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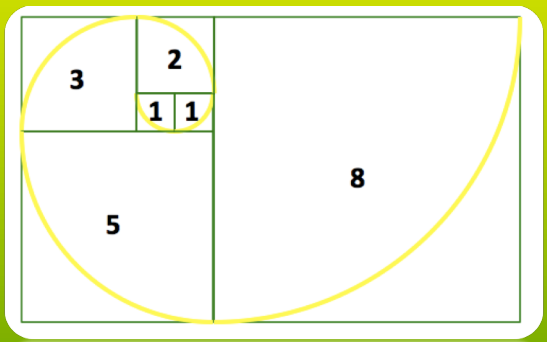
Leo's Pattern

Exploring Addition, Subtraction and More Using the Fibonacci Sequence



By Brad Fulton
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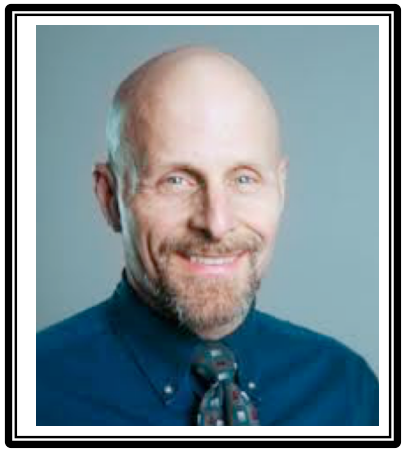
Brad Fulton

Educator of the Year



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- ♦ Consultant
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- ♦ Conference speaker

Known throughout the country for motivating and engaging teachers and students, Brad has co-authored over a dozen books that provide easy-to-teach yet mathematically rich activities for busy teachers while teaching full time for over 30 years. In addition, he has co-authored over 40 teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

Seminar leader and trainer of mathematics teachers

- ♦ 2005 California League of Middle Schools Educator of the Year
- ♦ California Math Council and NCTM national featured presenter
- ♦ Lead trainer for summer teacher training institutes
- ♦ Trainer/consultant for district, county, regional, and national workshops

Author and co-author of mathematics curriculum

- ♦ Simply Great Math Activities series: six books covering all major strands
- ♦ Angle On Geometry Program: over 400 pages of research-based geometry instruction
- ♦ Math Discoveries series: bringing math alive for students in middle schools
- ♦ Teacher training seminar materials handbooks for elementary, middle, and secondary school

Available for workshops, keynote addresses, and conferences

All workshops provide participants with complete, ready-to-use activities that require minimal preparation and give clear and specific directions. Participants also receive journal prompts, homework suggestions, and ideas for extensions and assessment.

Brad's math activities are the best I've seen in 38 years of teaching!
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"I can't begin to tell you how much you have inspired me!"
Sue Bonesteel, Math Dept. Chair, Phoenix, AZ

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Brenda McGaffigan, principal, Santa Ana, CA

"Absolutely engaging. I can teach algebra to second graders!"
Lisa Fellers, teacher

References available upon request

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Thanks and happy teaching,

Brad

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Leo's Pattern

Exploring Addition, Subtractions and More Using the Fibonacci Sequence

Overview:

Based on the famous Fibonacci sequence, this activity helps students develop numbers sense as they work with addition and subtraction. Decimals and integers are easily incorporated, and estimation skills are honed. It is a nice activity for the teacher since much of the work is self-correcting. Problem solving strategies can be taught, and students can even use algebra as a tool.

Required Materials:

☒ Paper

Optional Materials:

☐ Activity/Homework Master

☐ Calculators

Preface:

The Fibonacci sequence is a number pattern that has amazed mathematicians for centuries. It is based on the simple fact that 1 + 1 = 2. Leonardo Fibonacci is famous for his work with this pattern. **Each pair of adjacent numbers adds up to the subsequent number**:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597…

That is, not only does 1 + 1 = 2, but 1 + 2 = 3, 2 + 3 = 5, 3 + 5 = 8 and so on. The series goes on toward infinity.

