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Working with Mixed Numbers

$$3\frac{1}{2} + 2\frac{2}{3}$$

Addition, Subtraction,
Multiplication, and Division

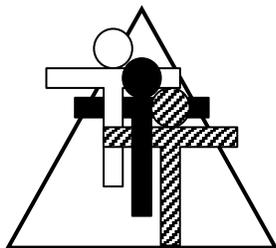
By Brad Fulton

Educator of the Year, 2005

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Known throughout the country for motivating and engaging teachers and students, Brad has co-authored over a dozen books that provide easy-to-teach yet mathematically rich activities for busy teachers while teaching full time for over 30 years. In addition, he has co-authored over 40 teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

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- ◆ 2005 California League of Middle Schools Educator of the Year
- ◆ California Math Council and NCTM national featured presenter
- ◆ Lead trainer for summer teacher training institutes
- ◆ Trainer/consultant for district, county, regional, and national workshops

Author and co-author of mathematics curriculum

- ◆ Simply Great Math Activities series: six books covering all major strands
- ◆ Angle On Geometry Program: over 400 pages of research-based geometry instruction
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Ruler Addition and Subtraction

Teaching Mixed Number Addition and Subtraction Through Measurement

Overview:

This is a powerful model for teaching addition and subtraction of common and mixed fractions. Students will also practice measurement as they learn to use a ruler. It provides a visual model of both addition and subtraction and can be used to lead directly into a traditional algorithm for addition and subtraction of fractions. This activity can be used with students new to the concepts of mixed number addition and subtraction or with students requiring remediation.

Required Materials:

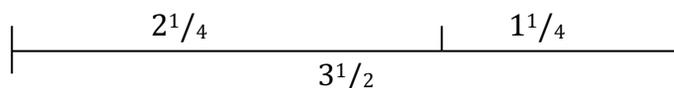
- Paper
- Copies of rulers
- Display of rulers

Optional Materials:

- Activity master

Procedure:

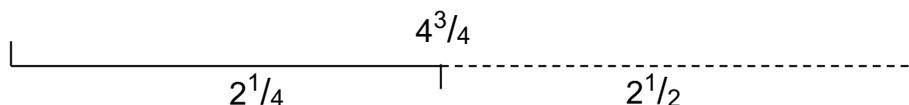
- 1 Have students construct a $2\frac{1}{4}$ inch line. (Using a ruler from the activity master is easier for students than using a standard ruler marked in sixteenths.) Next have them construct a $1\frac{1}{4}$ inch line connected to it and label it as shown:



- 2 Have them measure it to see that it is $3\frac{1}{2}$ inches long and label the answer below the line. Remind them that $3\frac{1}{2}$ is equal to $3\frac{2}{4}$
- 3 Make up additional problems using common and mixed numbers. Use denominators of halves and fourths. (Smaller divisions will lead to confusing answers when students measure inaccurately.) Begin with simple problems and make them more complex as the students' progress. Some samples are:

$$1\frac{1}{2} + 2\frac{1}{4} \quad 1\frac{1}{4} + 1\frac{1}{2} \quad 2\frac{3}{4} + 2\frac{3}{4} \quad 2\frac{3}{4} + 3\frac{3}{4}$$

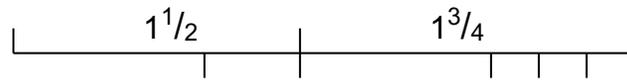
- 4 To model subtraction, have students construct a line that is $4\frac{3}{4}$ inches long. Measure off $2\frac{1}{2}$ inches of it from the right side. How much is left? ($2\frac{1}{4}$ inches).



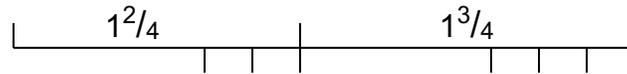
- 5 Have students practice more subtraction problems as they did in step three above.
- 6 After some practice, you can introduce a traditional algorithm for teaching addition and subtraction of fractions. It is a good idea to introduce the algorithm alongside a problem constructed with the ruler. The algorithm can be introduced using the following problem.

$$1\frac{1}{2} + 1\frac{3}{4}$$

Have the students construct the problem using their rulers, marking off the one inch and the half inch, and the one inch and the three quarters.



Next have them mark off the two quarters in the half inch as shown and write:



$$1\frac{2}{4} + 1\frac{3}{4} = 2\frac{5}{4} = 2 + 1\frac{1}{4} = 3\frac{1}{4}$$

This model illustrates the use of a common denominator.

- 7 You may wish to show the students the following problem and compare it to their rulers so they see the value of using an algorithm.

$$2\frac{1}{3} + 2\frac{4}{5} =$$



Journal Prompts:



Explain how you would solve this problem using a ruler:

$$2\frac{1}{4} + 1\frac{3}{4} - 3\frac{1}{2} =$$

What types of denominators are easiest to use with this method? Why?

Homework:



Use one of the activity masters or make up some problems using common or mixed fraction addition and subtraction. Have students solve them by making diagrams using a ruler or by using a traditional algorithm. Try using denominators that are powers of two: 2, 4, and 8. The powers of two can not only be modeled on the ruler, they are conceptually easier fractions for beginning learners. However, younger learners will struggle with measuring accurately to the sixteenth of an inch. A ruler marked in thirds of an inch is provided to illustrate that the method works with denominators that are not powers of two.

