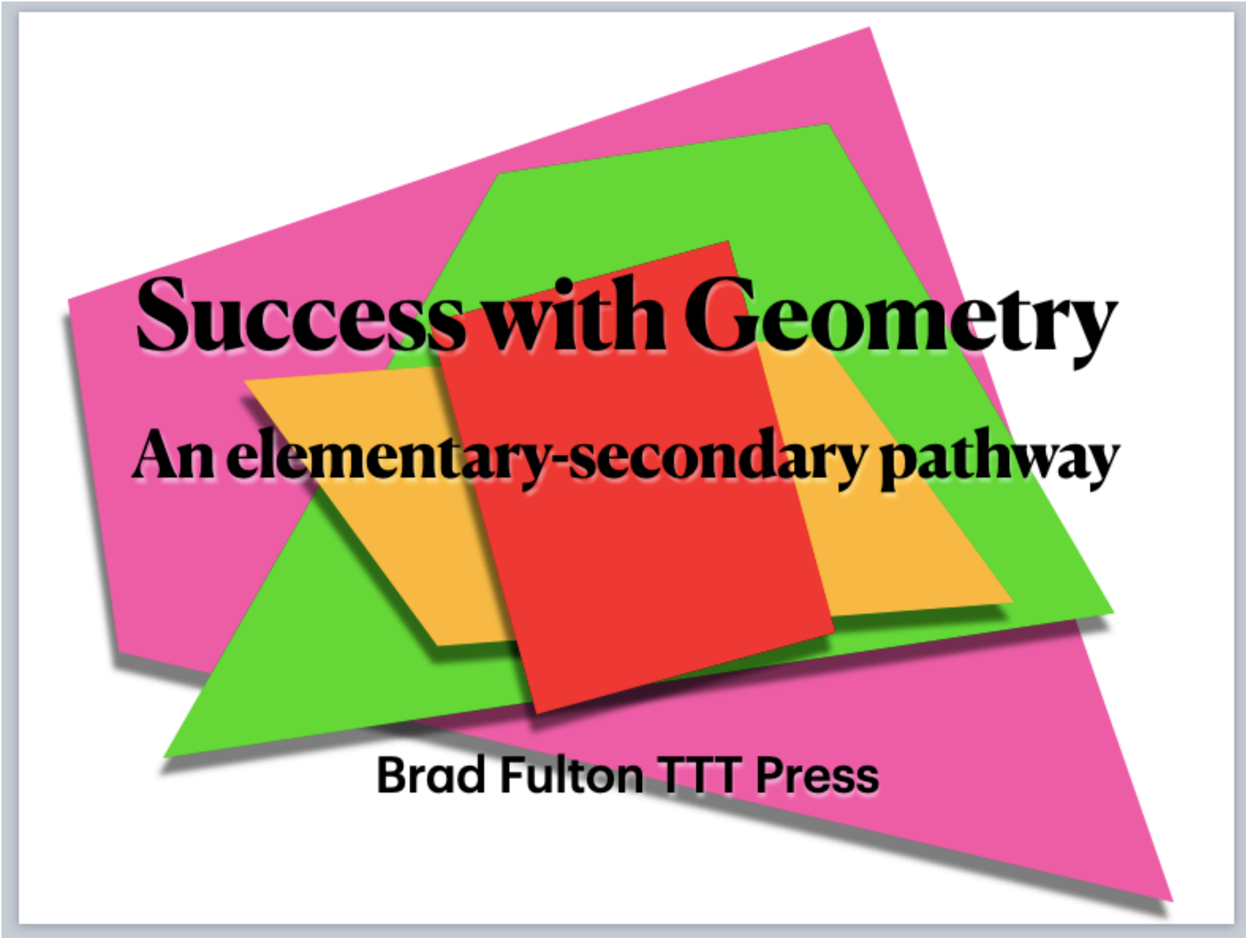

Based on the VanHiele model

By Brad Fulton
Educator of the Year, 2005
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Brad Fulton

Educator of the Year

- ♦ Consultant
- ♦ Educator
- ♦ Author
- ♦ Seminar leader
- ♦ Teacher trainer
- ♦ Conference speaker

Known throughout the country for inspiring and engaging educators and students, Brad has co-authored over a dozen books that provide easy-to-teach yet mathematically rich activities for busy teachers while teaching full time for over 35 years. In addition, he has co-authored over 40 teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