The Fibonacci sequence has some unusual properties. The numbers are very common in the natural world. For example, the number 4 is missing, and this is a rare number in nature. (Four-leaf clovers are uncommon.) Starfish have 5 arms, an octopus has 8, and pinecones have 8 spirals in one direction and 13 in the other. Their needles come in groups of 1, 2, 3, or 5. A daisy has 21 petals, so "she loves you". The ratio of consecutive Fibonacci numbers approaches the golden ratio. For example, 1597 ÷ 987 = 1.61803445. The golden ratio by comparison is around 1.61803399.

A search on the Internet will yield a multitude of websites dedicated to the many fascinating properties of this sequence. To engage your students, you may wish to share some of these.

The following activity uses the structure of the Fibonacci sequence to explore addition and subtraction of different types of numbers. As the teacher, you will decide on which level to begin with your students, and on which level to end.

Procedure:

1. Display of Leo's Pattern A,1. Explain the structure of the pattern, and tell students that each one of the patterns on the page follows the same rule: Add the first two terms to get the third term, add the second and third term to get the fourth term, and continue in this way. Two consecutive terms always add to get the following term. Patterns defined this way are called recursive patterns.

2. Show the class the rest of the patterns from set A. These patterns are examples of the recursive rule and show students that the rule can be used with many different sets of numbers.

3. Next ask the students to complete pattern B1. Set B patterns are missing the second term. Students should see that the second term is the *difference* between the third and first terms. Many students will be able to finish these patterns in their heads. You may create patterns as needed for more practice. Decimals, fractions, and negative integers make great entries for the terms.

4. After your students feel comfortable with the rule, patterns from set C may be introduced. Some of your students will feel challenged for the first time with set C patterns. Two numbers are missing between the numbers that are supplied in these patterns. This is a good time to introduce the problem solving technique of Guess and Check. With pattern C1, students may try to guess the number that comes after the 3, and then add the first two terms to get term three, and then add terms three and four to see if they sum to 17. If they guessed wrong the first time, they need to adjust their guess for term two, and try again.

   For pattern C1, this should not take too long, and students have a sense of accomplishment when they guess the right number.

5. Estimation skills and numbers sense will increase with the complexity of the problems and with the amount of practice. Your students will begin to notice patterns as they work. For example, if a student tries to put a four in the second blank of C1, they hit an 11 instead of a 17 in blank four. This tells the student that the guess was too low. If a 12 is tried, the student lands on 27, which is too high. Now the student knows that the correct answer is greater than four and less than 12. In fact, if the guess is decreased by one, and an 11 is tried, the target decreases by two to 25. This is double the guess. Since we want to decrease the target *eight* more to 17, we must decrease our guess *four* more to 7. This results in the correct answer.

   3, ___, ___, 17, ___, ___

   3, **4**, 7, 11 (low)

   3, **12**, 15, 27 (high)

   3, **11**, 14, 25 (high)

   3, **7**, 10, 17 ☺

6. More advanced students can use algebra instead of the Guess and Check strategy to solve these problems. Lets use the following problem as an example:

   5, _____ , _____ , _____ , _____ , _____ , _____ , _____ , _____ , 241

Since we do not know the value of the number in the second position, we will represent it with a variable:

$$5,\ \underline{x},\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_,\ 241$$

Each subsequent number in the sequence is the sum of the previous two. This means that the value of the third position is the sum of 5 and x:

$$5,\ \underline{x},\ \underline{5 + x},\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_,\ 241$$

Similarly, the value of the fourth position is the sum of x and $5 + x$:

$$5,\ \underline{x},\ \underline{5 + x},\ \underline{5 + 2x},\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_,\ \_\_\_\_,\ 241$$

Continuing in this way yields the following series:

$$5,\ \underline{x},\ \underline{5 + x},\ \underline{5 + 2x},\ \underline{10 + 3x},\ \underline{15 + 5x},\ \underline{25 + 8x},\ \underline{40 + 13x},\ \underline{65 + 21x},\ 241$$

(Did you notice that the coefficients of the variables are Fibonacci numbers: $1x$, $1x$, $2x$, $3x$, $5x$, $8x$, $13x$, and $21x$?)

If we were to go one more step in this series, the final position would be the sum of $40 + 13x$ and $65 + 21x$ or $105 + 34x$. This means that:

$$105 + 34x = 241$$

Solving this equation yields:

$$105 - 105 + 34x = 241 - 105$$

$$34x = 136$$

$$34x/34 = 136/34$$

$$x = 4$$

We can now substitute 4 in place of x in the second position:

$$5,\ \underline{4},\ \underline{9},\ \underline{13},\ \underline{22},\ \underline{35},\ \underline{57},\ \underline{92},\ \underline{149},\ 241$$

Journal Prompts:

If the first term of Leo's Pattern is 4 and the fourth term is 0, what must be true of the second term? Why?

