yos.

1. Last year there were 34 yo-yos in the box of spares. There were 27 experts, and each one brought 5 main yo-yos. How many yo-yos were there altogether?

$$y = 5 \bullet 27 + 34$$

2. This year each expert brought 6 yo-yos. The number of experts attending was 22. In all, there were 154 total yo-yos. How many were in the box?

$$154 = 6 \bullet 22 + b$$

3. Two years ago there were 182 yo-yos altogether, and 58 were in the box of spares. How many main yo-yos did each of the 31 experts bring?

$$182 = m \bullet 31 + 58$$

4. Three years ago there were 214 yo-yos altogether. The box contained 46 spares. Each expert was told to bring 7 main yo-yos. How many experts attended?

$$214 = 7x + 46$$

5. In the future all experts are to bring 8 main yo-yos. The box of spares will have 32 yo-yos. How many yo-yos will there be altogether?

$$y = \_\_\_ \bullet \_\_\_ + \_\_\_$$

Student Worksheet

The Great Yo-yo Festival

Name________________________________

Date____________________ Class______

x	y

Formula:

____

Homework

The Great Yo-yo Festival

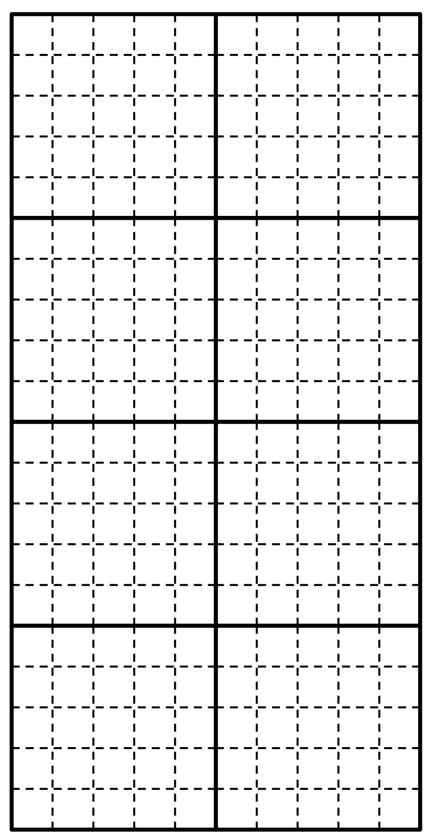
Name________________________________

Date______________________ Class______

At this year's yo-yo festival, every expert must bring 4 main yo-yos. There will be 12 spare yo-yos in the box. First complete the t-table and graph. Write a formula for the function in the form of $y = mx + b$. Then answer the questions below.

x	y
0	
1	
2	
3	
4	
5	
.	
.	
.	
17	

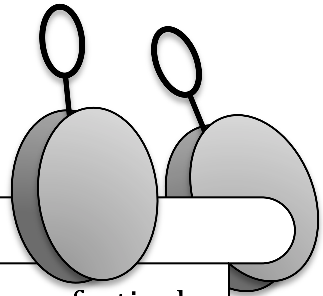
Formula: ______________________



1. If 7 experts arrive, how many yo-yos will there be at the festival?
   ____
2. If 10 experts arrive, how many yo-yos will there be?
   ____
3. What if 15 experts arrive?
   ____
4. If 27 experts arrive, how many yo-yos will be at the festival
   ____
5. If there are 44 total yo-yos, how many experts are attending?
   ____
6. If there are only 16 yo-yos at the festival, how many experts are attending?
   ____
7. There are 84 yo-yos at the festival. How many experts are attending?
   ____