Seminar leader and trainer of mathematics teachers

- 2005 California League of Middle Schools Educator of the Year
- California Math Council and NCTM national featured presenter
- Lead trainer for summer teacher training institutes
- Trainer/consultant for district, county, regional, and national workshops

Author and co-author of mathematics curriculum

- *Simply Great Math Activities* series: six books covering all major strands
- *Angle On Geometry Program*: over 400 pages of research-based geometry instruction
- *Math Discoveries* series: bringing math alive for students in middle schools
- Teacher training seminar materials handbooks for elementary, middle, and secondary school

Available for workshops, keynote addresses, and conferences

All workshops provide participants with complete, ready-to-use activities that require minimal preparation and give clear and specific directions. Participants also receive journal prompts, homework suggestions, and ideas for extensions and assessment.

Brad's math activities are the best I've seen in 38 years of teaching!
Wayne Dequer, 7th grade math teacher, Arcadia, CA

"I can't begin to tell you how much you have inspired me!"
Sue Bonesteel, Math Dept. Chair, Phoenix, AZ

"Your entire audience was fully involved in math!! When they chatted, they chatted math. Real thinking!"
Brenda McGaffigan, principal, Santa Ana, CA

"Absolutely engaging. I can teach algebra to second graders!"
Lisa Fellers, teacher

VanHiele research on geometry

By Brad Fulton

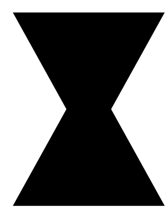
The Dutch mathematicians Dina and Pierre VanHiele developed the seminal model on the acquisition of geometric understanding in the 1950's. Though their findings have been validated and supported for decades, it has been slow to find its way into the American education system. In elementary and even in middle school, geometry is often overly simplified when students are asked to memorize content without exploring and developing it. Other times it is passed over entirely, or merely relegated to the final chapter of the book – a no-man's land where teachers rarely find time to venture.

Thus for many students their first venture into the domain of geometry comes when they have to pass a high-level course in secondary school. This coupled with the fact that the part of the brain that is dedicated to geometry is not the same region that deals with numerical mathematics means that many students fail this required course.

However, the solution to this problem is clear and straightforward. Students who are taught consistently through the VanHiele model are much more likely to develop the necessary skills to succeed in geometry.

Level 0: Visualization

Children **recognize shapes** by appearance: square, circle, rectangle. A child may call a sphere or cylinder a circle at this point, not distinguishing between 2D and 3D shapes. For example, a coin is a circle to children at this level. Students may apply the term hexagon to an octagon.



Similarly, if a shape does not fit their classification scheme, they may reject it. A rotated square may be called a diamond or rhombus. An hourglass or bowtie shape may not be called a hexagon because it is not regular. A student may not be able to identify the base of a trinagle that has a horizontal side at the top and a vertex pointing downward.

These students see shapes as separate classifications and ignore their interrelationships. For example, they don't see a rectangle as a subset of parallelograms.

Many older students and even adults are at this level of geometric understanding. To move them beyond this stage, one good activity is the "This is/This isn't" activity. Given a set of shapes, you could say, "This *is* a polygon," or "This *is not* a polygon," until students note the similarities and differences among them.

Level 1: Analysis

At this level, students will **focus on the properties** inherent in shapes. These students realize that a rotated square is still a square. The characteristics and properties of a shape take precedence over its appearance.

They will begin to define a square by its properties, though they may not be able to do this perfectly. They might say a square has four congruent sides and neglect the fact that it also has four congruent angles.

To develop this stage, educators should expose students to activities that will illustrate the properties of shapes.

- Create any triangle and cut it out. Remove the vertices and set them upon a common point. How many degrees are there? (180º)
- Create any quadrilateral. Locate the midpoints of the side. Connect them to form a new quadrilateral. What is the name of this shape? (Parallelogram)
- Compare the diagonals of different quadrilaterals. What characteristics do they share?

Manipulative and computer-based activities are crucial.

- https://www.geogebra.org/
- https://www.geogebra.org/m/FAhtKpR5
- https://www.geogebra.org/m/VkxdAZrG
- https://www.geogebra.org/m/GFwZ5qdf#material/YT2AVyyp

Level 2: Abstraction

Students will begin to see how shapes relate to one another and can see that a square is therefore both a rhombus and a rectangle. They do this by seeing that properties of one shape may apply to another also.

