Liquid Functions

Brad Fulton
Educator of the Year, 2005
Teacher to Teacher Press
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Brad Fulton

Educator of the Year

- Consultant
- Educator
- Author
- Keynote presenter
- Teacher trainer
- Conference speaker

Known throughout the country for motivating and engaging teachers and students, Brad has co-authored over a dozen books that provide easy-to-teach yet mathematically rich activities for busy teachers while teaching full time for over 30 years. In addition, he has co-authored over 40 teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

Seminar leader and trainer of mathematics teachers

- 2005 California League of Middle Schools Educator of the Year
- California Math Council and NCTM national featured presenter
- Lead trainer for summer teacher training institutes
- Trainer/consultant for district, county, regional, and national workshops

Author and co-author of mathematics curriculum

- Simply Great Math Activities series: six books covering all major strands
- Angle On Geometry Program: over 400 pages of research-based geometry instruction
- Math Discoveries series: bringing math alive for students in middle schools
- Teacher training seminar materials handbooks for elementary, middle, and secondary school

Available for workshops, keynote addresses, and conferences

All workshops provide participants with complete, ready-to-use activities that require minimal preparation and give clear and specific directions. Participants also receive journal prompts, homework suggestions, and ideas for extensions and assessment.

Brad's math activities are the best I've seen in 38 years of teaching!
Wayne Dequer, 7th grade math teacher, Arcadia, CA

"I can't begin to tell you how much you have inspired me!"
Sue Bonesteel, Math Dept. Chair, Phoenix, AZ

"Your <u>*entire*</u> *audience was fully involved in math!! When they chatted, they chatted math. Real thinking!"*
Brenda McGaffigan, principal, Santa Ana, CA

"Absolutely engaging. I can teach algebra to second graders!"
Lisa Fellers, teacher

References available upon request

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If we make copies for our friends, can we honestly tell our students not to copy or take things that don't belong to them? (Ouch!)

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Thanks, and happy teaching,

Brad

OVERVIEW

Skills:

- Understanding linear and nonlinear functions
- Graphing
- T-tables
- Slope and intercept
- Measuring liquids

Liquid Functions

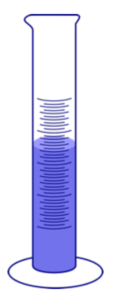
This engaging and kinesthetic activity will help students visualize and understand the principles and concepts involved in functions. You can adapt the lesson to study linear or nonlinear functions or include both. The materials needed are few and inexpensive, and can be adapted to the needs of your students. The activity can be used as an introduction to functions so that students get hands-on experience, or it can be used as an application of skills students have already learned in a unit of study on functions.

PROCEDURE

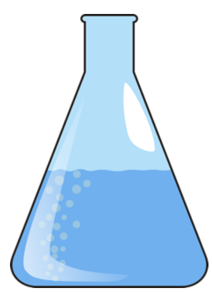
Materials:

- Various glass containers
- Water
- Liquid measuring device such as a graduated cylinder or test tubes
- Food coloring (optional)
- Centimeter rulers

1. Collect containers of various sizes. I like to use some with straight sides such as a graduated cylinder or a straight walled vase or beaker. I also use more interesting shapes such as Erlenmeyer and Florence flasks and curved vases and bottles. The best containers are those that have very consistent diameters such as the graduated cylinder or the opposite: those with radically changing diameters. A martini glass would be a prime example of this second type, though you might want to avoid using that in a classroom. A soda bottle on the other hand has various diameters along its vertical axis, but the widest and narrowest portions don't vary significantly. This results in a graph that appears fairly straight even though it is not linear.



Graduated cylinder



Erlenmeyer flask

Florence flask

2. I prefer to demonstrate this once using a simple container such as a graduated cylinder or beaker as students follow along. Then I give them more advanced containers on which to experiment

in their teams. I believe that it is important for students to work on this in pairs or small teams to generate discussion and dialog. Language is important in developing the critical thinking that we want to foster.

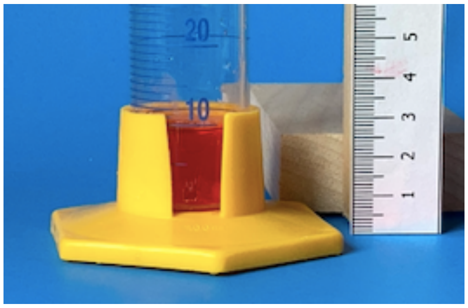
3. Give each student a copy of the t-table and graph for the first experiment. Ask them to make a prediction about how the graph will look. They can draw their prediction using colored pencil to distinguish it from the actual graph that they make using a different color. Notice that the horizontal axis does not have a scale. This is because this will vary according to the volume of the container. For example, when using a 100 mL graduated cylinder, each tick mark should be a multiple of ten. This makes the scale from 0 to 100 mL. Then students will add 10 mL of water each time. However, if the container is a 250 mL beaker, the scale should be numbered in increments of 25 and that amount of water will be added each time.