Name____________________________________

Date__________________________ Class______

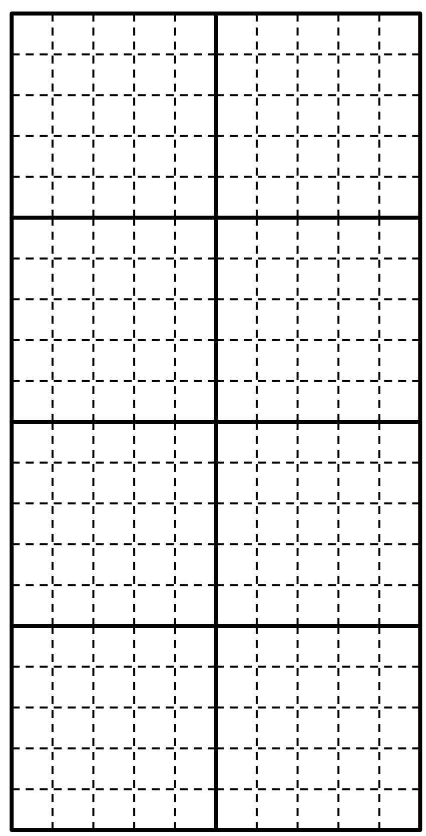


At this year's yo-yo festival, every expert must bring 9 main yo-yos. There will be 20 spare yo-yos in the box. First complete the t-table and graph. Write a formula for the function in the form of $y = mx + b$. Then answer the questions below.

x	y
0	
1	
2	
3	
4	
5	
.	
.	
.	
17	

8. If 11 experts arrive, how many yo-yos will there be at the festival? ______________
9. If 8 experts arrive, how many yo-yos will there be? ______________
10. What if 16 experts arrive? ______________
11. If 144 experts arrive, how many yo-yos will be at the festival ______________
12. If there are 38 total yo-yos, how many experts are attending? ______________
13. If there are 20 yo-yos at the festival, how many experts are attending? ______________
14. There are 110 yo-yos at the festival. How many experts are attending? ______________
15. If 256 experts arrive, how many yo-yos will be at the festival? ______________
16. If there are about 1,000 yo-yos at the festival, how many experts are attending? ______________

Formula: ________________________



Section 3: A Nutty Function .... 1 class period

This is one of my favorite activities for helping students grasp the concepts of slope, rise/run, y-intercept, and other aspects of functions. They will move seamlessly from a physical model to t-tables, graphs, and formulas only to find that they are right back at the physical representation once again.

Required Materials:

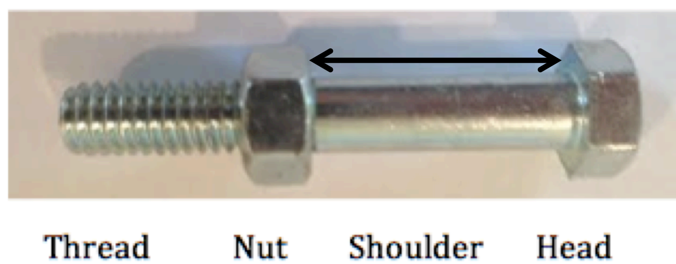
- ☒ Nutty Function Activity Master
- ☒ Nutty Function Student Worksheet
- ☒ Rulers
- ☒ Permanent markers
- ☒ Scissors
- ☒ Tape

Optional Materials:

- ☐ Calculators

Procedure:

1. Students should be working in pairs on this activity to promote mathematical discourse. Tell the students that algebra can be found anywhere, even in some pretty "nutty" places. Pass out the student worksheet as you display the activity master.
2. Explain that a bolt has four parts as shown here: the head, the shoulder, the thread, and the nut. The students will be measuring in millimeters the distance between the head and the nut as shown by the arrow.
3. After measuring, they will turn the nut five complete revolutions and remeasure. (This is shown on the activity master.) It will be helpful to mark the nut and head with a permanent marker to count full revolutions.



4. Each measurement should be recorded on the t-table. Each student should continue working until the nut is completely unthreaded.
5. Some students may notice that the measurements do not appear to be perfectly linear. That is, they may see an increase of 6 mm after five turns but 7 mm after five more. There are two explanations for this. First of all, any time we make measurements we are *approximating*. Counting is exact, but measurement is always rounded–in this case to the nearest millimeter. Secondly, the nut wiggles a bit on the threads to allow for tollerance. This is part of the manufacturing process. Although the measurements do not come out perfectly, it will still graph nicely due to the scales used. Suggest to the students that they try to find an approximate rate of increase. If it goes up by 6 mm, then 7, then 6, then 7, they can use 6.5 as their average.

x	y	
0	18	
5	24	+6
10	31	+7
15	37	+6
20	44	+7

6. Now ask the students to graph the results of their data. Be sure they understand that the horizontal axis counts by ones.