They will begin to reason about shapes and their properties, though this is often based on **inductive reasoning** (recognition and generalizations of patterns and similarities based on observations). To develop this level of ability, lead the students to make a discovery such as the fact that the vertices of a quadrilateral add up to 360º. Then have them test this repeatedly with various types of regular and irregular quadrilaterals.

Although students at this stage of development show a high level of understanding, they fail to reason deductively or to understand the need for postulates, conjectures, and theorems. They follow hunches and intuition more than proof. Again, geometry software can be of great help in developing these generalizations.

Level 3: Deduction

This is the classic stage of high school geometry. They can reason deductively (based on absolute truths that can be proven). These students can follow or create a deductive proof given certain initial conditions.

To help students in this stage, begin with the simplest of proofs. For example, if we accept that all triangles have an interior angle measurement of 180º, then we can prove that quadrilaterals must have an interior angle measurement of 360º since any quadrilateral can be divided into two triangles. Similarly, any pentagon can be subdivided into three triangles for an interior angle sum of 540º. Continuing this way, it can be shown algebraically that the formula for the interior angle sum of any polygon is 180(n–2) where n represents the number of sides.

It may be helpful to students if you compare this stage to a court trial. We cannot base guilt and innocence on hunches and simple observations of patterns: “The last three people who got caught speeding had red cars, so if you have a red car, you are guilty of speeding.” In a court proceeding, guilt must be proven, even if it is obvious. We depend upon evidence such as fingerprints or DNA that cannot be refuted. Though in most cases, inductive reasoning will get us through, there are times when we want to be absolutely sure. An astronaut going into space wants assurance that the rocket will get work there.

Level 4: Rigor

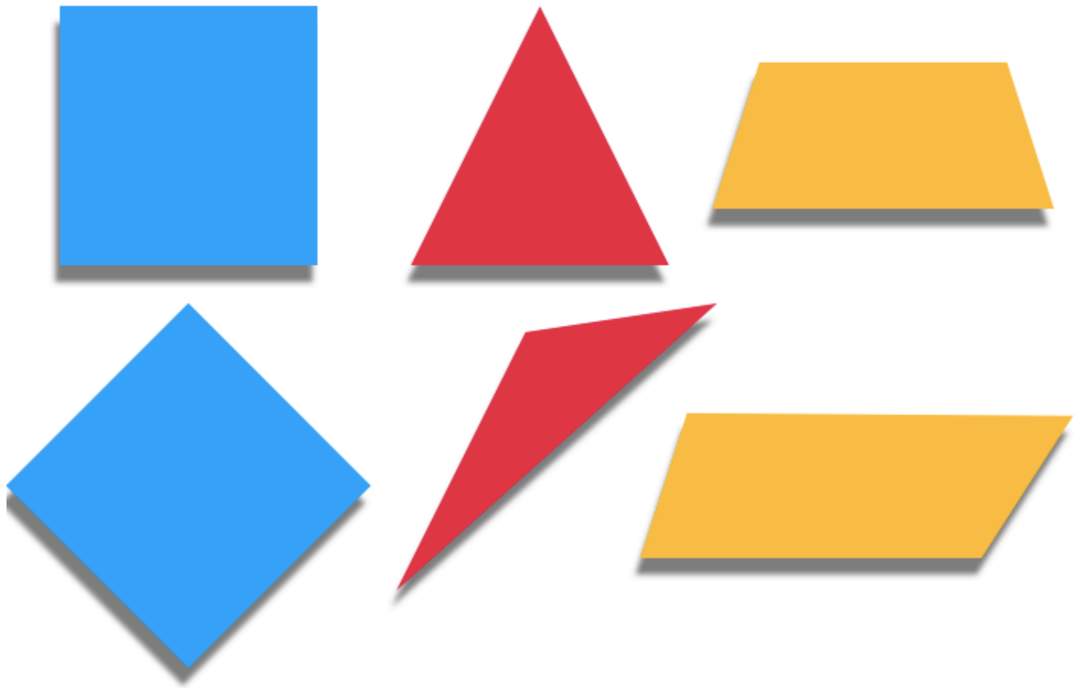
At this level, we can explore beyond plane geometry. For example, lines of latitude are perpendicular to the equator but don’t produce parallel lines. Instead they converge in both directions due to the curvature of the earth’s surface.

We would also find more rigorous proofs at this level, such as proof by negation.

