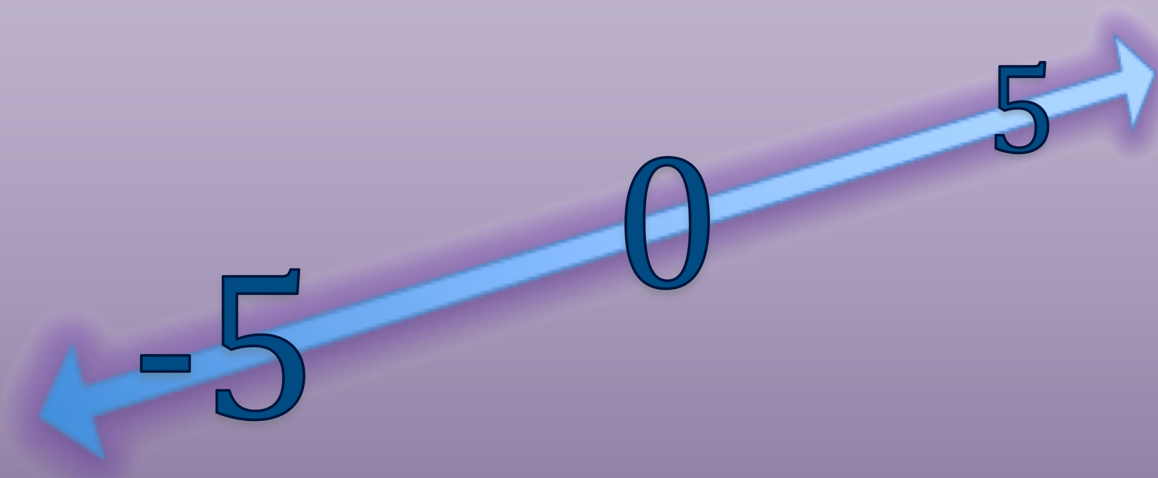
# Winning Ways With Integers!

## Successful Strategies for Teaching Signed Numbers





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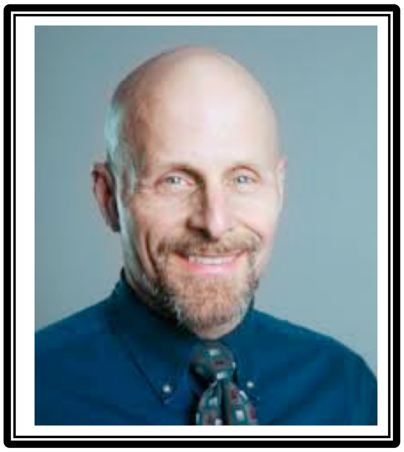

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# Winning Ways with Integers

**Overview:**

Integers are a common tripping point with students who are moving into advanced mathematics. They often understand the concepts we are teaching but make errors with signed numbers that prevent them from being successful. This handout is a collection of 30 years of the best practices I have found for teaching integer calculations. You'll find strategies that appeal to all types of learners: mathematical, linguistic, and visual and kinesthetic.

**Required Materials:**

☒ Plastic chips or counters

**Optional Materials:**

☐ Calculators

**Strategy 1: Chips method**

Using visual aids to represent positive and negative numbers is a common approach, but caution should be used. Some students find it helps them understand how negative numbers operate. However, if we don't present this strategy correctly, it can lead to confusion. We'll discuss this more at the conclusion of this section.

**Procedure:**

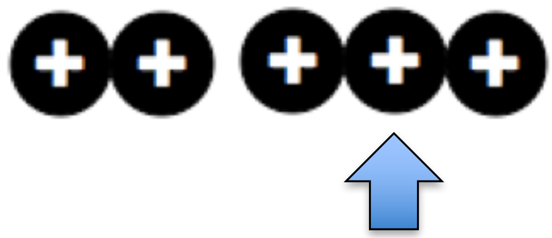
Use objects to represent positive one and negative one. Often this is done with colored chips. Red is often used to denote negativity. Blue is commonly used for positive values, but this is not critical. You should avoid using green as some students may have red/green color blindness.


An inexpensive option is to use tile spacers. These small plastic crosses can be purchased in bulk at any building supply or home improvement store. They come in packages by the hundreds so that you can have plenty for every student. Plus, they don't make a lot of noise on desktops and are easily replaced if lost. They already look like plus signs. The top and bottom can be easily cut off to create negative signs.

When I teach with this method, I always start with problems my students can easily understand. Even with my $8^{th}$ graders, I would begin with something as simple as 2 + 3. The reason for this is that negative numbers are non-intuitive. Many of our students are not understanding what we teach about integers but are simply memorizing the rules and algorithms. To them, calculations with negative numbers are contrived; we have made them up to confuse them. By beginning with a problem such as 2 + 3, I earn their trust that the procedure is valid and did not come from my invention. The following pages will show examples of solving problems involving integer addition and subtraction.

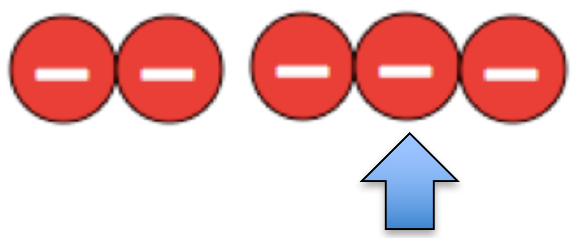
2 + 3

We begin with two positive chips on the field. Then we *add* three more positive chips. We see that we have a sum of +5.



–2 + –3

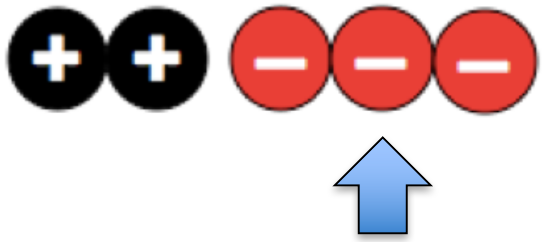
We have two negative chips on the field. We add three negative chips and see that the field now has a value of –5.



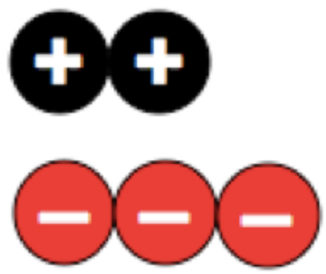
Now let's add opposite signs.

2 + –3

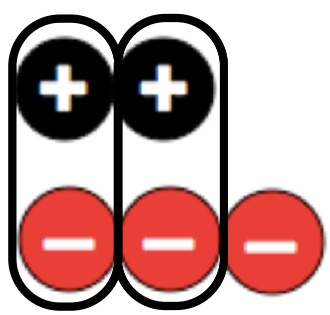
We begin with two positive chips on the field. Then we add three negative chips.



This seems to present a problem, but we simply rearrange them like this:



Now we create *zero pairs*.

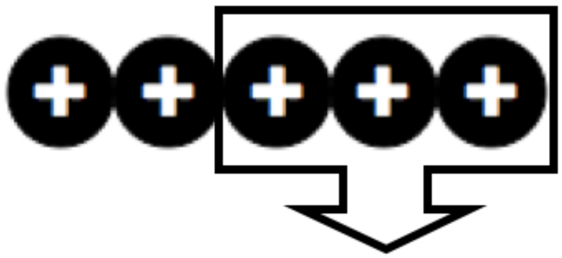


We can see that the net value of the field is –1.