Taking a Closer Look:



You can assign problems using more than two mixed numbers as in the journal prompt and have students construct them using rulers or solve them using an algorithm.

Assessment:



If students are working in groups, it will be easy for them to self-assess their work by comparing the lengths of their lines. An answer key for the activity masters is provided here.

Answer Key:

Ruler Fractions: Addition

- 1) $3\frac{3}{4}$ in. 2) $5\frac{1}{4}$ in. 3) $5\frac{1}{2}$ in. 4) $6\frac{1}{4}$ in.
5) $5\frac{3}{8}$ in. 6) $5\frac{5}{8}$ in. 7) $6\frac{1}{8}$ in. 8) 6 in.

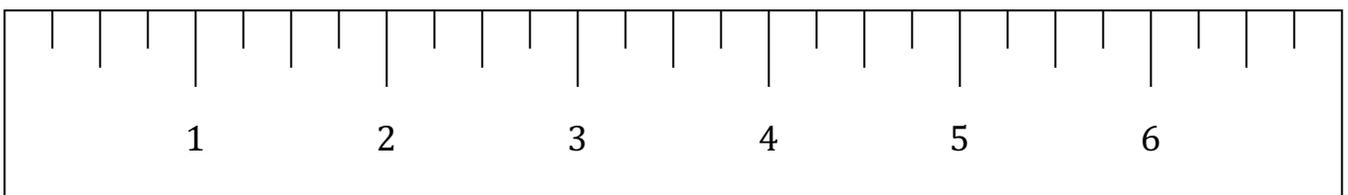
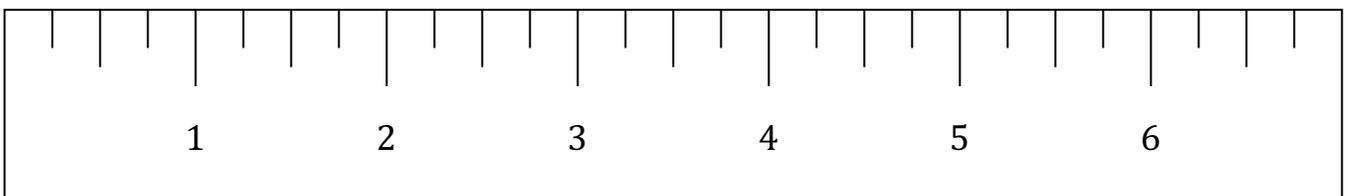
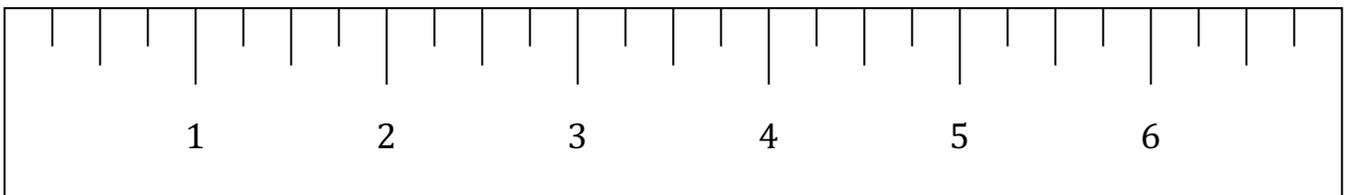
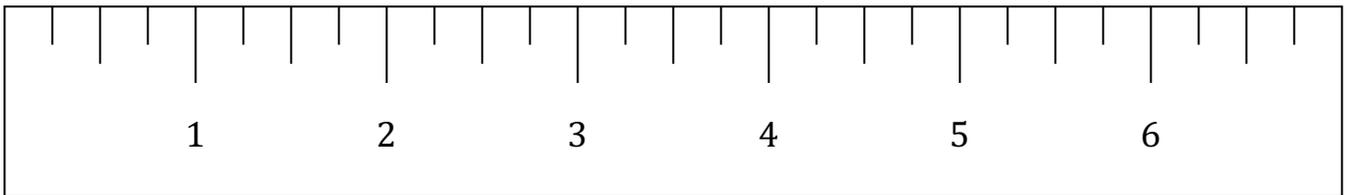
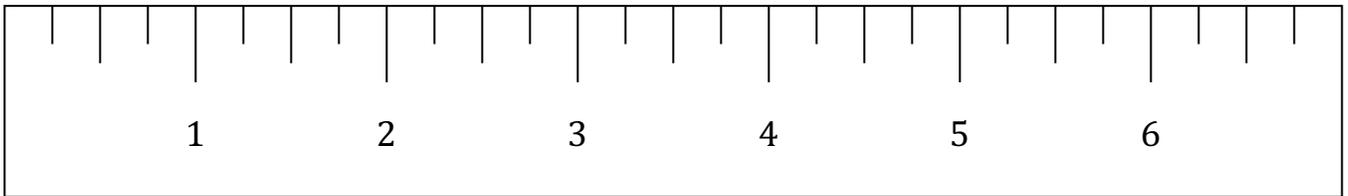
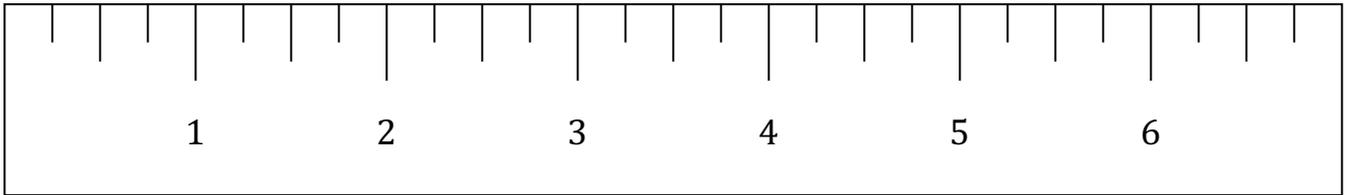
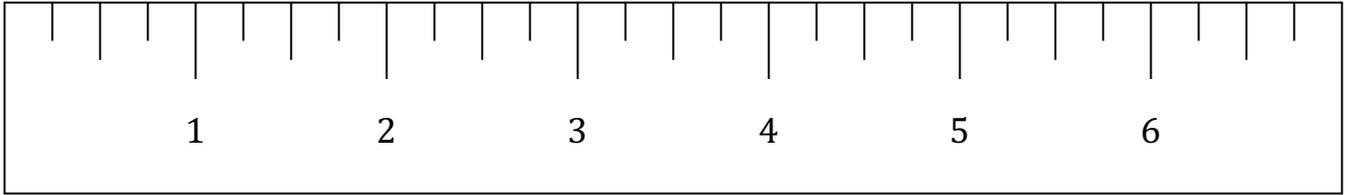
Ruler Fractions: Subtraction