This is scale is easy to determine. Simply fill the container and then pour it into a beaker. If it fills a beaker with 400 mL of water, then the horizontal scale will count by one tenth of that: 40, and that is how many mL of water will be added each time. For the purpose of the demonstration explained below, I'll assume that we are using a 100 mL graduated cylinder. The scale would be numbered 0, 10, 20, 30…100, and we'll add 10 mL each time.

4. Have them record the amount of water and depth in centimeters. To begin, there is no water and it obviously has no depth. Thus, the table and graph begin at the point (0, 0).

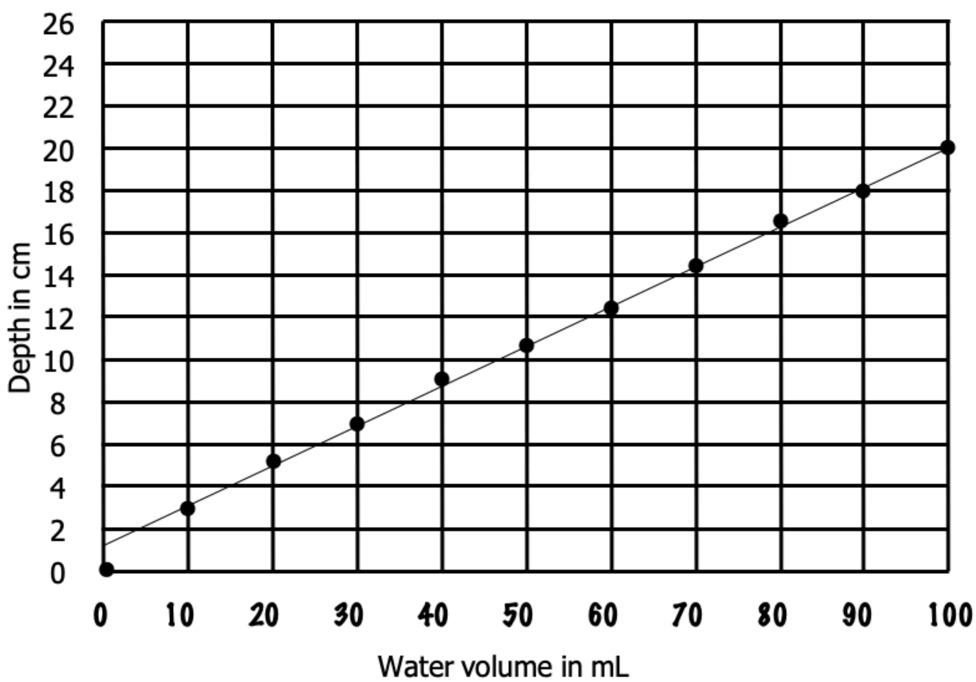
5. This should be recorded on the t-table and the graph. Now add 10 mL of water (if using a 100 mL container). This should be measured **<u>accurately</u>**. They can use another graduated cylinder or a test tube.

6. Continue to add water in consistent increments and record the data on the t-table and the graph. On the following page we see the results for my measurements with the 100 mL graduated

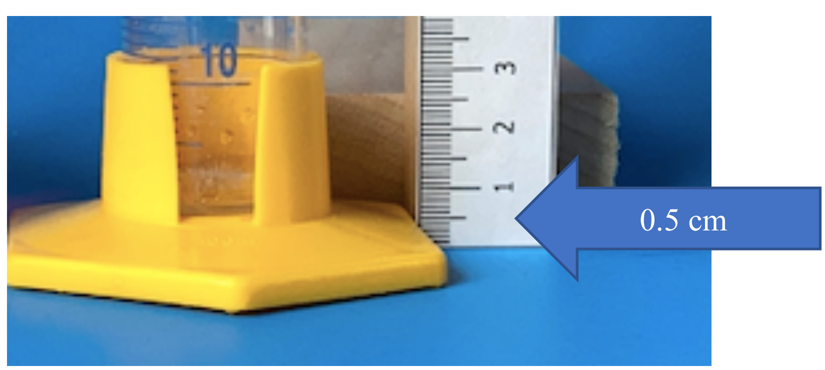


cylinder. It is important to note that measurement is always an approximation. Counting can be exact, but your reading of a centimeter scale and mine might be a bit different. Using the suggested scales, this will not be a significant factor.

x	y
0	0
10	3
20	5.5
30	7
40	9.2
50	10.5
60	12.2
70	14.3
80	16.3
90	18
100	20



7. Notice that as stated above, the data appears extremely linear. The only exception is the y-intercept at (0, 0). There is a good reason for this, and you may want to ask the students to explain the problem. (It is due to the fact that the ruler is measuring from the desk, but the water is not on the desk. It is sitting on the bottom of the inside of the graduated cylinder which is seated in a plastic stand. The plastic stand and the thickness of the glass bottom of the cylinder is about half a centimeter, which comes out right on the line.