Sadly, though most students are at level 0, or at best, level 1, high school geometry is taught at levels 3 and 4. And unlike some subjects, students must proceed through these levels sequentially; they cannot skip steps and find success. It is best to imagine the five levels as rungs on a ladder. Students must have every rung in place to ensure they can reach the upper heights.

Fortunately, to some degree the movement from one rung to the next is not dependent solely on age but is accelerated by experiences. That means that as we provide these opportunities to students in elementary and middle school, they are more likely to find success in high school geometry.

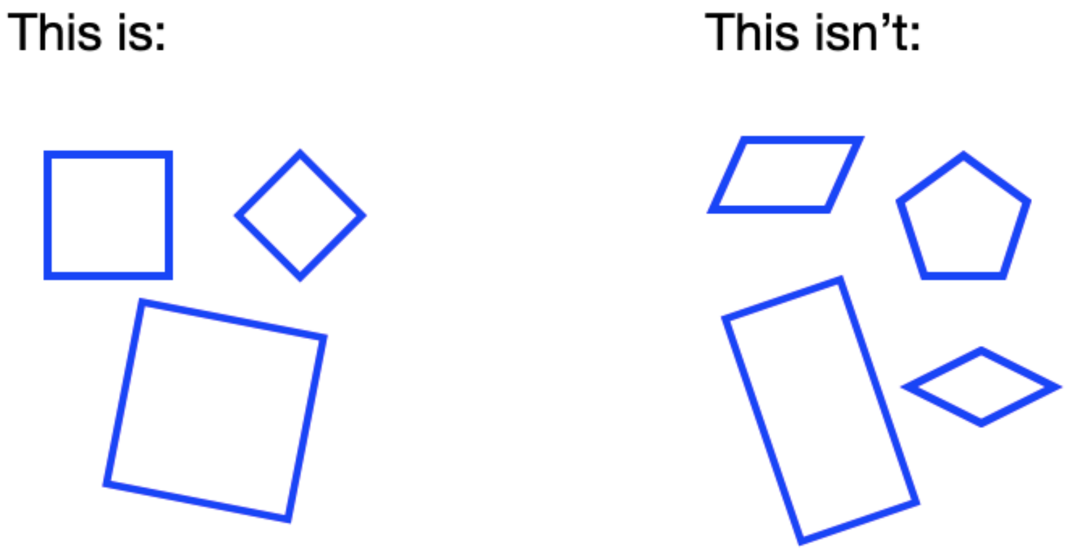
Level 0



Students learn to recognize shape by imitation and association. We have to break the habit of drawing predictable and limited representations of shapes. Push the envelope.

Moving from level 0 to 1

Visualization to Analysis

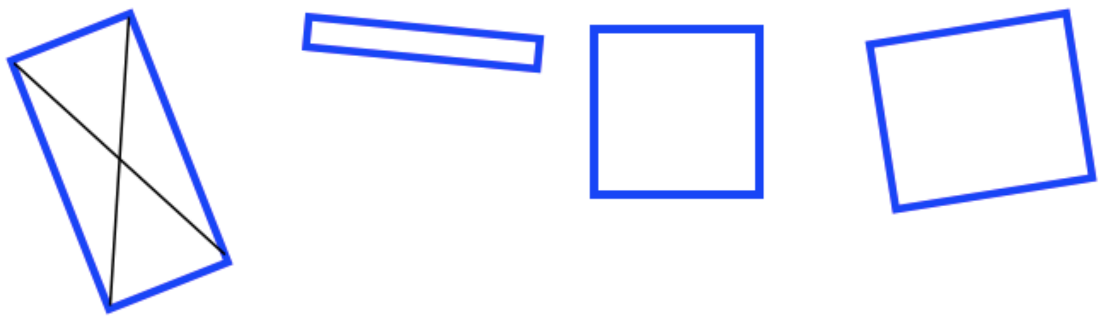


Students also learn by comparison and contrast. What *does* fall under the heading and what *doesn’t*? Students should be looking for the *properties* of shapes to move to level 1.

Moving from level 1 to 2

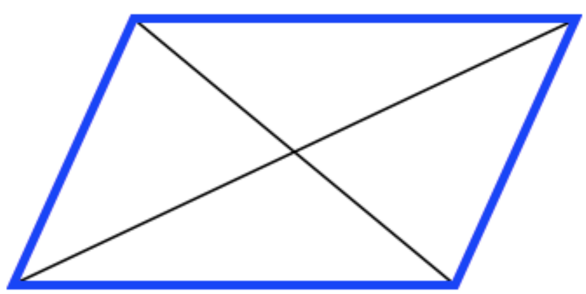
Analysis to Abstraction

- What do all the rectangles have in common?
- Construct their diagonals. What properties do you notice?