- 1) $3\frac{1}{4}$ in. 2) $2\frac{1}{4}$ in. 3) $2\frac{1}{2}$ in. 4) $\frac{3}{4}$ in.
5) $3\frac{1}{8}$ in. 6) $2\frac{1}{2}$ in. 7) $1\frac{3}{4}$ in. 8) $2\frac{5}{8}$ in.

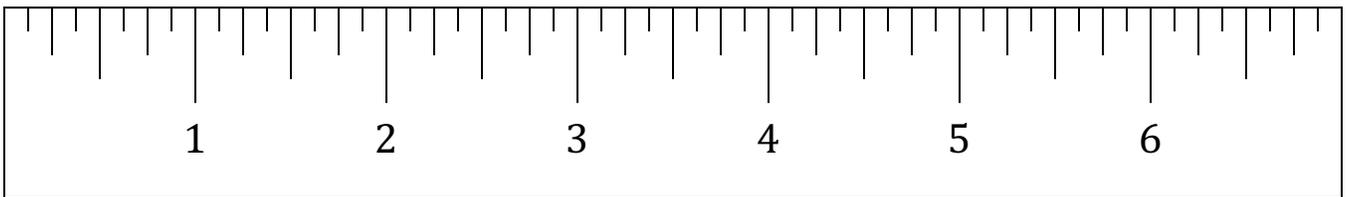
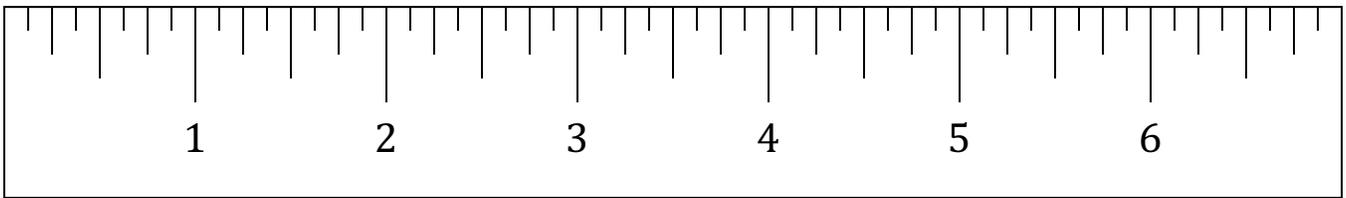
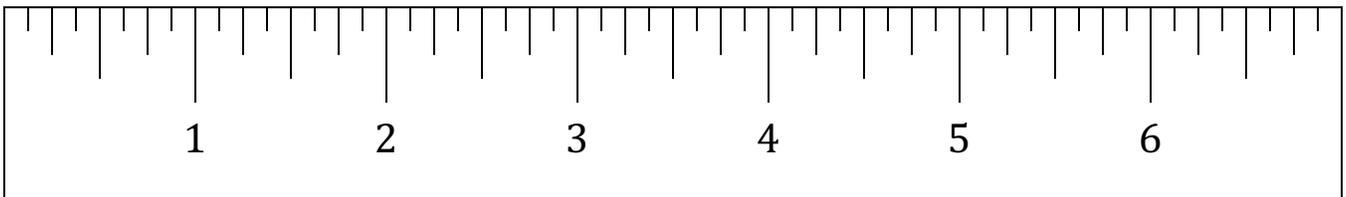
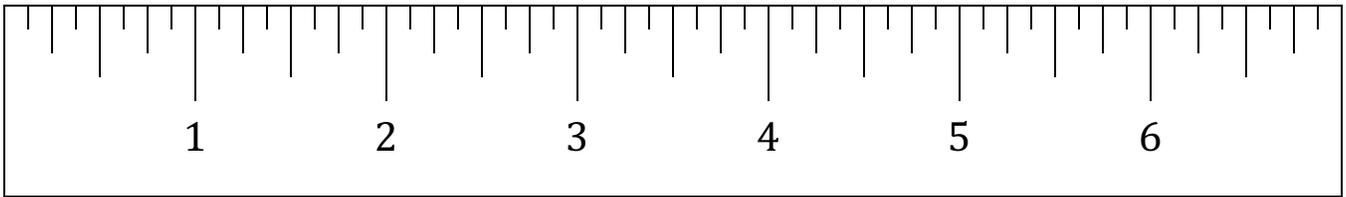