8. Ask the students if they had predicted that the graph would be linear. You can also ask them to find the formula for the function in slope-intercept form. The slope can be calculated by taking the final height of 20 and subtracting the initial height which was technically not zero but 0.5 cm. This gives a rise of 19.5 over a run of 100 mL for a slope of 0.195. The y-intercept is 0.5 cm. This results in a formula of

$$y = 0.195x + 0.5$$

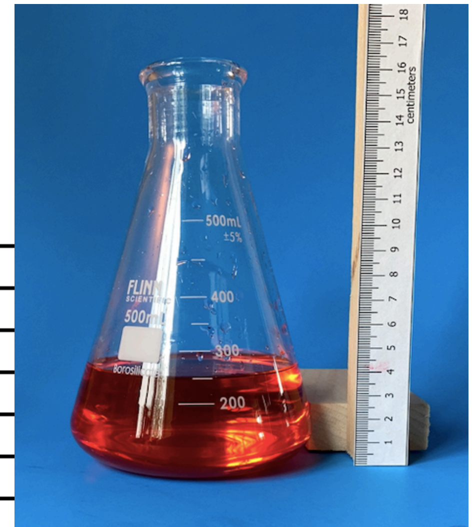
When we test this formula, we find that it is very accurate, but it does reveal that measurement is never perfect. Inputting 70 mL should give a height of 14.3. Actually, we get

$$0.195(70) + 0.5 = 14.15$$

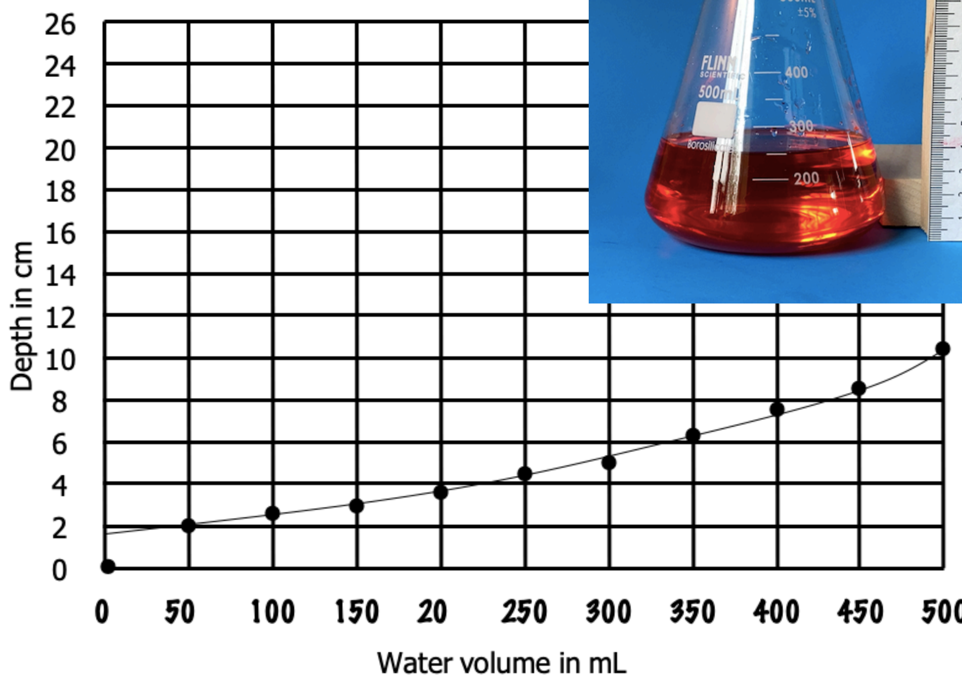
That's only off by 0.15 cm or 1.5 mm.

9. Students can now begin to explore the functions of other containers on their own. For beginning students, you may want to stick to those that have consistent widths such as beakers or straight-walled vases. More advanced students can explore more complex shapes. Some examples are shown on the following pages.

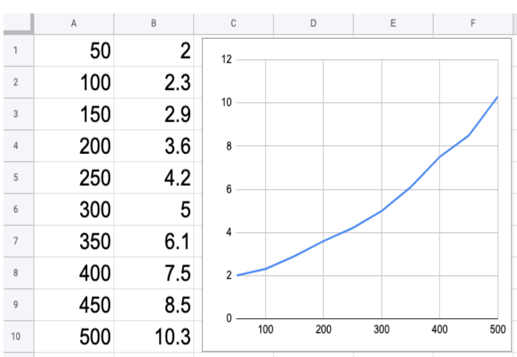
Erlenmeyer flask:



x	y
0	0
50	2.0
100	2.3
150	2.9
200	3.6
250	4.2
300	5.0
350	6.1
400	7.5
450	8.5
500	10.3



Notice that again, the first data point (0, 0) is not accurate due to the thickness of the glass bottom of the flask. You may want to tell students to ignore this first point and estimate the y-intercept visually.