However they counted by five revolutions, so they will only record data on every fifth line. Similarly the vertical axis counts by 4 mm. Students will see that the line appears fairly linear because of the scale we have used.

7. Next they should try to find a formula for the function. This is where it will get interesting. For example, in the t-table shown above, it would seem that the function has a y-intercept of 18 and a slope of about 6.5. This yields the formula

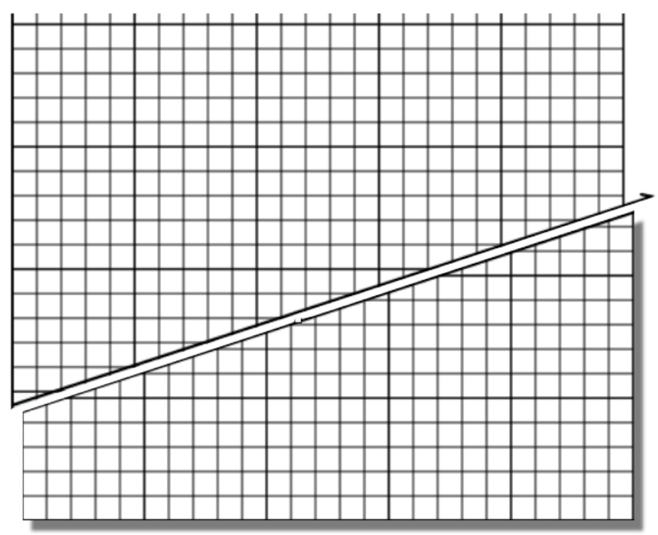
$$y = 6.5x + 18$$

However, when we test this formula with $x = 10$ we get $6.5(10) + 18 = 83$, a far cry from the 31 predicted in the t-table. This is because the rate of increase is *not* 6.5; it is 6.5 for every 5 turns.

8. This focuses students on the fact that they must compare rise to *run* to measure slope. Thus slope is really $6.5 \div 5$ or 1.3 in this case. This changes our formula to:

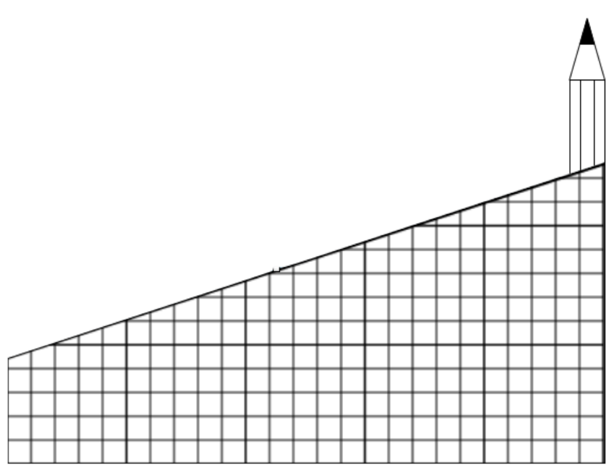
$$y = 1.3x + 18$$

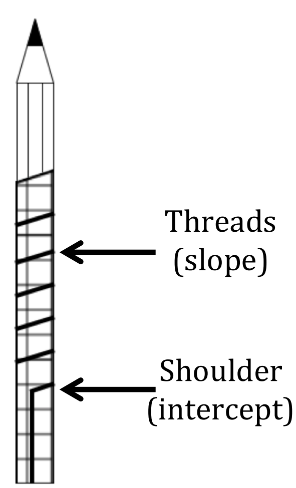
When we substitute we get $1.3(10) + 18 = 31$.