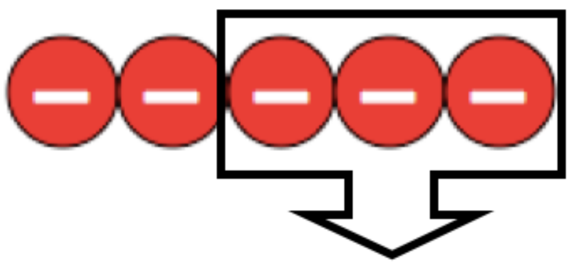
As we move into subtraction, it starts well.

5 – 3

We begin with five positive chips on the field, then we take away three of them. The net value of the field is +2.



This also works when both numbers are negative.
–5 — –3
We put five negative chips on the field and take away three of them. This leaves us with –2 on the field.



This would be a good time to illustrate that the same result occurs with this problem:
–5 + 3
This illustrates why we take subtraction of a negative and turn it into addition of a positive.
–5 — –3 = –5 + +3

The math gets messy when we try to subtract unlike signs.
5 — –2

We begin with five positive chips on the field.

But wait! Now we have to take away two red negative chips that aren’t there. We overcome this obstacle by introducing some zero pairs.

Notice that the *net value of the field is still* +5. Now we can take away two negative chips. The net value will then be +7.

At this point, many of my students look at me like I have just pulled a rabbit out of a hat.

Research by Dr. John Wilkins of California State University Dominguez Hills suggests that the chips strategy is fine for teaching addition of integers but has limited value in teaching subtraction and other operations. I suspect there are a number of reasons for this.

First of all, we don't typically think of zero pairs when doing addition and subtraction. When I look in my checkbook and find that I only have $1, I don't think that I really have a field of $1,000 positive dollars and $999 negative dollars and that they create 999 zero pairs with a net value of the field of $1. That's simply not how we conceptualize numbers. We could do it that way, but it's cumbersome and non-intuitive.

In fact, when thinking about subtraction, this strategy is probably more confusing for your students than some of our other traditional procedures. In my class, the only students who follow this convoluted way of thinking about subtraction are the abstract mathematical thinkers who already understand integers.

Another problem with this approach is that in my opinion we haven't truly used a concrete visual manipulative. When we introduce manipulatives it is for the purpose of making an abstract concept become real and visible. However, I suggest that even with physical chips it is still an abstraction. Here is why I believe that.

We have decided that a hyphen preceding a number makes it into its opposite. We have used a grammatical symbol to denote this. There is nothing inherently negative about hyphens. It is a symbol.

When we used colored chips, I admit we have a visual model, but we have decided that a specific color – red in this case – denotes a number's opposite. But this too is not inherently negative. If a blue car is a car, a red car doesn't become a non-car. If a blue car collides with a red car, the net result isn't zero! What we have done is simply replace a grammatical symbol with a colored visual symbol. It is still a symbolic approach. Now instead of understanding one symbolic representation, they have to comprehend two.

With that said, I still use this approach with my students. I do this partly because I know it is the method of choice for most of their teachers and will be used in subsequent coursework. I also know that it does work for some of my students. **And here is the important point of all the strategies in this handout: you don't catch all the fish with one worm.**

I explain to my students that I will show them many strategies throughout the year. Some may make integers more understandable, and some may be confusing. They have permission to retain what works for them and toss the rest. The important thing is to find a tool that works for them.

**Strategy 2: The Number Line Walk**

This is by far, the method that seems to catch the most fish for me. It is a very visual and kinesthetic model that appeals to a wide spectrum of the students in my class. Even my abstract symbolic thinkers find it helpful. Plus, it is very engaging in the way I present it to them. They actually get excited to add and subtract integers!

This strategy is based on physical movement along a number line. This can be done on a number line on the student's desk or with the number line we often see hung above the board at the front of a classroom. However, I prefer to actually *walk* along the number line, and for that reason, my number line hangs from by ceiling tiles. These are 2' square, so each number is 2' from the next one. This corresponds to a typical step. On my number line, the positive numbers are black and the negatives are red. This corresponds to being "in the black" or "in the red". Zero is white on a colored background as it is neither positive or negative.

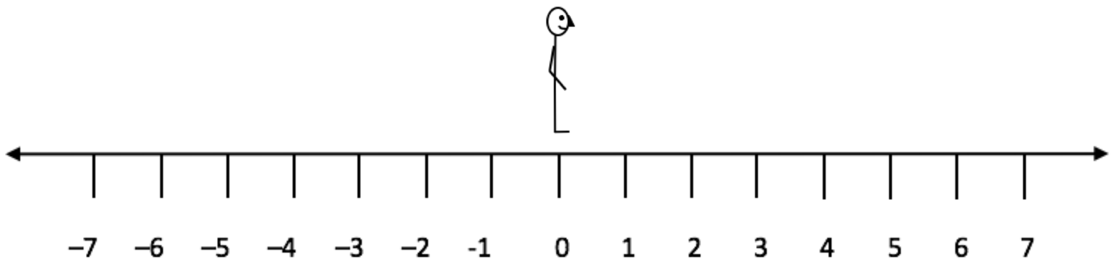
To add and subtract numbers, simply act out these three steps:

1. Begin at zero and face the positive numbers.
2. For positive numbers walk forward, and for negative numbers walk backwards.
3. For subtraction, turn 180°.

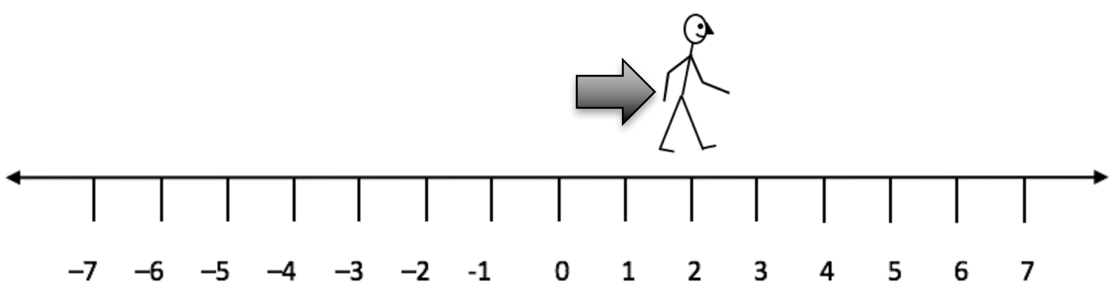
On the following pages, I will illustrate the same problems we used in the chips model.

2 + 3

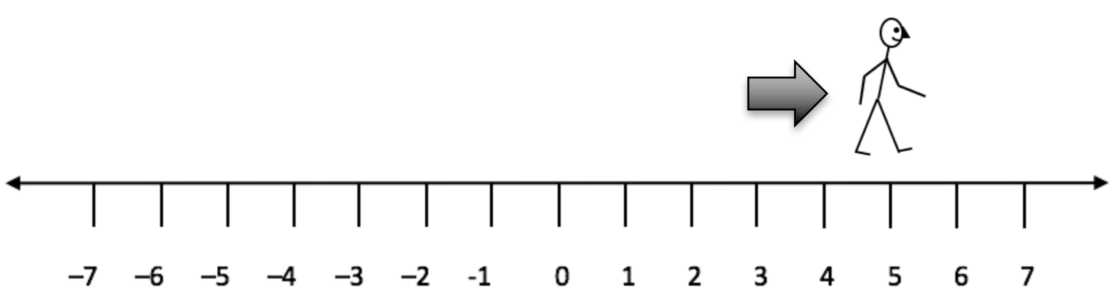
Start at 0 facing the positive direction.