	A	B
1	50	2
2	100	2.3
3	150	2.9
4	200	3.6
5	250	4.2
6	300	5
7	350	6.1
8	400	7.5
9	450	8.5
10	500	10.3

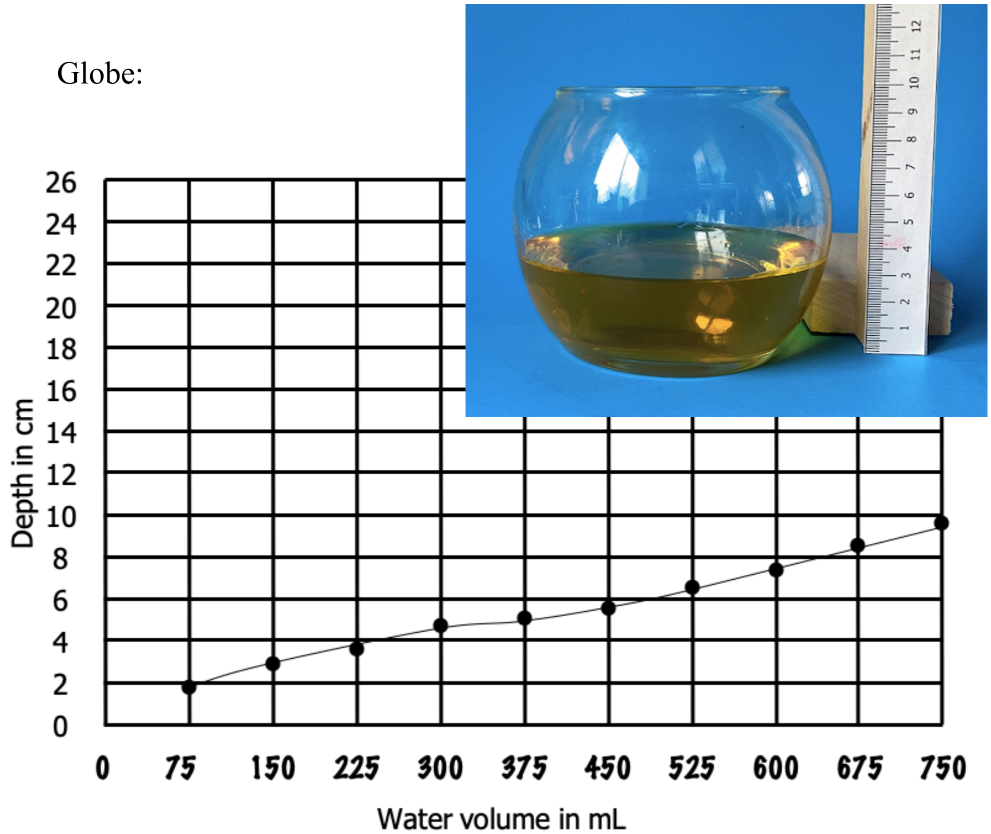
We also see that this arrangement of data points is not linear. The closer we get to the narrower top of the Erlenmeyer flask, the more drastic the change in the water level. This would be even more apparent if a different vertical scale such as 0–12 cm was used*. In the margin is a picture of how this looks when graphed on Google Sheets. Using resources such as Google Sheets, Excel, or Desmos graphing software would be a very good way to integrate technology into this activity for students who are ready.

You could also point out that the data point (300, 5.0) appears to have a degree of inaccuracy in the measurement as the graphed point is a bit below the curve.

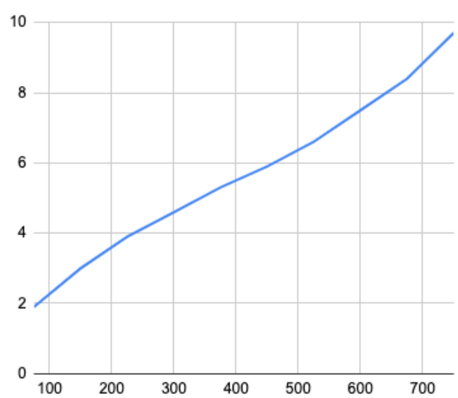
***Helpful tip:**
Two versions of the student graphing page are provided. One has a vertical scale from 0 to 26 cm. The other copy has a blank vertical scale so students can decide what is most appropriate.

Globe:

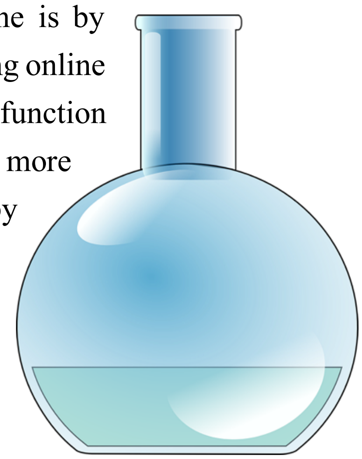
x	y
75	1.9
150	3.0
225	3.9
300	4.6
375	5.3
450	5.9
525	6.6
600	7.5
675	8.4
750	9.7



Now we see a slight S-curve to the line. This is not extremely obvious, especially to students who are not accurate with their measurements. In fact, they might think that it is a linear function.