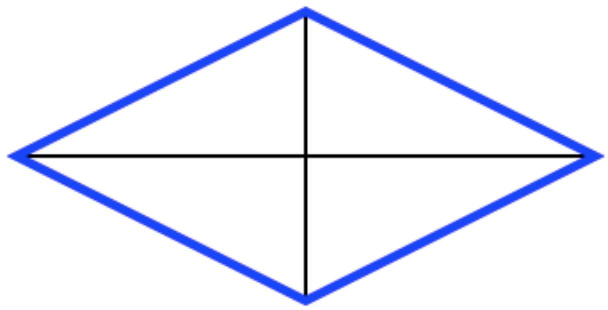


Students should study the *properties* of shapes to move up to the abstraction level. When studying a square, let's not limit ourselves to the definition: a quadrilateral with congruent angles and sides. Squares also have two congruent diagonals that are perpendicular bisectors. No other shape does that.

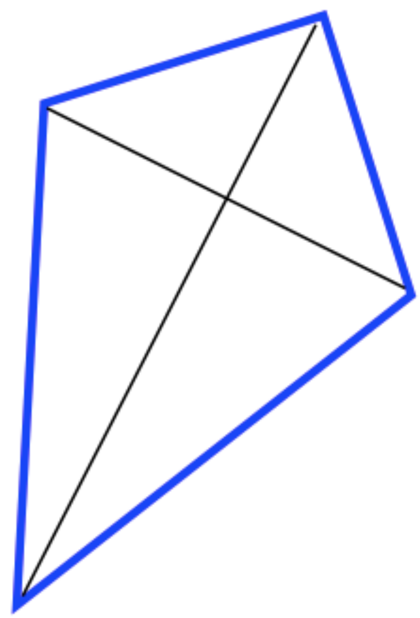
- What about the parallelograms?



- What about the rhombi?

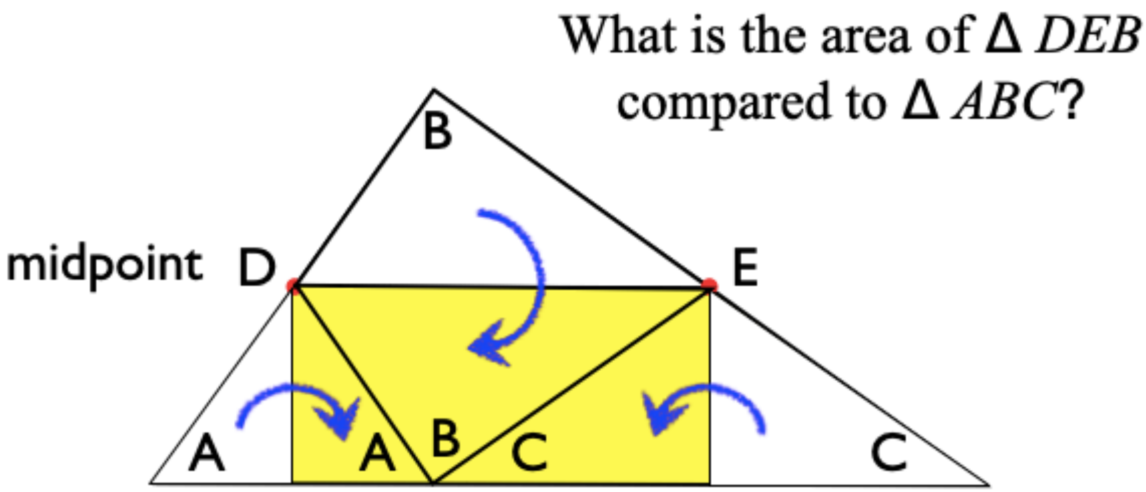


- What about the kites?