Take two steps forward (because the 2 is positive). You are now at +2.



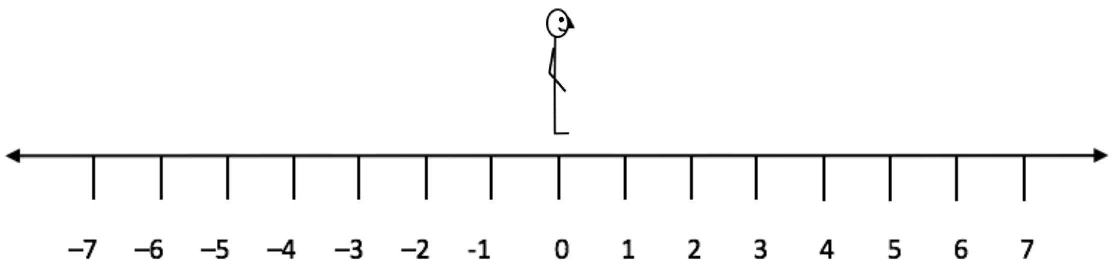

Take three steps forward (because the 3 is positive).



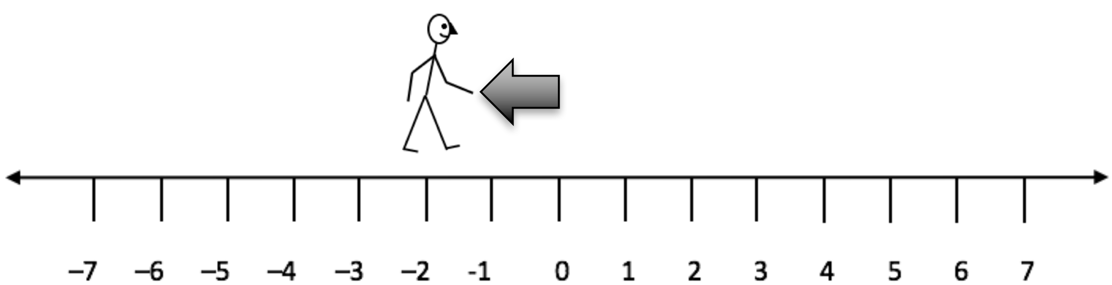

You are now at +5.

−2 + −3

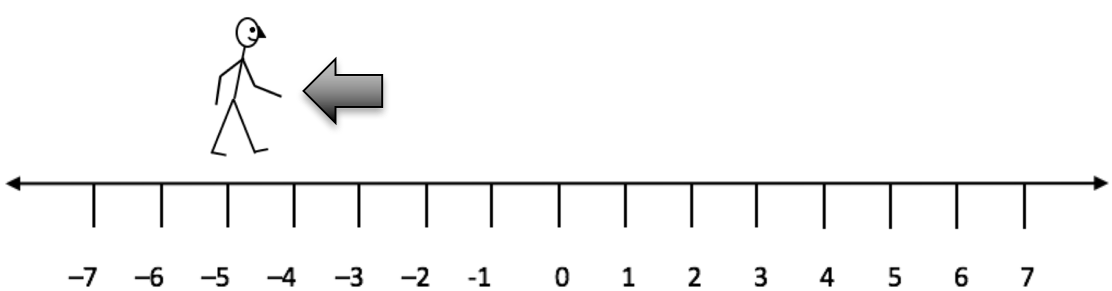
Start at 0 facing the positive direction.




Take two steps backwards (because the 2 is negative).
You are now at −2.



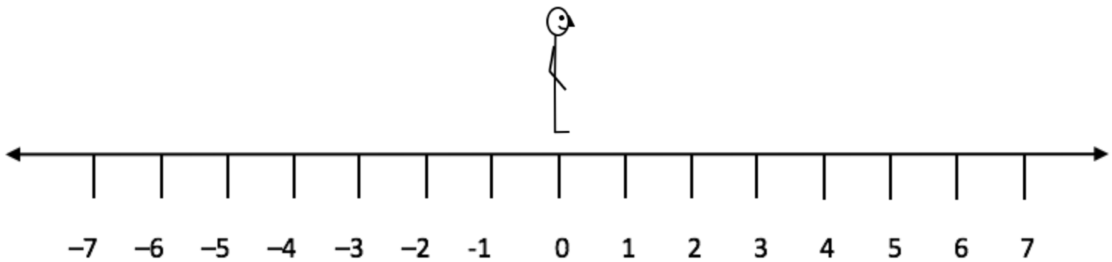

Take three steps backward (because the 3 is negative).



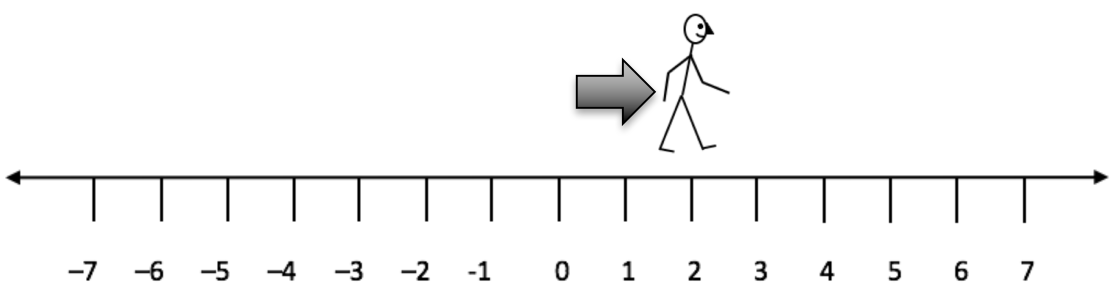

You are now at −5.
Notice that you didn't turn around; this is not subtraction. You simply walked backwards according to rule 2.

2 + –3

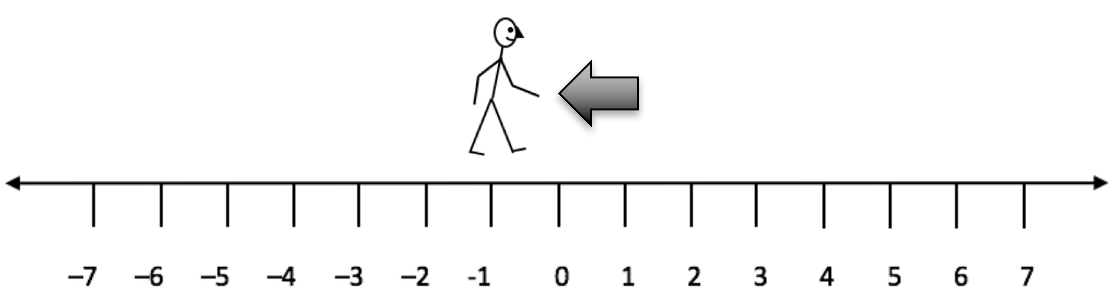
Start at 0 facing the positive direction.