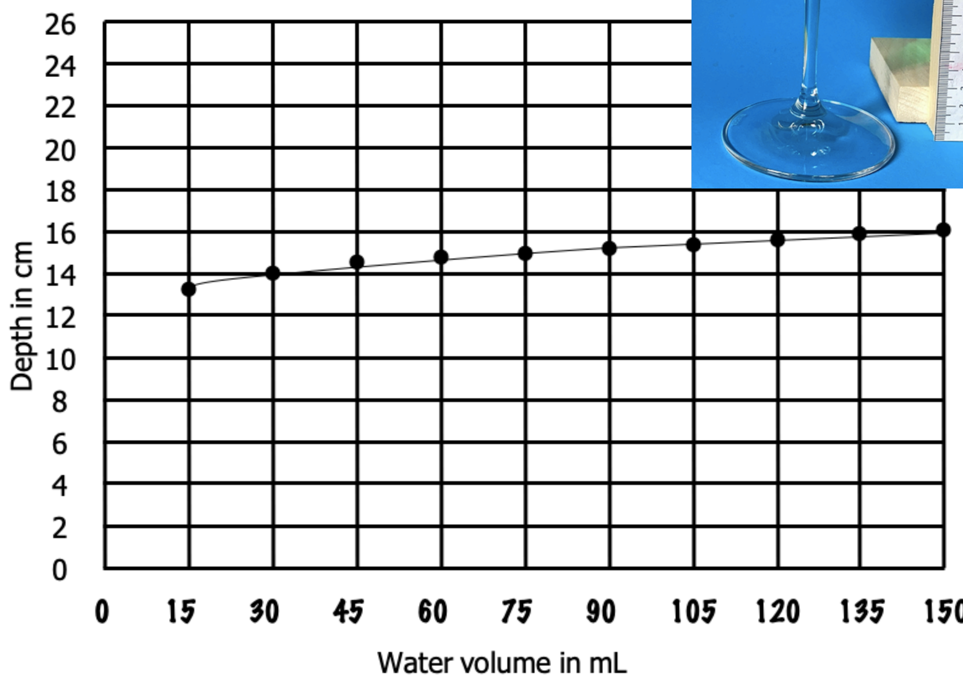
This can be alleviated two ways. One is by adjusting the scale on the y-axis or using online software. In the margin is the same function graphed on Google Sheets. A more pronounce curve can be obtained by using a Florence flask which has greater variation in its diameter across its height.



One last suggestion. You will notice that the globe in the picture has yellow food coloring in the water. If you want to avoid crude jokes from your students, use any color except yellow!

Martini glass:

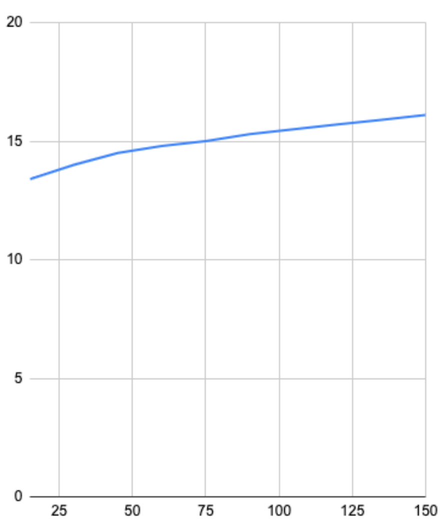
x	y
15	13.4
30	14.0
45	14.5
60	14.8
75	15.0
90	15.3
105	15.5
120	15.7
135	15.9
150	16.1



Now there is a slight flattening of the curve as the martini glass is filled. It seems to be the opposite of the graph of the Erlenmeyer flask. Why is this? It is due to the fact that the Erlenmeyer flask gets narrower as we move upward, and the martini glass gets wider. Did the students predict this?

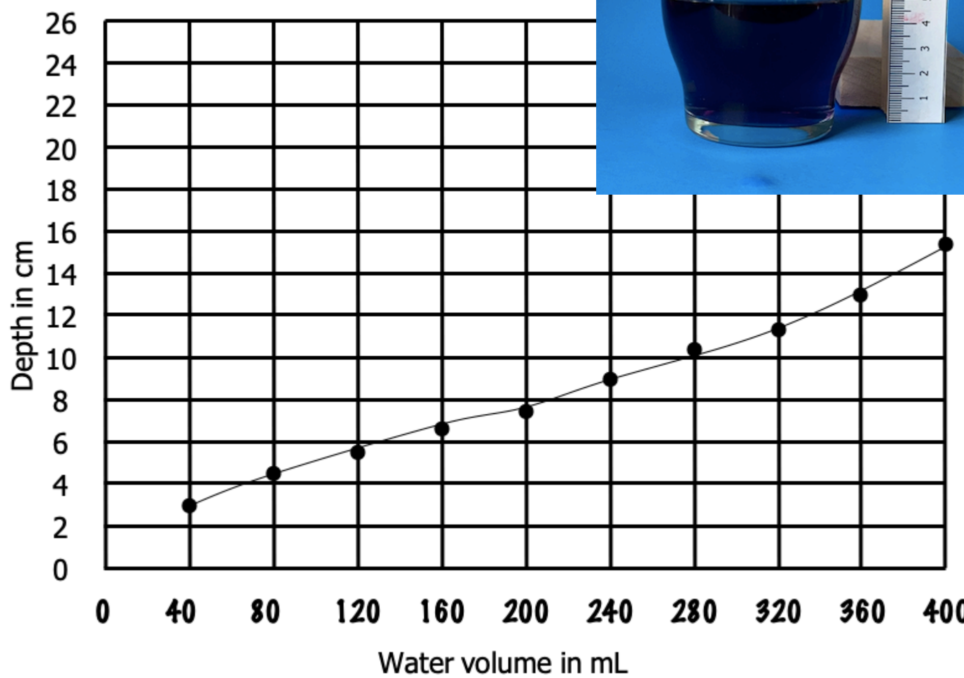
Also, we seem to have a much higher y-intercept. How do your students explain this? In this case, the stem of the martini glass is about 12 cm tall.