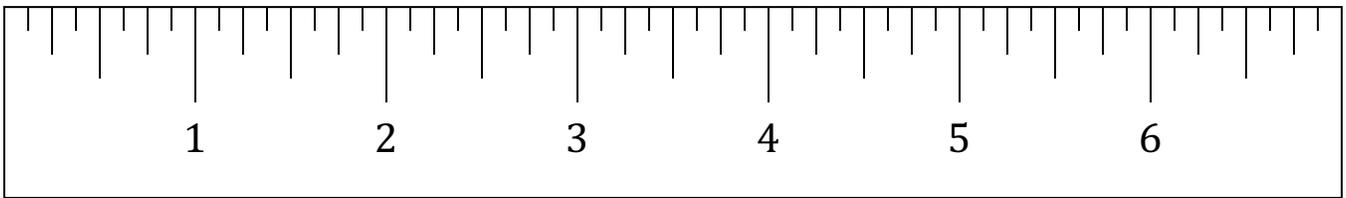
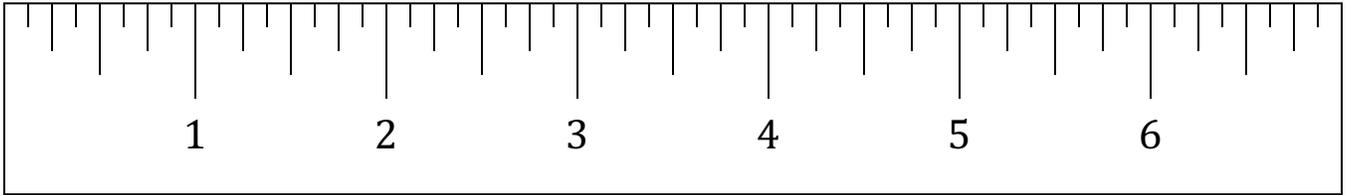
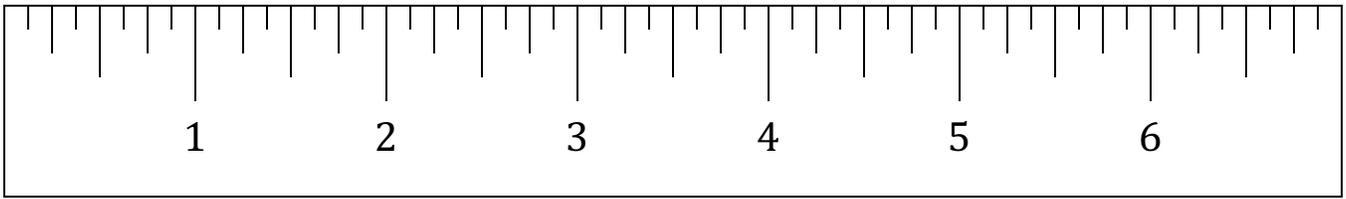
Ruler Fractions: Addition and Subtraction

- 1) $1\frac{3}{4}$ in. 2) $3\frac{3}{4}$ in. 3) $5\frac{1}{4}$ in. 4) $4\frac{3}{4}$ in.
5) 4 in. 6) $2\frac{7}{8}$ in. 7) $2\frac{5}{8}$ in. 8) $3\frac{3}{8}$ in.