Take two steps forward (because the 2 is positive). You are now at +2.



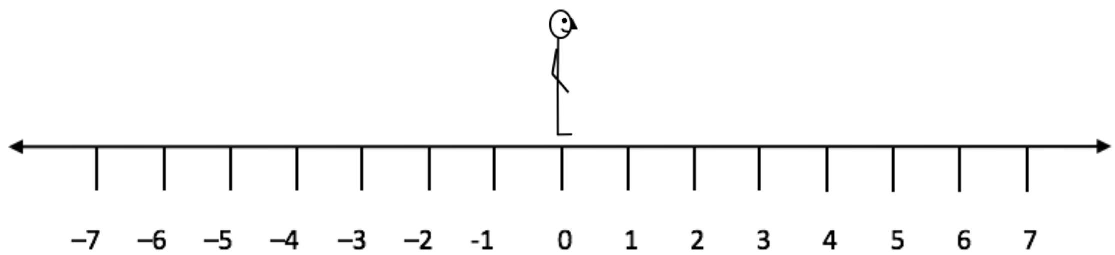

Take three steps backwards (because the 3 is negative).



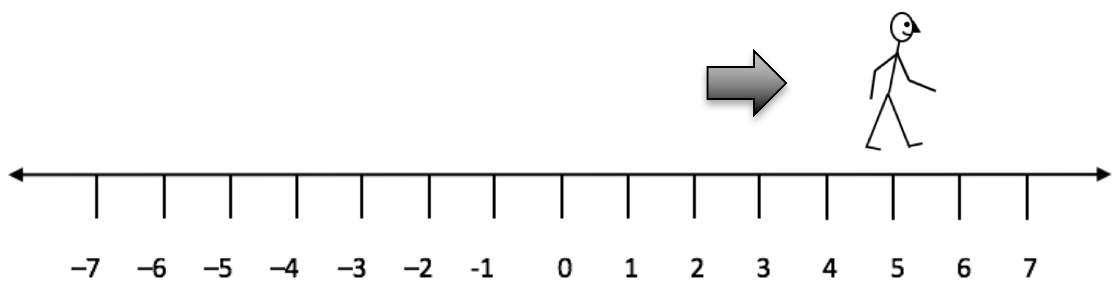

You are now at –1.
Notice that unlike the chips method, this strategy does not require the concept of zero pairs.

5 — 3

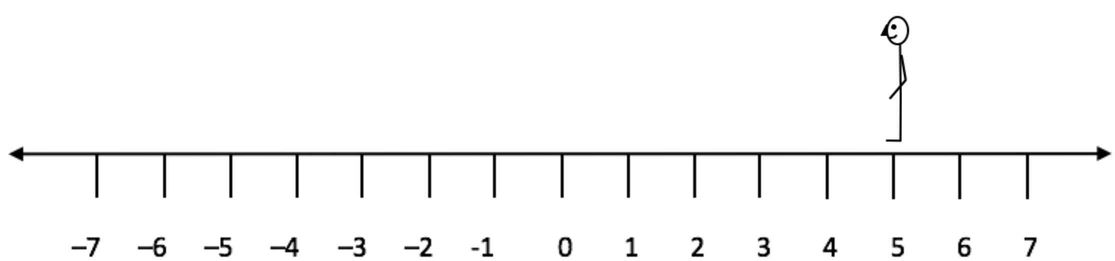
Start at 0 facing the positive direction.



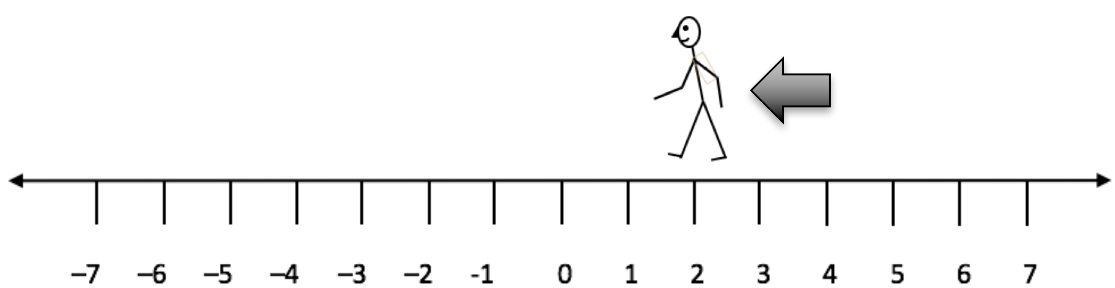

Take five steps forward (because the 5 is positive). You are now at +5.




Turn around (because there is a subtraction sign).



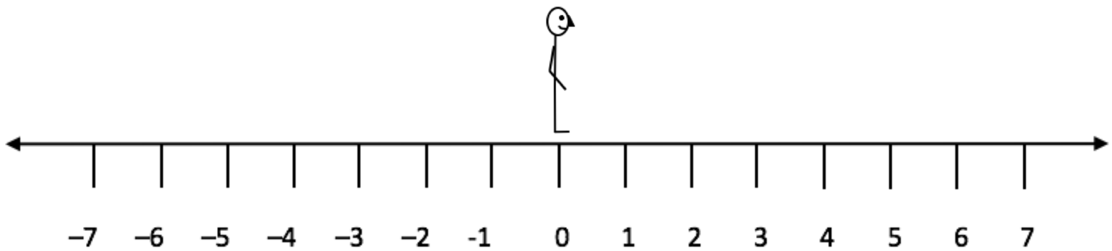

Take three steps forward (because the 3 is positive).



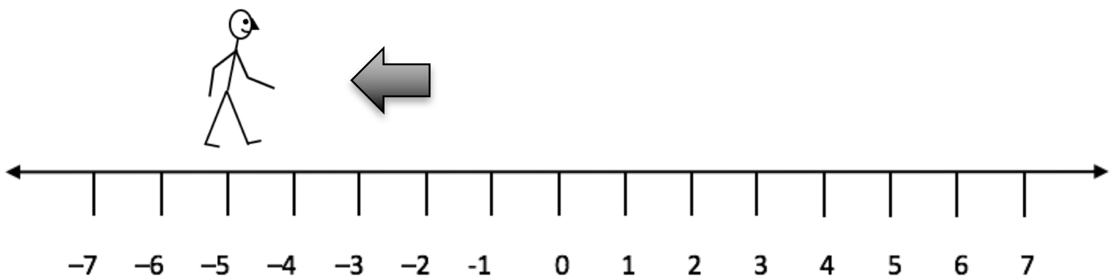

You are now at +2.

–5 — (–3)

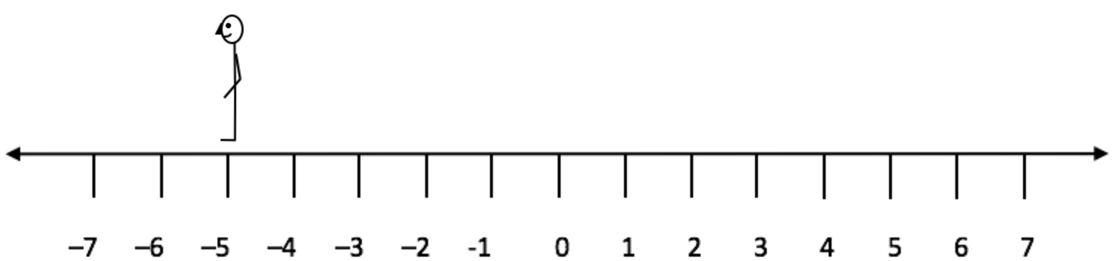
Start at 0 facing the positive direction.