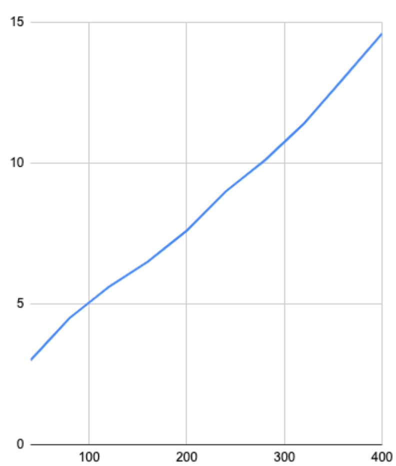
In the margin is the same data graphed on Google Sheets. You will have to decide if your students are mature enough to handle working with a martini glass without making inappropriate remarks.



Curved vase:

x	y
40	3.0
80	4.5
120	5.6
160	6.5
200	7.6
240	9
280	10.1
320	11.4
360	13.0
400	14.6

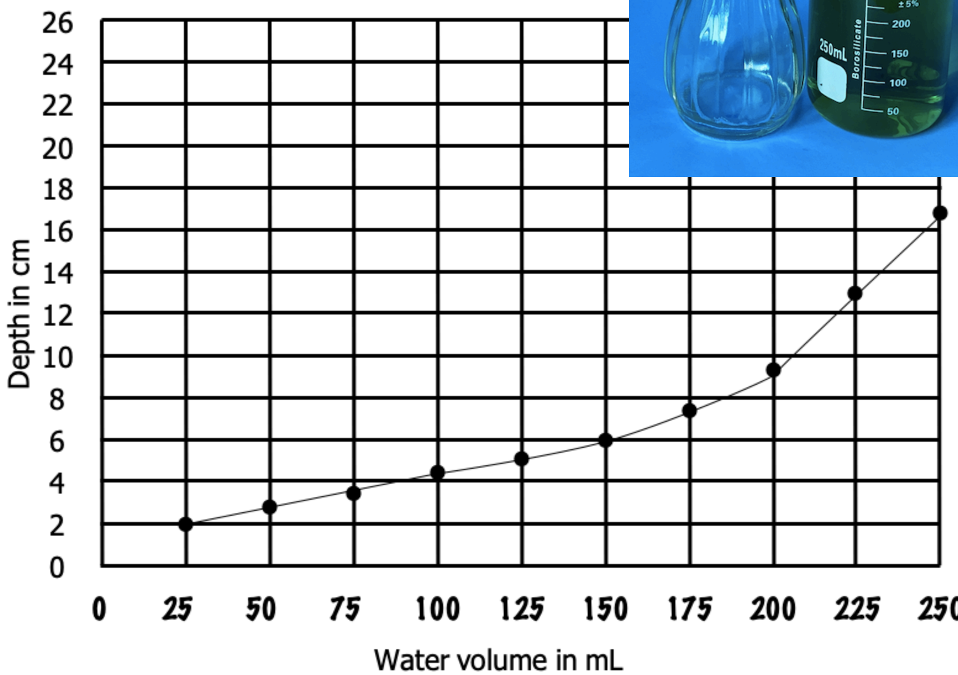


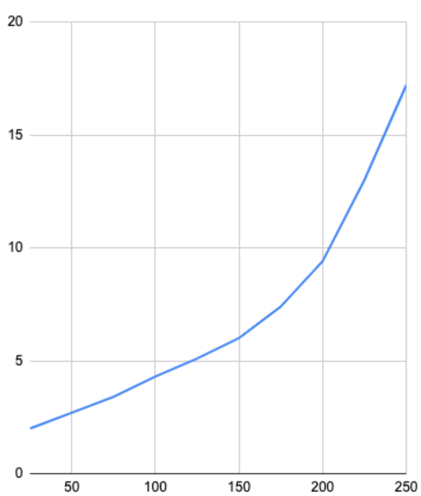


In this case, the line on the graph is harder to interpret. Even when graphed on Google Sheets, it is not so easy to visualize how the graph should look if the measurements were perfect. This is largely due to the fact that there is not as much variation between the widest and narrowest cross sections of the vase.

Tall vase:

x	y
25	2.0
50	2.7
75	3.4
100	4.3
125	5.1
150	6.0
175	7.4
200	9.4
225	13.0
250	17.2





Here we see an interesting result. The graph is curving upward with a steeper slope until we get to 200 mL. Then it appears to be linear. Can your students see what is causing this? It is the fact that the sides of the vase are curved toward the base, but the neck has a constant diameter. In the margin is the same function shown on Google Sheets.

Liquid Functions

Name________________________________

Date_____________________ Class______

Add water to your container and record the depth in centimeters on the t-table and the graph. Choose what increments of water you will add each time.