9. At this point students have taken a physical object and found the algebra using a t-table, graph, and formula. Now it is time to turn the process around and find the object in the algebra. Have them cut out their graph along the *x* and *y* axes, along the line they graphed and back to the *x* axis as shown. Then have them tape the right edge to their pencil and wind it around. When they finish, they will see that the bolt reappears! The line becomes the thread and the y-intercept is the shoulder!

10. There is no homework with this assignment as students may not have access to nuts and bolts at home. You





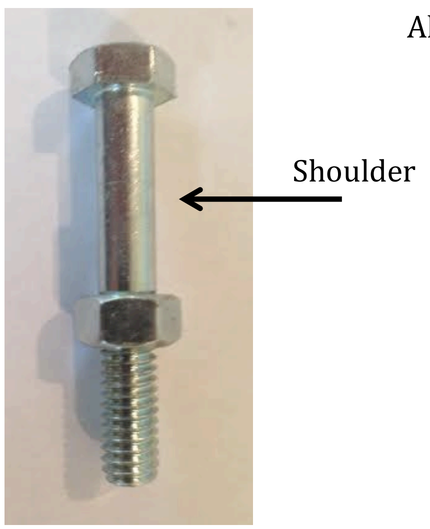
may wish to have students take a nut and bolt home, but therre are other extensions to this activity that allow students to gain mastery.

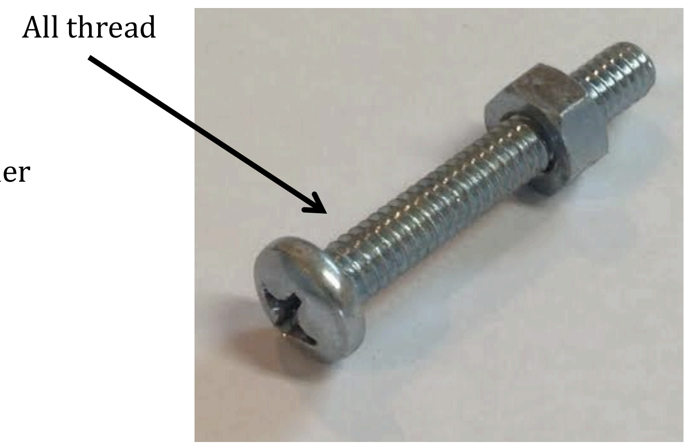
a. Have students exchange bolts and compare their new graphs, t-tables, and formulas to their originals.

b. Ask students to match a set of formulas or graphs to a set of bolts.