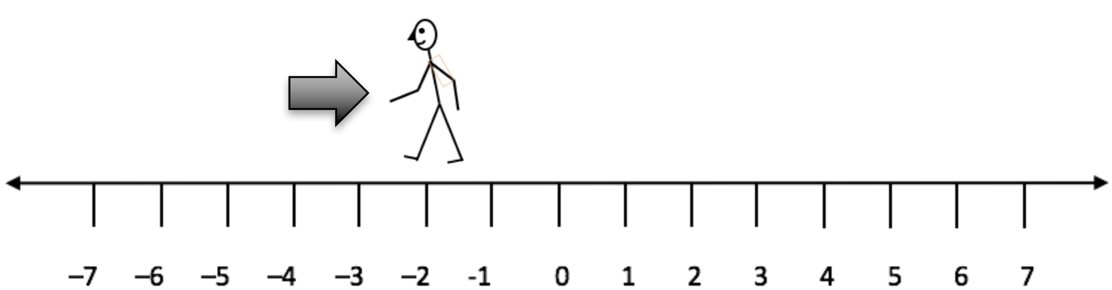

Take five steps backward (because the 5 is negative). You are now at –5.




Turn around (because there is a subtraction sign).



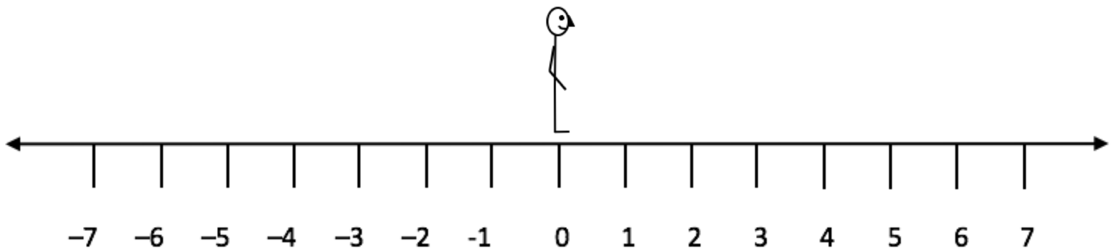

Take three steps backward (because the 3 is negative).



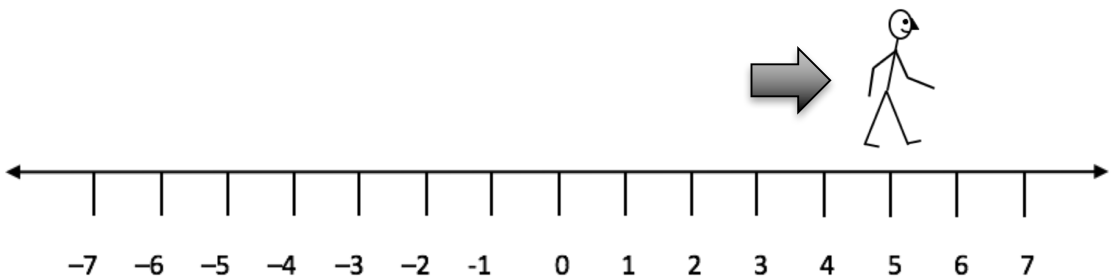

You are now at –2.

5 — (–2)

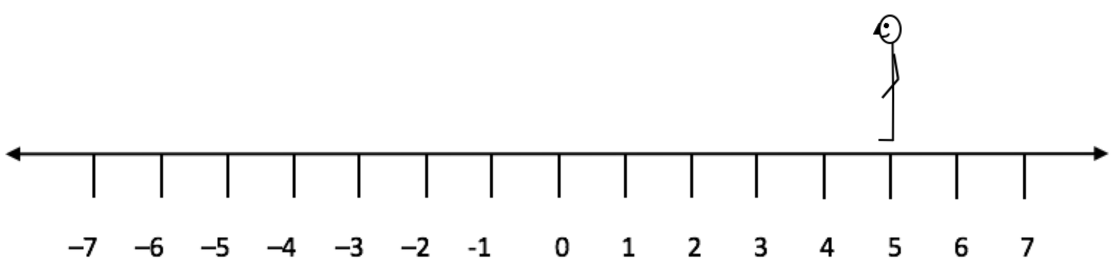
Start at 0 facing the positive direction.



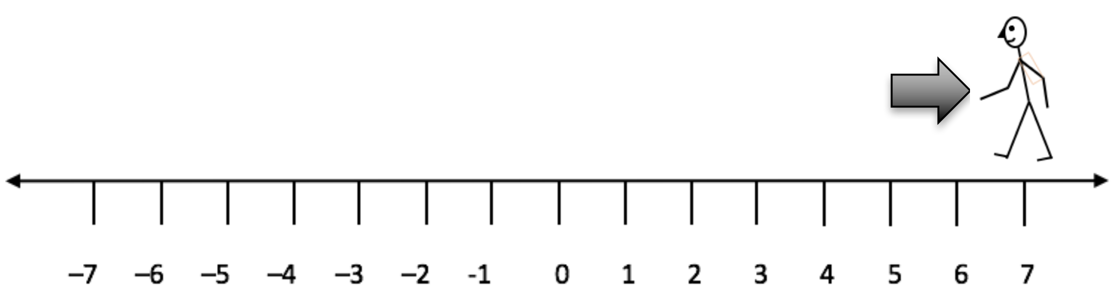

Take five steps forward (because the 5 is positive). You are now at +5.




Turn around (because there is a subtraction sign).




Take two steps backward (because the 2 is negative).




You are now at +7

This would be a good time to demonstrate the fact that subtracting a negative is a positive action. After demonstrating the previous problem. I start it over. However, once I have landed on +5 I ask, “Is there an easier way to get to +7 without turning around and walking backwards two steps?” Students will see that I could more easily *take two steps forward*. I perform that action and ask them how it would be written. They suggest 5 + 2. I then write that below the previous problem like this:

$$5 - (-2) = 7$$
$$5 \quad + 2 \quad = 7$$

This helps students see why we take the subtraction sign and the negative sign and replace them with an addition symbol. I sometimes drive home the point by saying, “If I took away an office referral from you, is that a positive or a negative experience?” They understand that subtracting something negative from your life is a positive thing.

I then practice this strategy using a series of problems. First, I walk out sample problems that I have written on the board. These might be my own problems or suggestions from my students.

During this phase, students must record the problem, predict where I will end up on the number line and write the answer. Then they talk me through the problem while I follow their directions. We compare their prediction with my actual landing spot.

Next, I allow students to volunteer to come up and walk the number line using problems that I create. They are very eager to do this. In fact, I often find that the students who are most willing to engage are those who are least willing to engage on days of more traditional math instruction. I suspect that this is because they are kinesthetic and physical learners who are finding that they understand this type of instruction.