1. Sketch your container on the right.

2. Sketch what you think the graph will look like.

3. Explain why you believe your sketch is reasonable.

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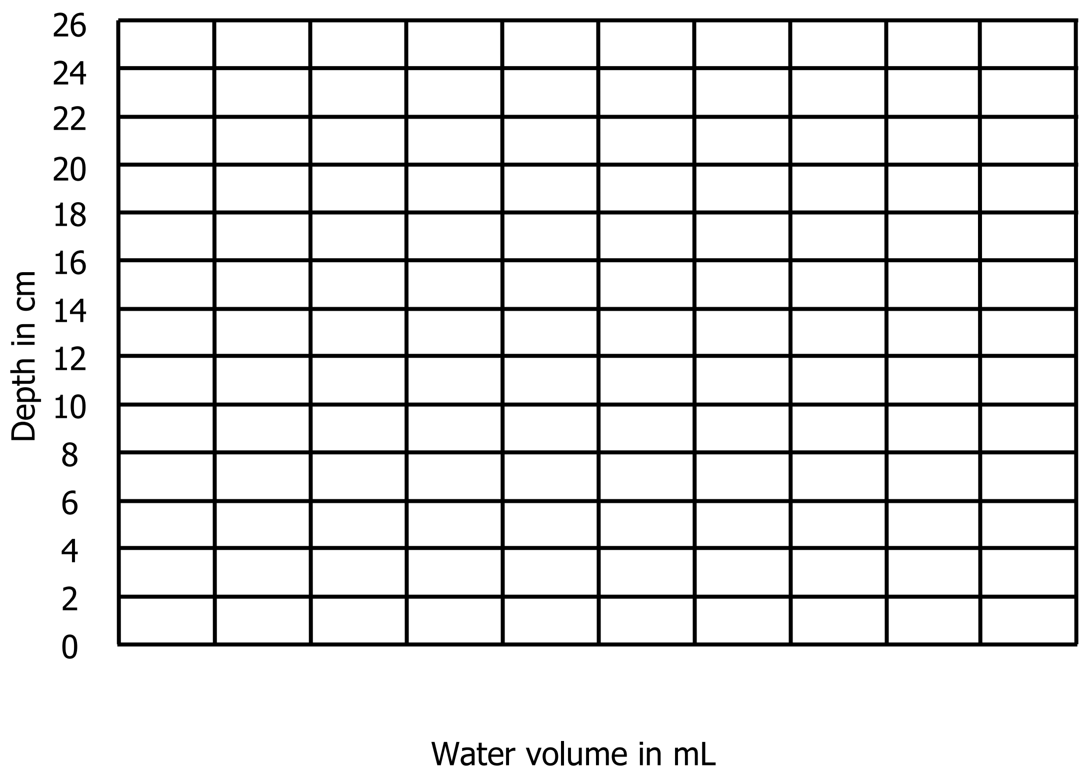
x	y



4. How does your preliminary sketch compare to the actual graph? Explain any discrepancies.

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Liquid Functions

Name________________________________

Date_____________________ Class______

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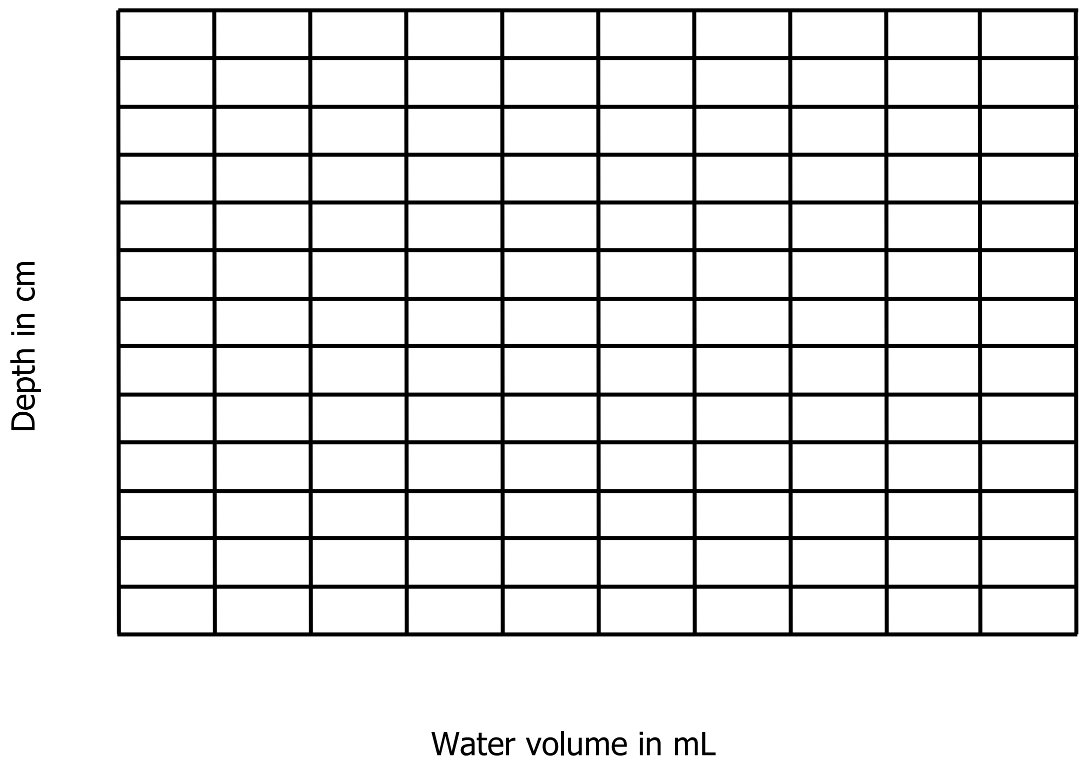
x	y