If the first term of Leo's Pattern is 2 and the fourth term is 5, how would you find the second term?

Describe to a student who was absent how to find the second and third terms of sequence C3 by the Guess and Check method.

Homework:

You can assign one of the activity/homework masters for homework or create ten new patterns for students to solve using the blank masters. Give terms 1 and 2 or 1 and 4 only. Or you may choose to give the last two terms in the series to teach subtraction concepts. You may wish to include decimals, fractions, or negative numbers as solutions.

You can also ask ten students create problems, write them on the board, and then assign the set for homework.

Taking a Closer Look:

There is a lot of algebra lurking in Leo's Patterns as shown in step 6 above. For a more complete treatment of the algebraic solutions, refer to our algebra book, *Simply Great Math Activities: Algebra Readiness, Volume 2*.

If you are teaching the pre-algebra concept of combining like terms, you can insert binomials into the series as shown here:

2x+7, 6x-1, _____ , _____ , _____ , _____ , _____ , _____

Assessment:

Use the answer key below or have students check each other's work.

Answer Key:

A1	5	8	13	21	34	55	89
A2	7	11	18	29	47	76	123
A3	12	19	31	50	81	131	212
A4	10	12	22	34	56	90	146
A5	6	10	16	26	42	68	110
A6	18	26	44	70	114	184	298
A7	1	21	22	43	65	108	173
A8	13	47	60	107	167	274	441
A9	7	49	56	105	161	266	427
A10	46	104	150	254	404	658	1062

B1	2	4	6	10	16	26
B2	3	6	9	15	24	39
B3	2	7	9	16	25	41
B4	1	5	6	11	17	28
B5	2	9	11	20	31	51
B6	8	1	9	10	19	29
B7	0	14	14	28	42	70
B8	19	0	19	19	38	57
B9	16	16	32	48	80	128
B10	74	37	111	148	259	407

C1	3	7	10	17	27	44
C2	4	8	12	20	32	52
C3	6	3	9	12	21	33
C4	5	2	7	9	16	25
C5	7	2	9	11	20	31
C6	8	11	19	30	49	79
C7	16	2	18	20	38	58
C8	0	24	24	48	72	120
C9	34	13	47	60	107	167
C10	43	3	46	49	95	144

D1	5	-2	3	1	4	5
D2	2	-5	-3	-8	-11	-19
D3	4	-6	-2	-8	-10	-18
D4	14	-10	4	-6	-2	-8
D5	-12	0	-12	-12	-24	-36
D6	13	-13	0	-13	-13	-26
D7	-23	8	-15	-7	-22	-29
D8	35	-21	14	-7	7	0
D9	31	-25	6	-19	-13	-32
D10	-31	25	-6	19	13	32

E1	9	-5	4	-1	3	2
E2	12	-12	0	-12	-12	-24
E3	-16	11	-5	6	1	7
E4	-14	9	-5	4	-1	3
E5	0	-6	-6	-12	-18	-30
E6	2	-3	-1	-4	-5	-9
E7	28	-25	3	-22	-19	-41
E8	-6	5	-1	4	3	7
E9	-17	10	-7	3	-4	-1
E10	-59	34	-25	9	-16	-7

F1	3	-6	-3	-9	-12	-21
F2	9	-11	-2	-13	-15	-28
F3	-6	4	-2	2	0	2
F4	-8	4	-4	0	-4	-4
F5	15	-7	8	1	9	10
F6	0	-8	-8	-16	-24	-40
F7	22	-14	8	-6	2	-4
F8	-11	0	-11	-11	-22	-33
F9	-26	16	-10	6	-4	2
F10	16	-15	1	-14	-13	-27