Then I have a volunteer come up. They wait for me to give them a problem, but instead I suggest they think of a problem in their mind and walk it out. The seated students have to write the problem and answer. Finding the answer is easy because they student is ends up standing under the answer. Writing the problem allows them to translate the physical movement into mathematical language.

Since my number line has two-foot spaces, I can’t extend it very far within the walls of my classroom. This is actually a good thing, as it forces students to think abstractly at some point. They imagine that I will end up in one of the adjacent classrooms that are extensions of the positive and negative ends of my number line.

I catch a lot of fish with this strategy. It is the favorite for most of my students because they can see why integers behave the ways they do. I also notice that during a test it is not uncommon for students to look up at the number line and move their hand through the air as they imagine themselves walking the line.

The only downfall to this method is that it only works with pairs of numbers. Unless more rules are added, it isn't reliable with problems that have three or more terms. That is where my next strategy comes in handy.

**Strategy 3: The Boxes Method**

This strategy is less intuitive and more procedural than the other two, but it is very efficient, it's simple and quick, works with any amount of terms, and therefore becomes the method of choice for many of my students. For this example, we will tackle a more complex problem:

$$6 + -4 — 8 + 5 — -3 — 7 — -2$$

First we draw boxes around each term ***and its associated signs***:

$$\boxed{6}\boxed{+ -4}\boxed{— 8}\boxed{+ 5}\boxed{— -3}\boxed{— 7}\boxed{— -2}$$

Notice that a number annexes all the signs that precede it. Then we separate the numbers into two groups: positive and negative. The 6 is obviously positive. The 4 is negative due to the sign that precedes it. The 8 is also negative because of the subtraction sign. This strategy reinforces that subtraction signs behave like negative signs. The 5 has a positive sign. The three is positive because it has a subtraction sign *and* a negative sign. This reinforces that two negatives make a positive. The seven has a subtraction sign and is therefore negative. The two is positive since it has a subtraction sign and a negative sign. The problem then looks like this:

| positive terms: | negative terms: |
|---|---|
| 6 | 4 |
| 5 | 8 |
| 3 | 7 |
| 2 | |

Then we add the two columns:

| positive terms: | negative terms: |
|---|---|
| 6 | 4 |
| 5 | 8 |
| 3 | 7 |
| 2 | |
| 16 | 19 |

Lastly, we ask ourselves, which team will win the battle? The negative team will win because it has three more team members than the positive side. This tells us that the answer is –3.

This strategy not only makes easy work of long integer problems, it works with addition and subtraction in the same problem. It also reinforces concepts such as the fact that two adjacent negative signs are positive. Finally, it breaks down every problem into an addition problem in which students ultimately focus on the absolute values of the two sums. The first time I showed this to my students, one of them cautiously asked, “Is it all right if we use that method?” Perhaps he thought it was so easy that it must be illegal or something!

**Strategy 4: The Pattern Approach:**

This is not so much a strategy for solving problems as much as an instructional model that demonstrates why integers behave the ways they do. Too often, students simply memorize rules and procedures with integers with little conceptual understanding. They quote, "Two negatives make a positive," and apply that rule to operations such as addition where it doesn't apply. When asked why two negatives have that effect, they say, "That's what I was told."

Of the ways our brain stores information, rote memorization is the most temporary and most finite. It doesn't last long – often becoming lost before the test arrives. The brain also has very limited space for memorization. That's why we write down a grocery list.

There is evidence however, that learning stored with understanding is connected more permanently in our brain. Some researchers also suggest that this type of memory, called locale memory, may be infinite in magnitude.

Because our brain is so good at recognizing and trusting patterns, even among very young or struggling learners, I use this characteristic to show students why multiplying two negatives *must* result in a positive product or why two negatives make a positive in subtraction.

I begin with a problem they all know to be true. Let's assume we want to demonstrate why multiplying positive and negative factors yields a positive product. I write the following problem on the board:

$$3 \times 3 = 9$$

Then I begin to modify it by decreasing the second factor. Each time I do this, I ask them for the product. This leads to this diagram:

$$3 \times 3 = 9$$

$$3 \times 2 = 6$$

$$3 \times 1 = 3$$

Then I ask them what problem we should write next. They recognize the pattern and suggest:

$$3 \times 3 = 9$$

$$3 \times 2 = 6$$

$$3 \times 1 = 3$$

$$3 \times 0 = 0$$

Then I ask them what patterns they recognize. They notice that as the second factor decreases by 1, the product decreases by 3. Because this is based on patterning, their brain accepts and understand the results and stores this in locale memory instead of memorizing the problems. I ask them what we should write next. They suggest 3 x -1. I ask them what the product must be and they realize that since we are skipping down the number line by 3 each time, the product would have to be -3:

3 x 3 = 9

3 x 2 = 6

3 x 1 = 3

3 x 0 = 0

3 x -1 = -3

Then we continue the pattern two more steps to get:

3 x 3 = 9

3 x 2 = 6

3 x 1 = 3

3 x 0 = 0

3 x -1 = -3

3 x -2 = -6

3 x -3 = -9

They then realize that there are no new multiplication facts to learn. Even when numbers are negative, 3 x 2 is still six, but when one is positive and one is negative, the product *must be* negative for the pattern to continue. Our brains like patterns and we want them to remain consistent. We conclude: a positive times a negative is negative.

Next I take this last problem and use it to generate a new pattern. Since we know that 3 x -3 = -9, I begin to decrease the first factor. 2 x -3 would have to be -6 since a positive multiplied by a negative is negative and 2 x 3 = 6. We continue to generate this pattern:

3 x -3 = -9

2 x -3 = -6

1 x -3 = -3

Now I ask them what problem we should write next. They realize that it would have be 0 x -3 since the lead factor is decreasing by one each step. I ask what the products are doing. While some students suggest the numbers are getting smaller, one look at the number line shows that we are skipping back up toward larger numbers. The products are *increasing* by 3 each time and the next product must be zero. This yields this set of problems:

$$3 \times -3 = -9$$
$$2 \times -3 = -6$$
$$1 \times -3 = -3$$
$$0 \times -3 = 0$$

When I ask them what problem comes next in this pattern, they see that it would have to be -1 x -3. They also recognize that the product would increase to positive 3.

$$3 \times -3 = -9$$
$$2 \times -3 = -6$$
$$1 \times -3 = -3$$
$$0 \times -3 = 0$$
$$-1 \times -3 = +3$$

I typically write the positive sign on the 3 for emphasis during this demonstration. We then continue the pattern as shown.

$$3 \times -3 = -9$$
$$2 \times -3 = -6$$
$$1 \times -3 = -3$$
$$0 \times -3 = 0$$
$$-1 \times -3 = +3$$
$$-2 \times -3 = +6$$
$$-3 \times -3 = +9$$

They conclude that multiplying two negatives must give a positive product for the sake of consistency of the pattern.

We could use a similar progression of problems to show that these same rules apply to division. We can also use this strategy to help students understand the complex principles of subtraction of negative numbers as shown here:

$$3 - 3 = 0$$
$$3 - 2 = 1$$
$$3 - 1 = 2$$
$$3 - 0 = 3$$
$$3 - (-1) = 4$$
$$3 - (-2) = 5$$
$$3 - (-3) = 6$$

It is simple to show why these last three problems lead us to understand that two adjacent negative signs in a problem can be turned into one addition sign.