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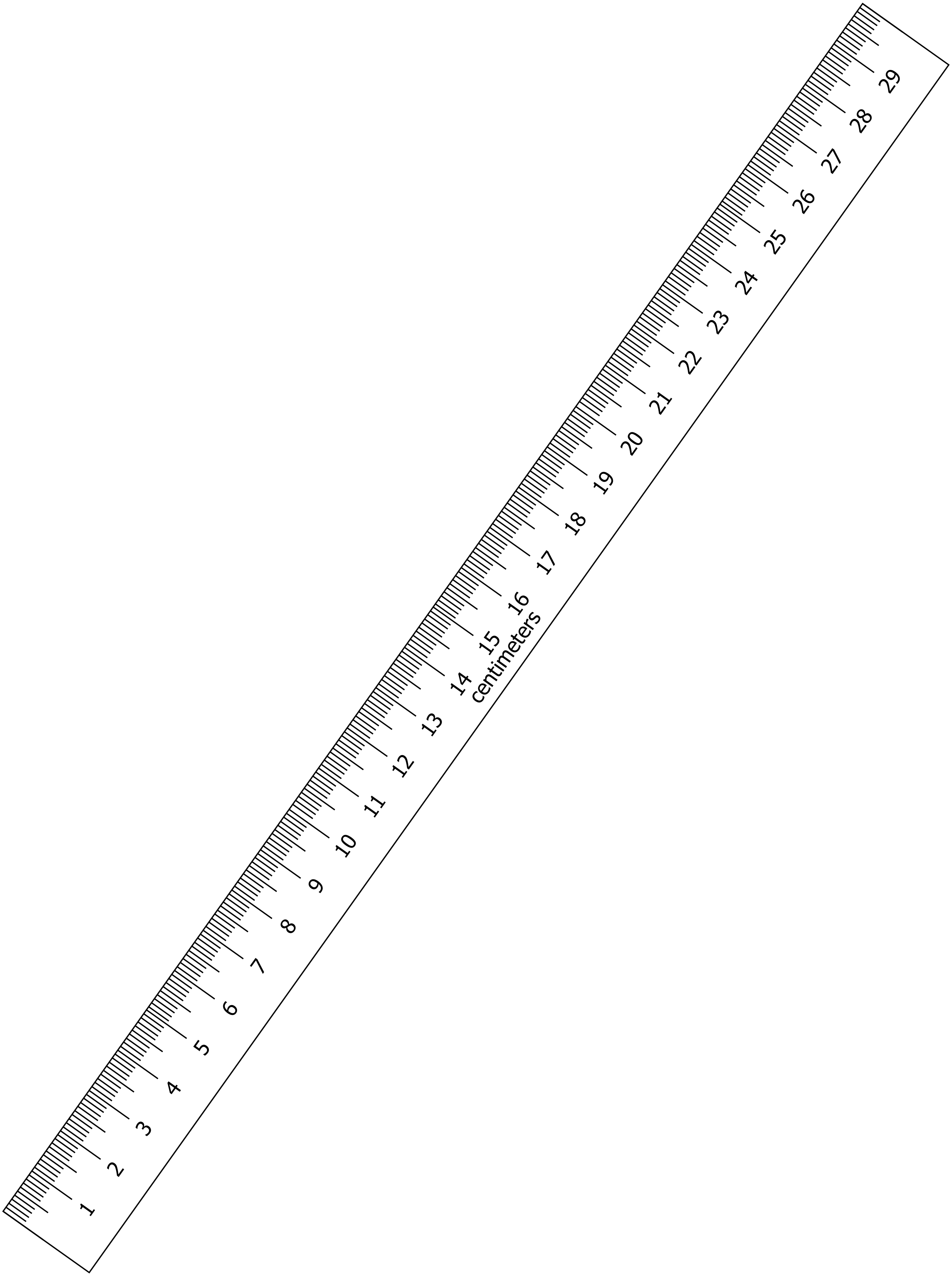
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centimeters

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- *Function Fun: Part 2 – Working with negative and fractional slopes and intercepts*
- *Function Fun: Part 3 – Applications of functions: The Great Yo-yo festival, Banking on Functions, and The Nutty Function*
- *Function Fun: Part 4 – Exploring quadratic functions*
- *Function Fun: Part 5 – the King's Pathway and King's Patio projects. Your students design their own functions!*
- *Milk Carton Apartments: a simple function activity for hands-on learners*
- *Losing Your Marbles: slope, intercept, domain, range, independent and dependent variables, rise and run, fractional slopes, line of fit – all in a visual model.*

Feel free to contact me if you have questions or comments or would like to discuss a staff development training or keynote address at your site.

Happy teaching,
Brad