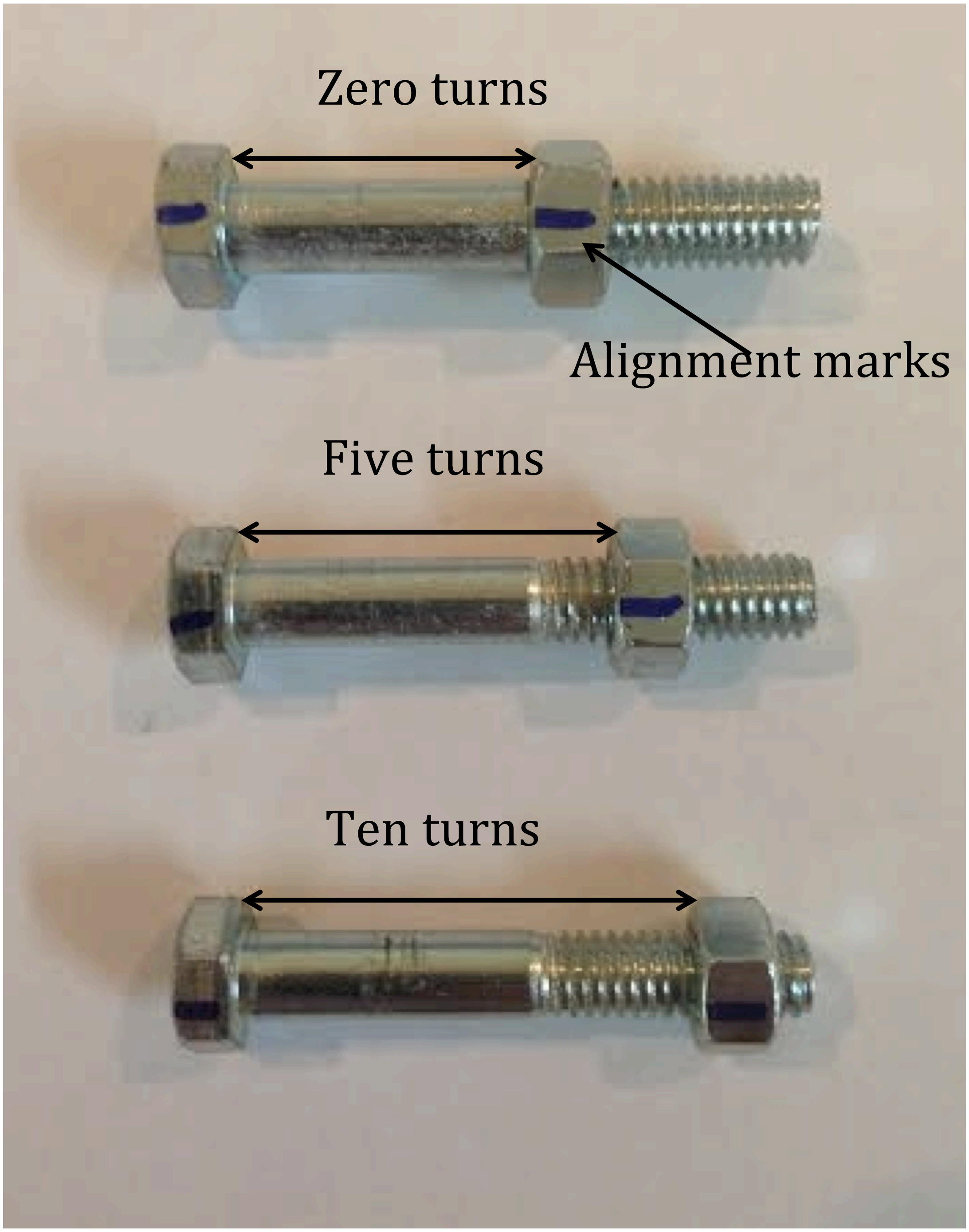
c. Compare bolts with fine threads to those with coarse threads. Some bolts have 24 threads per inch while others have 32. This results in higher and lower slopes.

d. Students can also compare bolts with a shoulder to those that are fully threaded as shown here. Bolts with a shoulder have a y-intercept that is not zero while full thread bolts graph through the origin. This is because the nut can thread all the way up against the head of the bolt.