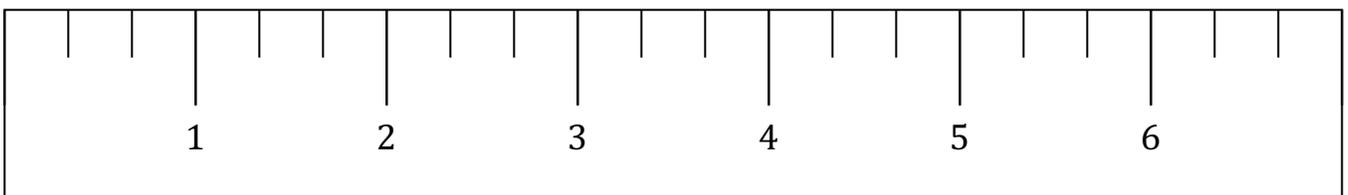
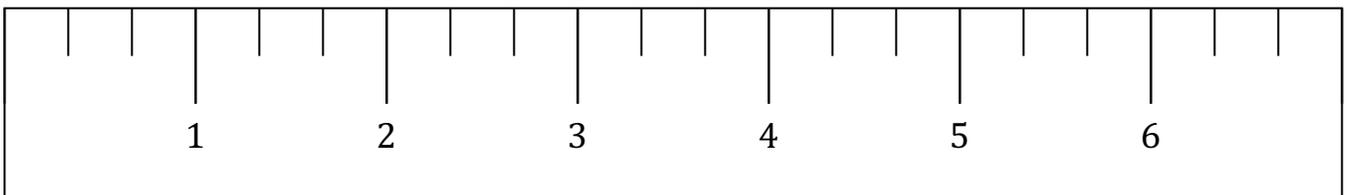
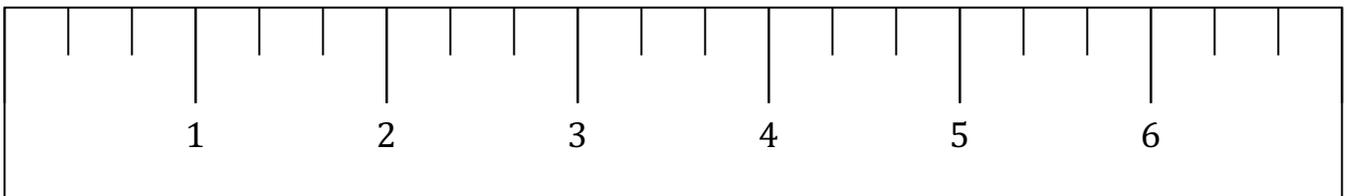
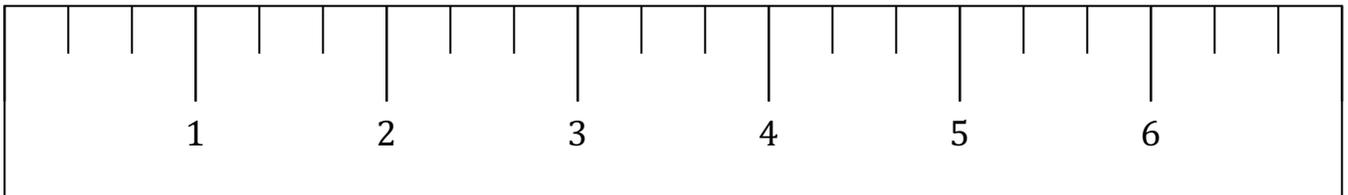
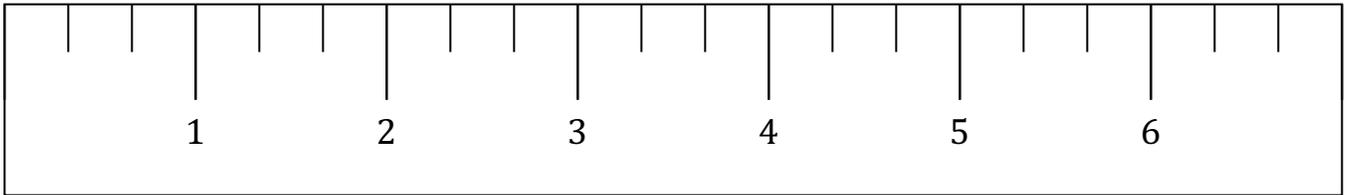
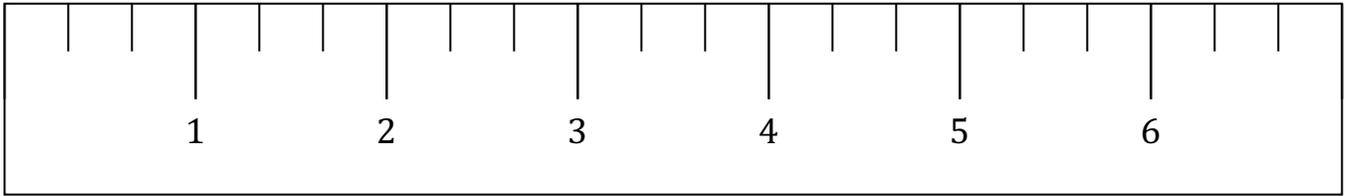
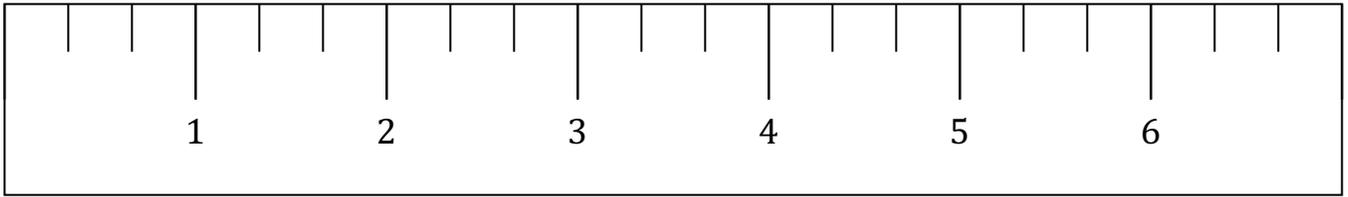
Copy master:
 $\frac{1}{2}$ and $\frac{1}{4}$ inch rulers



Copy master:
 $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ inch rulers



Copy master:
 $\frac{1}{3}$ inch rulers



Ruler Fractions: Addition

Name _____

Use your ruler to add the problems below. Diagram each problem on the line and write your answer after the problem.