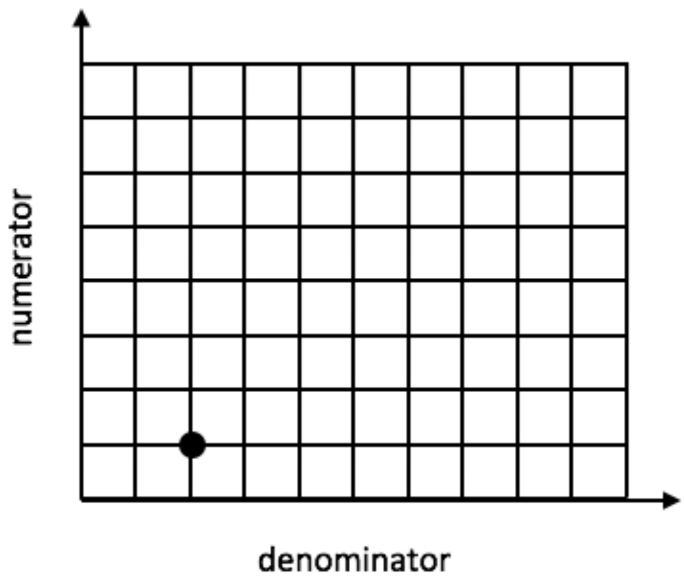
$$3 - (-3) = 6$$
$$3 \quad + \quad 3 \quad = 6$$

This would be a very impractical way to actually *solve* a problem. Imagine the table you would have to create to show that -37 – (-23) = -14! However, its value is in the way it helps students understand that the rules and procedures of integers in particular and of mathematics in general are not contrivances but are based on consistency and reasoning.

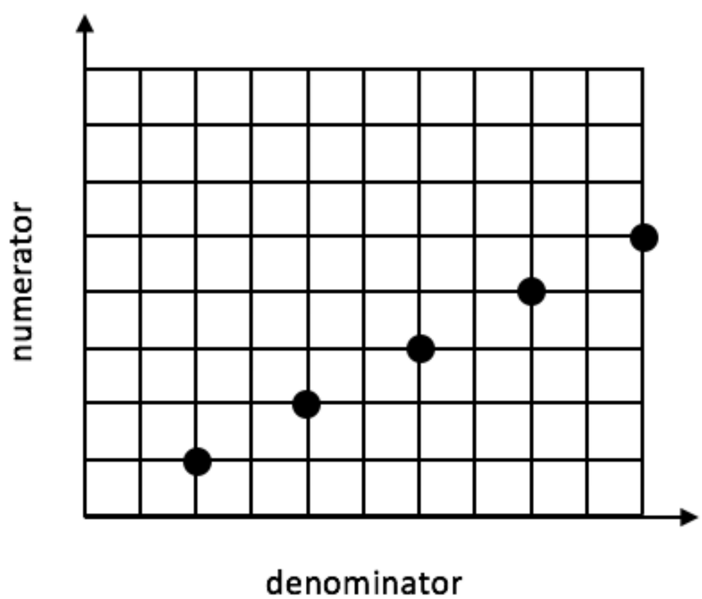
**Strategy 5: The Graphical approach:**

Like the former method, this is not a problem solving strategy but an instructional one. It demonstrates that dividing a negative number by another negative must result in a positive quotient.

I begin by graphing the fraction ${}^{1}/_{2}$. I do this on a graph in which the x-axis represents the denominator and the y-axis the numerator. Although it might seem best to switch these, you will see why we must do it this way. Thus the fraction ${}^{1}/_{2}$ would look like this on the graph:

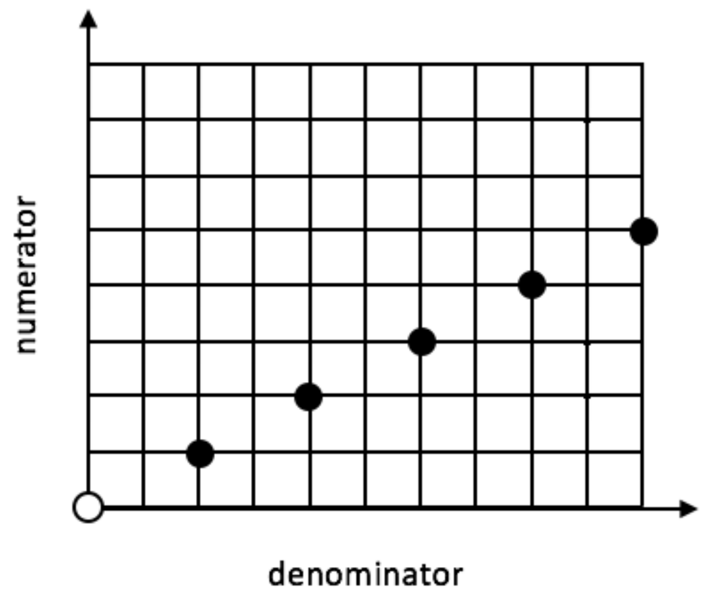



Next I ask the students for some fractions equivalent to ${}^{1}/_{2}$. They might suggest ${}^{2}/_{4}$, ${}^{3}/_{6}$, ${}^{4}/_{8}$, and ${}^{5}/_{10}$. I add these points to the graph and they see that they are collinear.

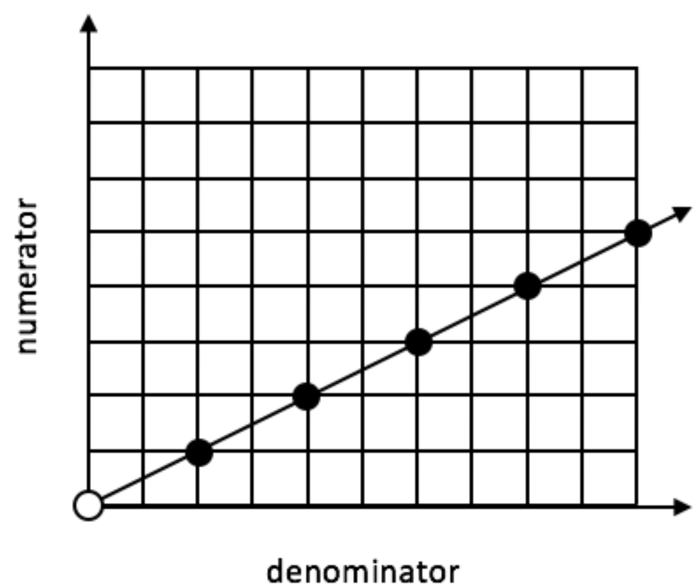


They see that all fractions equivalent to $^1/_2$ are on a line. This is because the line has a *slope* of $^1/_2$. That is why we set up the graph this way, since slope is change in *y* divided by change in *x*.