Nutty Function Activity Master

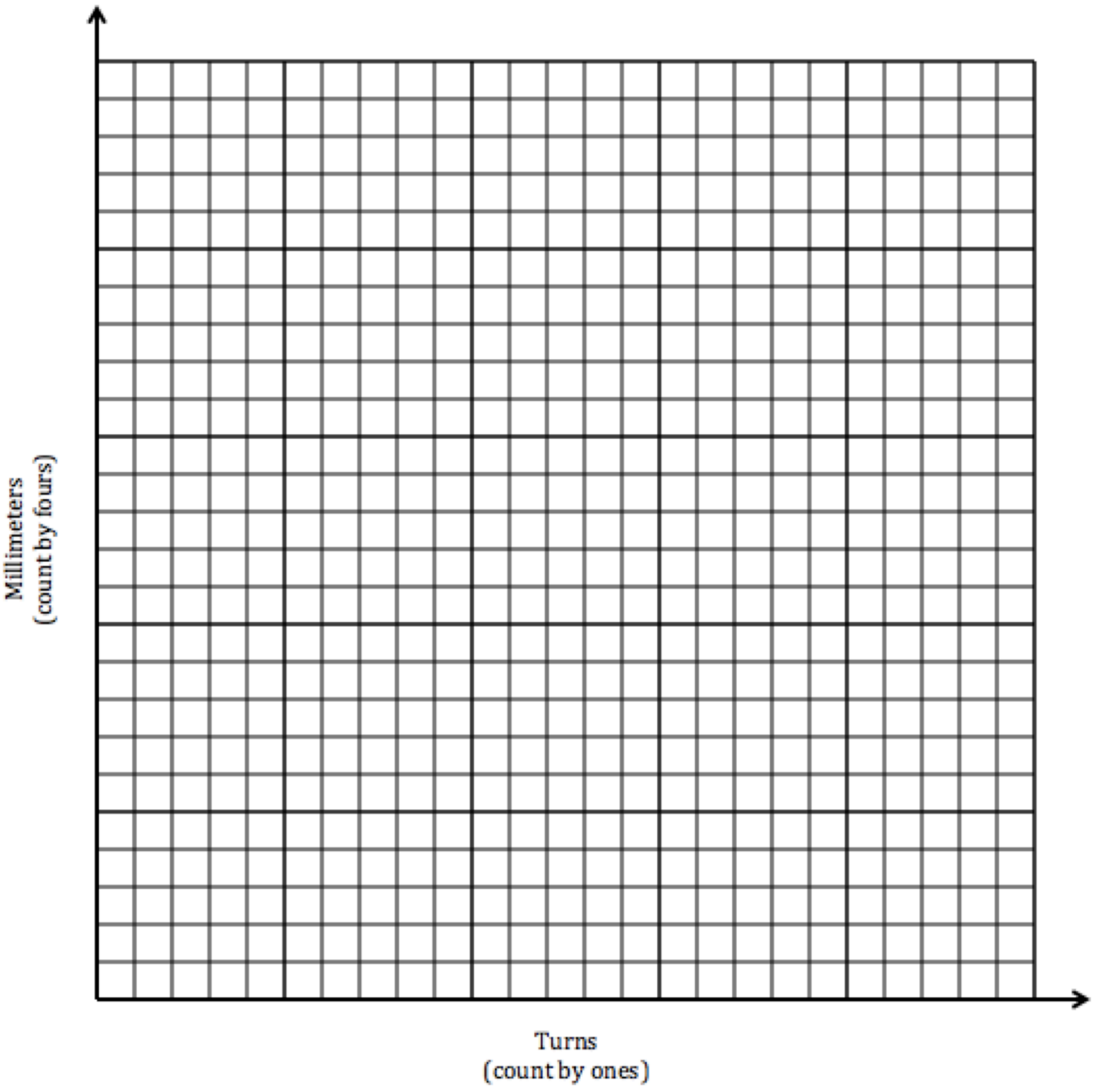


Nutty Function

Student Master

Name____________________________

Date___________________ Class_____



turns	distance

What is the slope of the function? _______________

What is the y-intercept? _______________

What is the formula for the function? _______________

If you could take 23 turns, how far would the nut travel? _______________

If you liked this activity, you might also like some of the other lessons available in my TeachersPayTeachers store. Simply search for “Brad Fulton”.

You can also find many free and inexpensive resources on my personal website, www.tttpress.com. Be sure to subscribe to receive monthly newsletters, blogs, and activities.

Similar activities include:

- *Function Fun: Part 1 – Understanding functions, slope, and y-intercept in a multi-representational approach*
- *Function Fun: Part 2 – Negative and fractional slopes and y-intercepts*
- *Function Fun: Part 4 – Exploring quadratic functions*
- *Function Fun: Part 5 – The King's Pathway and King's Patio projects. Your students design their own functions!*
- *Milk Carton Apartments: a simple function activity for hands-on learners*
- *Losing Your Marbles: slope, intercept, domain, range, independent and dependent variables, rise and run, fractional slopes, line of fit – all in a visual model.*

Feel free to contact me if you have questions or comments or would like to discuss a staff development training or keynote address at your site.

Happy teaching,

Brad