$$1\frac{1}{4} + 2\frac{1}{2} =$$

$$2\frac{3}{4} + 2\frac{1}{2} =$$

$$2\frac{3}{4} + 2\frac{3}{4} =$$

$$2\frac{5}{8} + 3\frac{5}{8} =$$

$$1\frac{3}{4} + 3\frac{5}{8} =$$

$$1\frac{3}{4} + 3\frac{7}{8} =$$

$$2\frac{1}{2} + 1\frac{1}{2} + 2\frac{1}{8} =$$

$$2\frac{1}{4} + 2\frac{1}{8} + 1\frac{5}{8} =$$

Ruler Fractions: Subtraction

Name _____

Use your ruler to subtract the problems below. Diagram each problem on the line and write your answer after the problem.

$$5\frac{1}{2} - 2\frac{1}{4} =$$

$$4\frac{3}{4} - 2\frac{1}{2} =$$

$$4\frac{1}{4} - 1\frac{3}{4} =$$

$$3\frac{1}{2} - 2\frac{3}{4} =$$

$$5\frac{1}{4} - 2\frac{1}{8} =$$

$$6\frac{1}{8} - 3\frac{5}{8} =$$

$$4\frac{1}{8} - 2\frac{7}{8} =$$

$$5\frac{1}{4} - 2\frac{5}{8} =$$

Ruler Fractions:

Name _____

Addition and Subtraction

Use your ruler to solve the problems below. Diagram each problem on the line and write your answer after the problem.

$$2\frac{1}{2} + 1\frac{1}{2} - 2\frac{1}{4} =$$

$$2\frac{1}{2} - 1\frac{1}{2} + 2\frac{3}{4} =$$

$$3\frac{3}{4} - 1\frac{1}{4} + 2\frac{3}{4} =$$

$$4\frac{1}{4} - 1\frac{3}{4} + 2\frac{1}{4} =$$

$$3\frac{1}{4} + 1\frac{1}{2} - \frac{3}{4} =$$

$$2\frac{1}{8} + 2\frac{3}{8} - 1\frac{5}{8} =$$

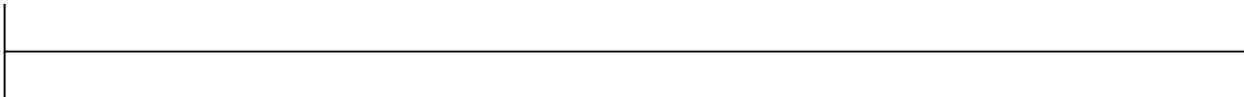
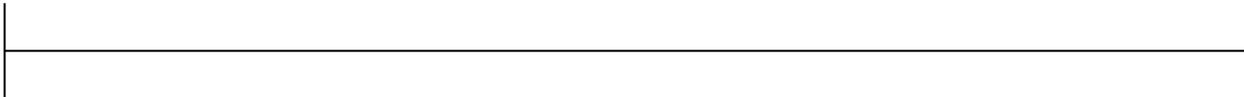
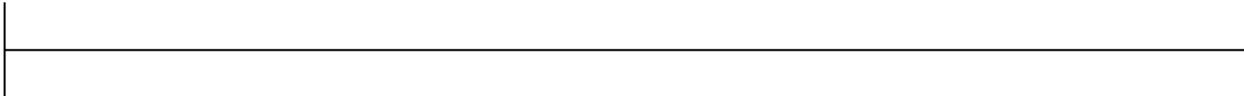
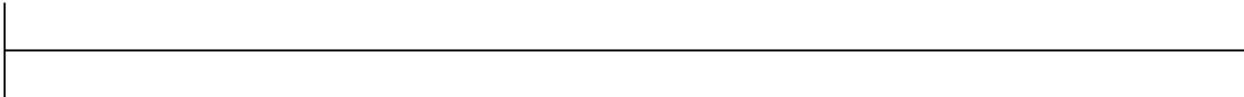
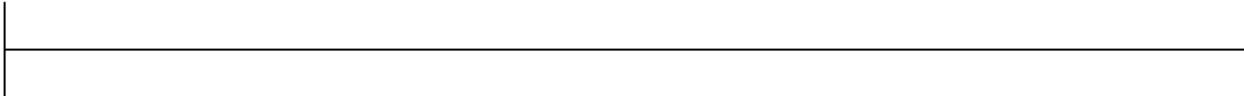
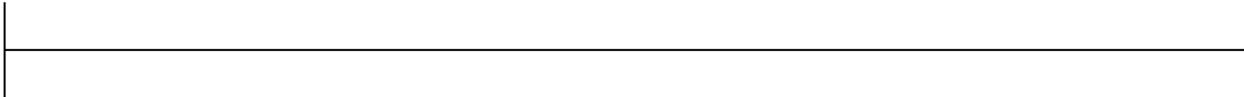
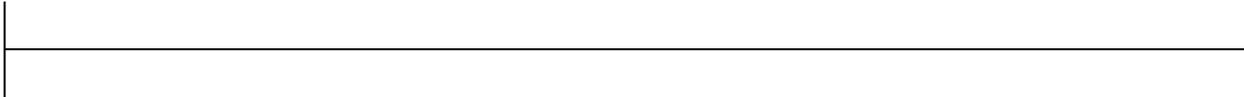
$$5\frac{1}{2} - 1\frac{3}{4} - 1\frac{1}{8} =$$

$$6\frac{1}{8} - 5\frac{1}{2} + 2\frac{3}{4} =$$

Ruler Fractions

Name _____

Use your ruler to solve the problems below. Diagram each problem on the line and write your answer after the problem.