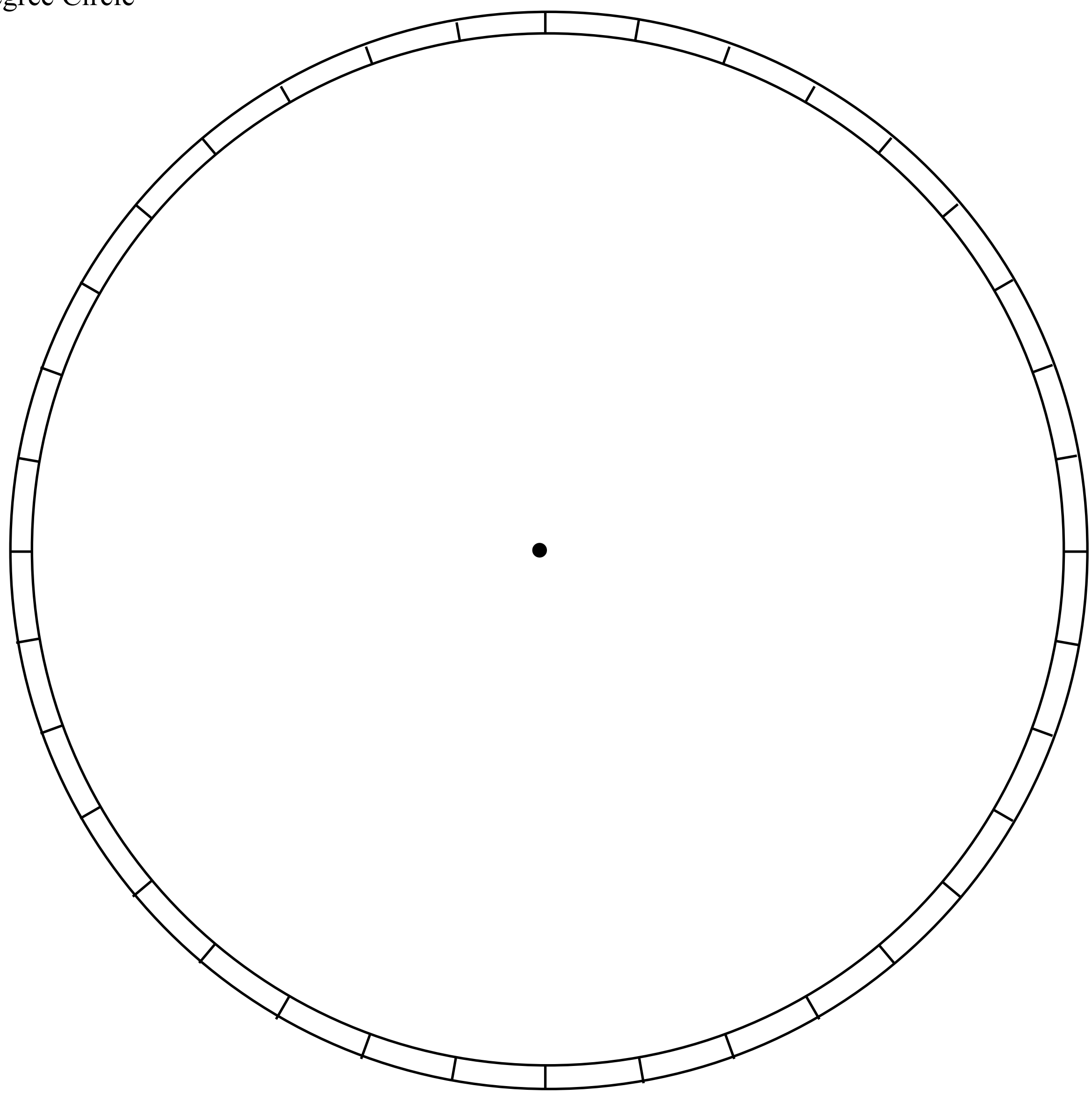
To move to level 2, students must note that properties are consistent. They will do this through *inductive* reasoning. By comparing multiple representations of shapes, they will test their conjectures. Have them work in groups with different shapes of the same category to see if the properties remain consistent. Then compare the data of the whole class. Did *every* triangle have an interior angle sum of 180º? Even scalene and obtuse triangles?

Start with any triangle:



What is the sum of $\angle A + \angle B + \angle C$?