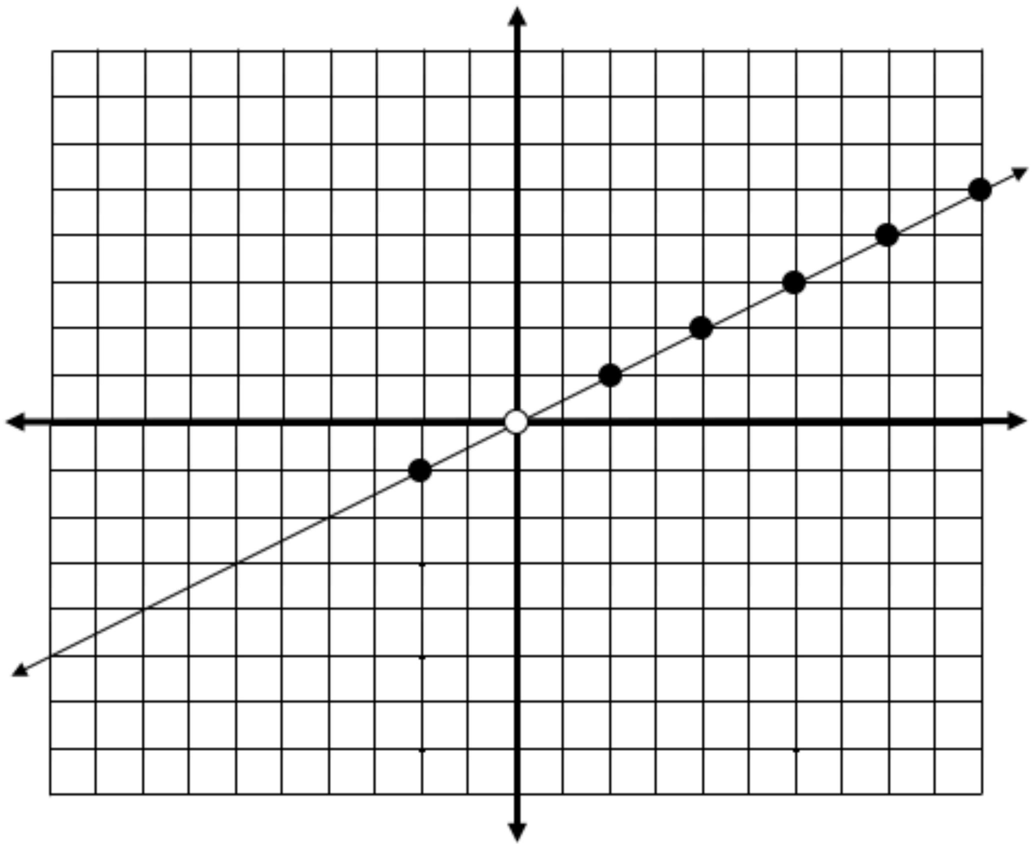
I ask them if there are any points on this line that are *not* equivalent to $^1/_2$. They may or may not realize that the point (0, 0) is collinear but is not equal to $^1/_2$. I put an open circle at that location.




I ask them if there are any other points equal to $^1/_2$ that would fit on this graph. Some student may realize that *any* other point on that line would be equivalent. That means that a complex fraction such as $^{½}/_1$ would be equivalent also. If enough such points were placed on the graph, you would eventually see the line shown below.



However, if *every* point on this line other than the origin are equivalent to $^1/_2$, and since a line goes infinitely in both directions, we can extend this into a four-quadrant graph to get this:



This means that the new point in the third quadrant must also be equivalent to $^1/_2$. We note that this point is (-2, -1) or written as a fraction $^{-1}/_{-2}$. This can be read as "negative one divided by negative 2", and we can conclude that $^{-1}/_{-2} = {}^1/_2$. Thus when we divide a negative by another negative, the value is positive.

**Concluding notes:**

Fluency with integers is a gatekeeper that will allow students to pass successfully into higher mathematics or hinder them from progress. Quite often, it is not the algebraic concepts that impede students – they can solve equations and other problems successfully as long as positive whole numbers are involved[1]. Rather it is their inability to conceptualize the abstract nature of integers and their operations that prevents them from success.

For this reason, a multi-faceted approach as demonstrated in this manual is recommended. Additionally, we cannot assume that a simple review of past strategies in the opening chapter of a textbook is sufficient to allow students to perform integer operations well throughout the rest of the year. To be fluent, they need frequent and ongoing instruction and practice.

I typically only present one strategy at a time. Then students are given the opportunity to practice using that. Later, I introduce other procedures and offer further practice sessions. I give students permission to use the strategies that make the most sense to them, but I ask them to try the new ones so their brain can make the important connections that unify all these approaches. Always remember, **you don't catch all the fish with one worm**

I try to incorporate integer practice one day per week in my 8th grade class. They are moving on to high school next year, and I want them to go in armed with an arsenal of tools for the work ahead.

Lastly, you should consider how much if any points should be attributed to practice time. We don't keep score during the basketball practice for any other reason than to provide feedback to the coach and player. Likewise, when students are practicing a skill such as this, you may not want to include their scores in their final grade but use them instead so you and they can measure their growth.

[1] For students struggling with fraction concepts, I have a handout titled "Fast Facts and Fractions" on my Teachers Pay Teachers store. In it I show how I helped my struggling students master their multiplication facts and all four fraction operations in ten minutes per day.

Common Core Connection:

Grade 6:

CCSS.MATH.CONTENT.6.NS.C.5
Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

CCSS.MATH.CONTENT.6.NS.C.6
Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

CCSS.MATH.CONTENT.6.NS.C.6.A
Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite.

CCSS.MATH.CONTENT.6.NS.C.6.B
Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

CCSS.MATH.CONTENT.6.NS.C.6.C
Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

CCSS.MATH.CONTENT.6.NS.C.7.C
Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write |-30| = 30 to describe the size of the debt in dollars*.

CCSS.MATH.CONTENT.6.NS.C.8
Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Grade 7:

CCSS.MATH.CONTENT.7.NS.A.1.A
Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged*.

CCSS.MATH.CONTENT.7.NS.A.1.B
Understand $p + q$ as the number located a distance $|q|$ from $p$, in the positive or negative direction depending on whether $q$ is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

CCSS.MATH.CONTENT.7.NS.A.1.C
Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

CCSS.MATH.CONTENT.7.NS.A.1.D
Apply properties of operations as strategies to add and subtract rational numbers.

CCSS.MATH.CONTENT.7.NS.A.2.A
Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

CCSS.MATH.CONTENT.7.NS.A.2.B
Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number.
If $p$ and $q$ are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

CCSS.MATH.CONTENT.7.NS.A.2.C
Apply properties of operations as strategies to multiply and divide rational numbers.

CCSS.MATH.CONTENT.7.NS.A.3
Solve real-world and mathematical problems involving the four operations with rational numbers.

If you liked this activity, you might also like some of the other lessons available in my TeachersPayTeachers store. Simply search for "Brad Fulton".

You can also find many free and inexpensive resources on my personal website, www.tttpress.com. **Be sure to subscribe to receive monthly newsletters, blogs, and FREE activities.**

Similar activities include:

- *Fast Facts and Fractions – Help your students learn their multiplication facts and master all fraction operations in five minutes a day!*
- *X Marks the Spot – A great way to develop number sense and gain valuable skills practice with whole numbers, fractions, integers, and algebraic terms.*
- *Pyramid Math – Practice addition and subtraction of whole numbers, fractions, and integers while developing number sense.*
- *Foursquare Addition – Another way to practice addition and subtraction of whole numbers, decimals, and integers while students self-assess.*

Feel free to contact me if you have questions or comments or would like to discuss a staff development training or keynote address at your site.

Happy teaching,

*Brad*