Fraction Arrays

Teaching Mixed Number Multiplication and Division with Arrays

Overview:

Powerful, but easy. Those words describe this activity. It's tough to find a model for something so confusing to students as multiplying mixed numbers, but here is one that not only powerfully illustrates the process, it also is easy for students to do. The model leads into the traditional algorithm for multiplying common and mixed fractions. It can be used as initial instruction in multiplication and division of mixed numbers or with students in an intervention setting.

Required Materials:

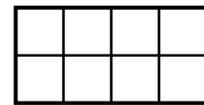
Centimeter grid paper masters

Optional Materials:

Activity master

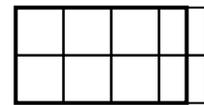
Procedure:

- 1 Give each student a sheet of centimeter grid paper with $\frac{1}{2}$ centimeter divisions. Explain that each bold square is one square centimeter. Have them outline a 2 cm. by 4 cm. rectangle. Ask them to find the area of the rectangle; that is, how many 1 cm. squares are inside? If you are using standard centimeter grid paper, have them outline the rectangle to show the eight square centimeters as shown. Have the students write the problem and answer, $2 \times 4 = 8$, below the rectangle as shown.



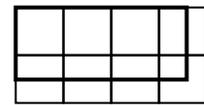
$$2 \times 4 = 8$$

- 2 Next have them outline a 2 by $3\frac{1}{2}$ centimeter rectangle and find its area. They will see it has seven square centimeters—six whole squares and two half squares. The students should write $2 \times 3\frac{1}{2} = 7$ below the rectangle.



$$2 \times 3\frac{1}{2} = 7$$

- 3 Now have them construct a $1\frac{1}{2}$ by $3\frac{1}{2}$ centimeter rectangle and find its area. They will be able to count the squares to see that $1\frac{1}{2} \times 3\frac{1}{2} = 5\frac{1}{4}$ square centimeters. There are three whole squares, four half squares making two more whole squares for a total of five, and one quarter square. Have them write this below the rectangle.



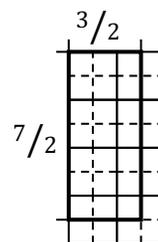
$$1\frac{1}{2} \times 3\frac{1}{2} = 5\frac{1}{4}$$

- 4 Have them solve a few more sample problems such as these:

$$2\frac{1}{2} \times 1\frac{1}{2} \qquad 1\frac{1}{2} \times 3\frac{1}{2} \qquad \frac{1}{2} \times 4\frac{1}{2}$$

$$2\frac{1}{2} \times 2\frac{1}{2} \qquad 1\frac{1}{4} \times 4$$

- 5 You may introduce a more traditional algorithm for multiplying mixed fractions. To do this, have the students sketch this example: $3\frac{1}{2} \times 1\frac{1}{2}$. Next, they should label the rectangle using improper fractions as shown.



They then can see that the problem is now $\frac{7}{2} \times \frac{3}{2}$. They can also see that there are twenty-one fourths in the rectangle. You can then simply show them that multiplying across easily provides the same answer:

$$\frac{7}{2} \times \frac{3}{2} = \frac{21}{4} = 5\frac{1}{4}$$

- 6 A $\frac{1}{2}$ by $\frac{1}{3}$ centimeter grid is provided also for additional practice of multiplying different denominators such as $2\frac{1}{2}$ by $2\frac{2}{3}$.



Journal Prompts:



A recipe for a one-layer cake calls for $2\frac{1}{2}$ cups of flour. You want to make a three-layer cake. Explain how to find out the amount of sugar you will need.

A recipe for a two-layer cake calls for $1\frac{1}{2}$ cups of sugar. You want to make a three-layer cake. Explain how to find out the amount of sugar you will need.

Write a note to a friend explaining your new way to multiply mixed numbers.

Good Tip!



If students struggle with the small regions represented on the centimeter paper, enlarge a section of the master on a copy machine. Increasing the size by 254% will convert the centimeter grid to inches!

Homework:



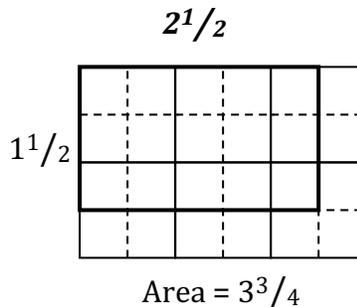
Assign some problems involving multiplying mixed numbers or use one of the activity masters from this chapter. Have students solve them using the traditional

algorithms or by sketching them. If you are going to have students sketch them, use only halves and thirds.

Taking a Closer Look:



Division of mixed numbers can be explored using this method although it is challenging. Ask students to construct a rectangle with an area of $3\frac{3}{4}$ square centimeters and one side of $1\frac{1}{2}$ centimeters. The other side is $2\frac{1}{2}$ cm. long.