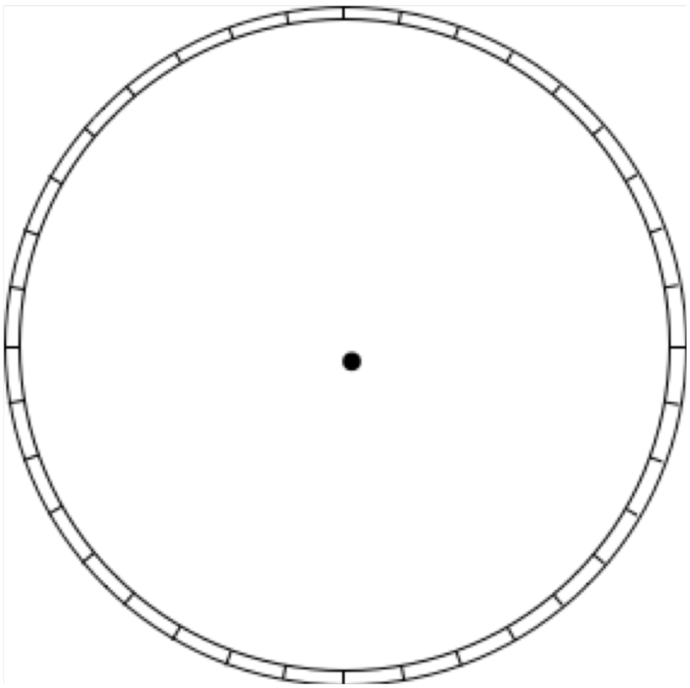
Degree Circle

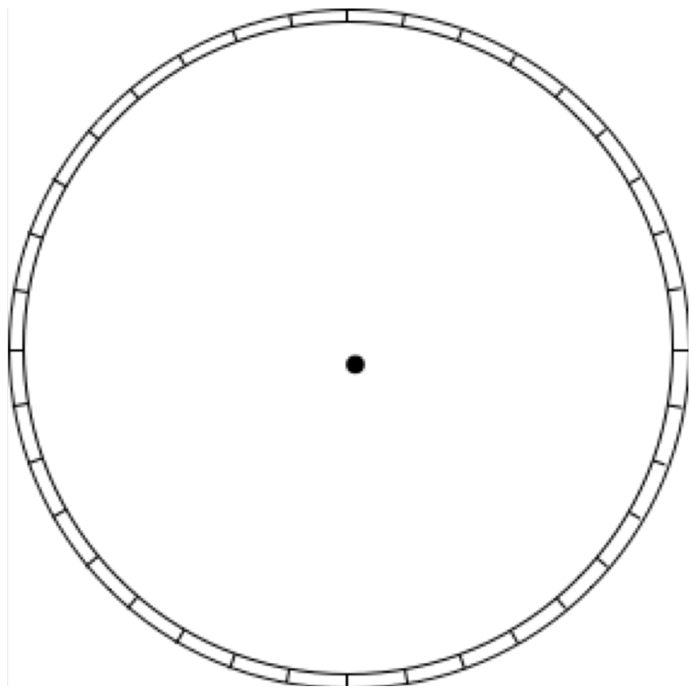


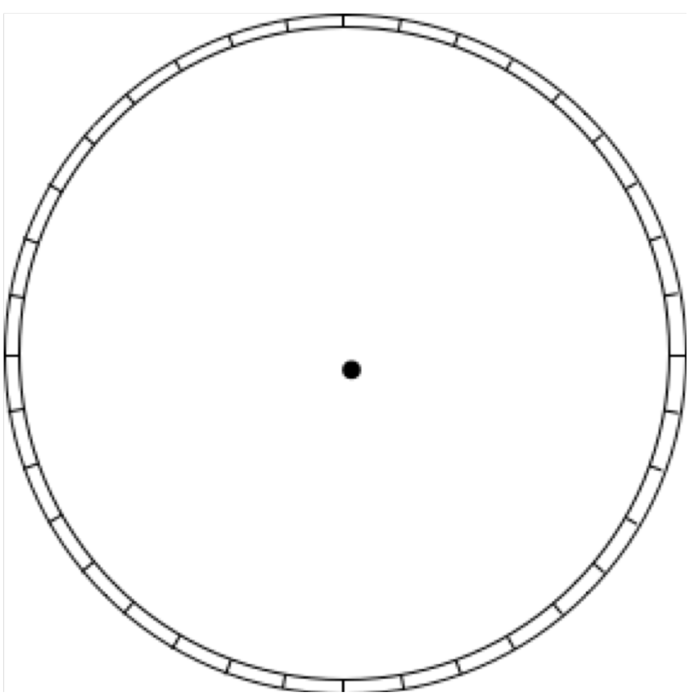
Degree Circles

Name____________________________

Date____________________ Class_____







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