G1	3	3.8	6.8	10.6	17.4	28
G2	6.2	0.7	6.9	7.6	14.5	22.1
G3	0.23	0.09	0.32	0.41	0.73	1.14
G4	0.5	0.09	0.59	0.68	1.27	1.95
G5	6.4	1.7	8.1	9.8	17.9	27.7
G6	7.93	0.17	8.1	8.27	16.37	24.64
G7	3.2	0.6	3.8	4.4	8.2	12.6
G8	8.3	8.3	16.6	24.9	41.5	66.4
G9	0.17	0.38	0.55	0.93	1.48	2.41
G10	0.01	0.52	0.53	1.05	1.58	2.63

H1	3	4 ½	7 ½	12	19 ½	31 ½
H2	3 ½	2	5 ½	7 ½	13	20 ½
H3	6 ½	1 ½	8	9 ½	17 ½	27
H4	4 ¼	1 ¾	6	7 ¾	13 ¾	21 ½
H5	½	¼	¾	1	1 ¾	2 ¾
H6	¾	1 ½	2 ¼	3 ¾	6	9 ¾
H7	½	2	2 ½	4 ½	7	11 ½
H8	1 ¼	1 ½	2 ¾	4 ¼	7	11 ¼
H9	2 ½	1	3 ½	4 ½	8	12 ½
H10	0	3 ½	3 ½	7	10 ½	17 ½

The Common Core Connection

Fourth grade

Number and Operations in Base Ten

B4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Fifth grade

Number and Operations in Base Ten

B7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Sixth grade

The Number System

B3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Seventh grade

The Number System

1C: Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

1D: Apply properties of operations as strategies to add and subtract rational numbers.

High school algebra

Arithmetic with Polynomials and Rational Expressions

D7: …add, subtract, multiply, and divide rational expressions.

Activity/Homework Master

Name_____

Date_____ Class_____

Leo's Pattern A

These patterns all follow the same rule: add two successive terms to get the next term. The original Fibonacci sequence is given, along with additional examples. Find the missing terms.

A1 1, 1, 2, 3, 5, 8, ___, ___, ___, ___, ___

A2 1, 3, 4, 7, 11, ___, ___, ___, ___, ___

A3 2, 5, 7, 12, 19, ___, ___, ___, ___, ___

A4 8, 2, 10, 12, ___, ___, ___, ___, ___

A5 2, 4, 6, 10, ___, ___, ___, ___, ___

A6 8, 18, 26, ___, ___, ___, ___, ___

A7 20, 1, 21, ___, ___, ___, ___, ___

A8 34, 13, 47, ___, ___, ___, ___, ___

A9 42, 7, 49, ___, ___, ___, ___, ___

A10 58, 46, 104, ___, ___, ___, ___, ___

Activity/Homework Master

Name_____

Date_____ Class_____

Leo's Pattern B

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

B1 2, ___, 6, ___, ___, ___

B2 3, ___, 9, ___, ___, ___

B3 ___, ___, ___, ___, 25, 41

B4 ___, ___, ___, ___, 17, 28

B5 ___, ___, ___, 20, ___, 51

B6 ___, ___, ___, ___, 19, 29

B7 ___, ___, ___, 28, ___, 70

B8 ___, ___, 19, 19, ___, ___

B9 ___, ___, 32, ___, 80, ___

B10 ___, ___, ___, 148, ___, 407

Activity/Homework Master

Name_____

Date_____ Class_____

Leo's Pattern C

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

C1 3, ___, ___, 17, ___, ___

C2 4, ___, ___, 20, ___, ___

C3 6, ___, ___, 12, ___, ___

C4 5, ___, ___, ___, 16, ___

C5 7, ___, ___, ___, ___, 31

C6 ___, ___, 19, ___, ___, 79

C7 ___, 2, ___, ___, ___, 58

C8 ___, 24, ___, ___, 72, ___

C9 ___, ___, 47, ___, ___, 167

C10 43, ___, ___, ___, ___, 144

Activity/Homework Master

Name_____

Date_____ Class_____

Leo's Pattern D

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

D1 5, -2, 3, ___, ___, ___

D2 2, -5, -3, ___, ___, ___

D3 4, -6, ___, ___, ___, ___

D4 14, -10, ___, ___, ___, ___

D5 -12, 0, ___, ___, ___, ___

D6 13, -13, ___, ___, ___, ___

D7 -23, 8, ___, ___, ___, ___

D8 35, -21, ___, ___, ___, ___

D9 31, -25, ___, ___, ___, ___

D10 -31, 25, ___, ___, ___, ___

Activity/Homework Master

Name_____

Date_____ Class_____

Leo's Pattern E

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

E1 ___, -5, 4, ___, ___ , ___

E2 ___, ___, ___, -12, -12, ___

E3 ___, ___, ___, ___, 1, 7

E4 ___, ___, ___, ___, -1, 3

E5 ___, ___, ___, ___, -18, -30

E6 ___, ___, ___, ___, -5, -9

E7 ___, ___, ___, ___, -19, -41

E8 ___, ___, ___, ___, 3, 7

E9 ___, ___, ___, ___, -4, -1

E10 ___, ___, ___, ___, -16, -7

Activity/Homework Master

Name_____

Date_____ Class_____

Leo's Pattern F

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

F1 ___, -6, ___, -9, ___, ___

F2 ___, ___, -2, ___, -15, ___

F3 ___, ___, -2, ___, 0, ___

F4 ___, 4, ___, ___, -4, ___

F5 15, ___, ___, 1, ___, ___

F6 ___, ___, -8, ___, ___, -40

F7 22, ___, ___, -6, ___, ___

F8 -11, ___, ___, -11, ___, ___

F9 -26, ___, ___, ___, ___, 2

F10 16, ___, ___, ___, ___, -27

Activity/Homework Master

Name_____

Date_____ Class_____

Leo's Pattern G

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

G1 3, 3.8, ___, ___, ___, ___

G2 6.2, .7, ___, ___, ___, ___

G3 .23, .09, ___, ___, ___, ___

G4 .5, .09, ___, ___, ___, ___

G5 ___, 1.7, 8.1, ___, ___, ___

G6 ___, .17, 8.1, ___, ___, ___

G7 ___, ___, ___, ___, 8.2, 12.6

G8 ___, ___, 16.6, ___, 41.5, ___

G9 ___, .38, ___, .93, ___, ___

G10 ___, .52, ___, ___, 1.58, ___

Activity/Homework Master

Name_____

Date_____ Class_____

Leo's Pattern H

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

H1 3, 4 ½, ___, ___, ___, ___

H2 3 ½, 2, ___, ___, ___, ___

H3 6 ½, 1 ½, ___, ___, ___, ___

H4 4 ¼, 1 ¾, ___, ___, ___, ___

H5 ½, ¼, ___, ___, ___, ___

H6 ¾, 1 ½, ___, ___, ___, ___

H7 ___, ___, ___, 4 ½, 7, ___

H8 ___, ___, ___, 4 ¼, 7, ___

H9 ___, ___, 3 ½, ___, 8, ___

H10 ___, ___, 3 ½, ___, 10 ½, ___

Activity/Homework Master

Name_____

Date_____ Class_____

Leo's Pattern

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

1 ___, ___, ___, ___, ___, ___

2 ___, ___, ___, ___, ___, ___

3 ___, ___, ___, ___, ___, ___

4 ___, ___, ___, ___, ___, ___

5 ___, ___, ___, ___, ___, ___

6 ___, ___, ___, ___, ___, ___

7 ___, ___, ___, ___, ___, ___

8 ___, ___, ___, ___, ___, ___

9 ___, ___, ___, ___, ___, ___

10 ___, ___, ___, ___, ___, ___

Activity/Homework Master

Name_____

Date_____ Class_____

Leo's Pattern

These patterns all follow the same rule: add two successive terms to get the next term. Find the missing terms.

1 ___, ___, ___, ___, ___, ___, ___, ___

2 ___, ___, ___, ___, ___, ___, ___, ___

3 ___, ___, ___, ___, ___, ___, ___, ___

4 ___, ___, ___, ___, ___, ___, ___, ___

5 ___, ___, ___, ___, ___, ___, ___, ___

6 ___, ___, ___, ___, ___, ___, ___, ___

7 ___, ___, ___, ___, ___, ___, ___, ___

8 ___, ___, ___, ___, ___, ___, ___, ___

9 ___, ___, ___, ___, ___, ___, ___, ___

10 ___, ___, ___, ___, ___, ___, ___, ___

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- *Building Number Sense:* Helpful tips and techniques for developing rich number sense at all grade levels.

Feel free to contact me if you have questions or comments or would like to discuss a staff development training or keynote address at your site.

Happy teaching,
Brad