Assessment:



If students work in groups, they can monitor their progress and results. Call on students as they work to see if the areas of their rectangles are correct. If you use the activity masters, you can check them with the answer key that follows.

Answer Key:

Multiplying Halves

- | | | | |
|----|-----------------|----|-----------------|
| 1 | $\frac{1}{4}$ | 2 | $7\frac{1}{2}$ |
| 3 | $3\frac{3}{4}$ | 4 | $11\frac{1}{4}$ |
| 5 | $8\frac{3}{4}$ | 6 | $12\frac{1}{4}$ |
| 7 | $10\frac{1}{2}$ | 8 | $2\frac{1}{4}$ |
| 9 | 10 | 10 | $2\frac{1}{2}$ |
| 11 | $2\frac{3}{4}$ | 12 | $1\frac{3}{4}$ |

Multiplying Thirds

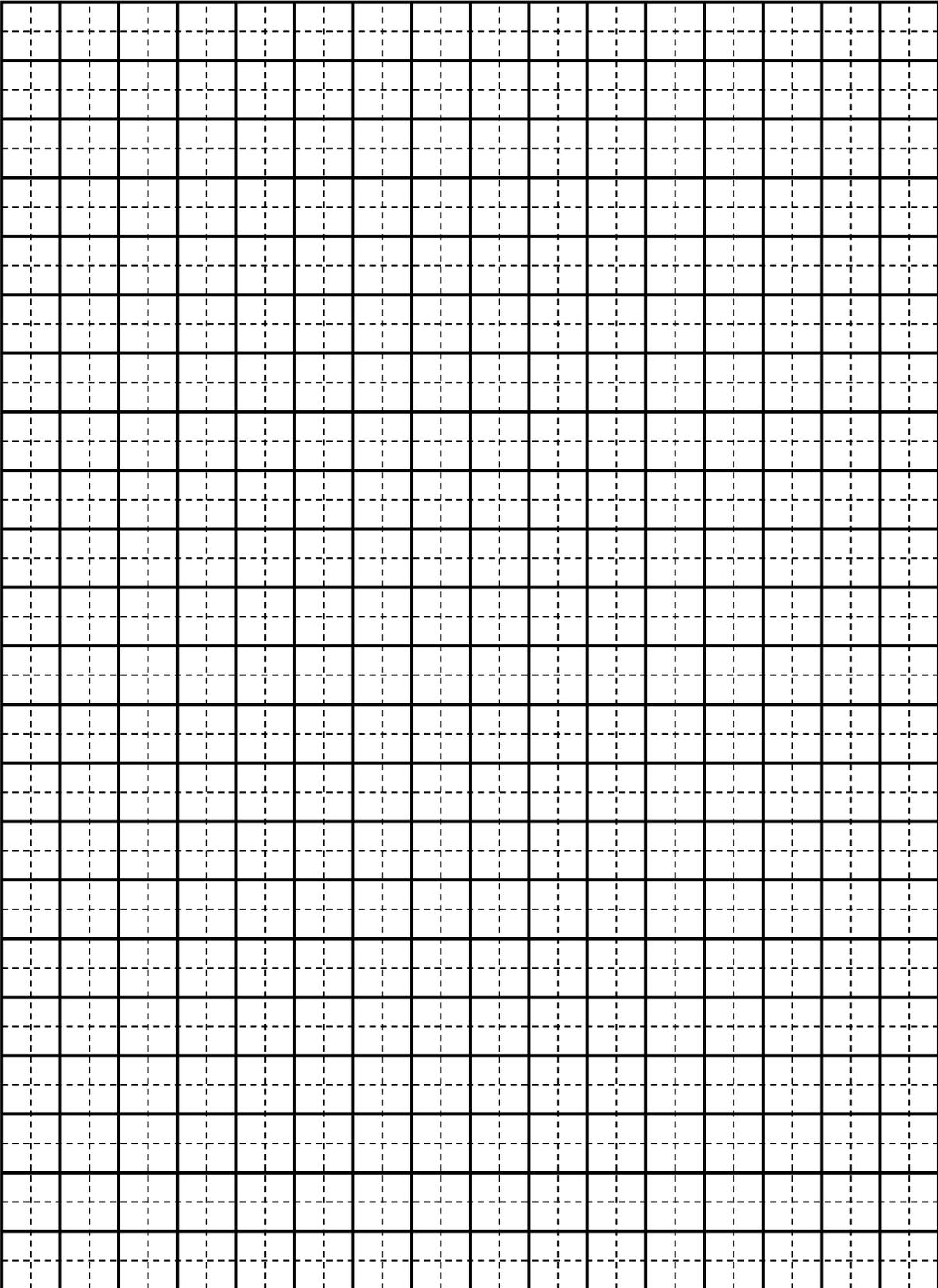
- | | | | |
|---|----------------|---|----------------|
| 1 | $\frac{1}{9}$ | 2 | $5\frac{4}{9}$ |
| 3 | 5 | 4 | $6\frac{2}{9}$ |
| 5 | $8\frac{2}{3}$ | 6 | $8\frac{8}{9}$ |

- | | | | |
|----|--------|----|--------|
| 7 | $27/9$ | 8 | $14/9$ |
| 9 | 10 | 10 | 4 |
| 11 | $35/9$ | 12 | $2/9$ |

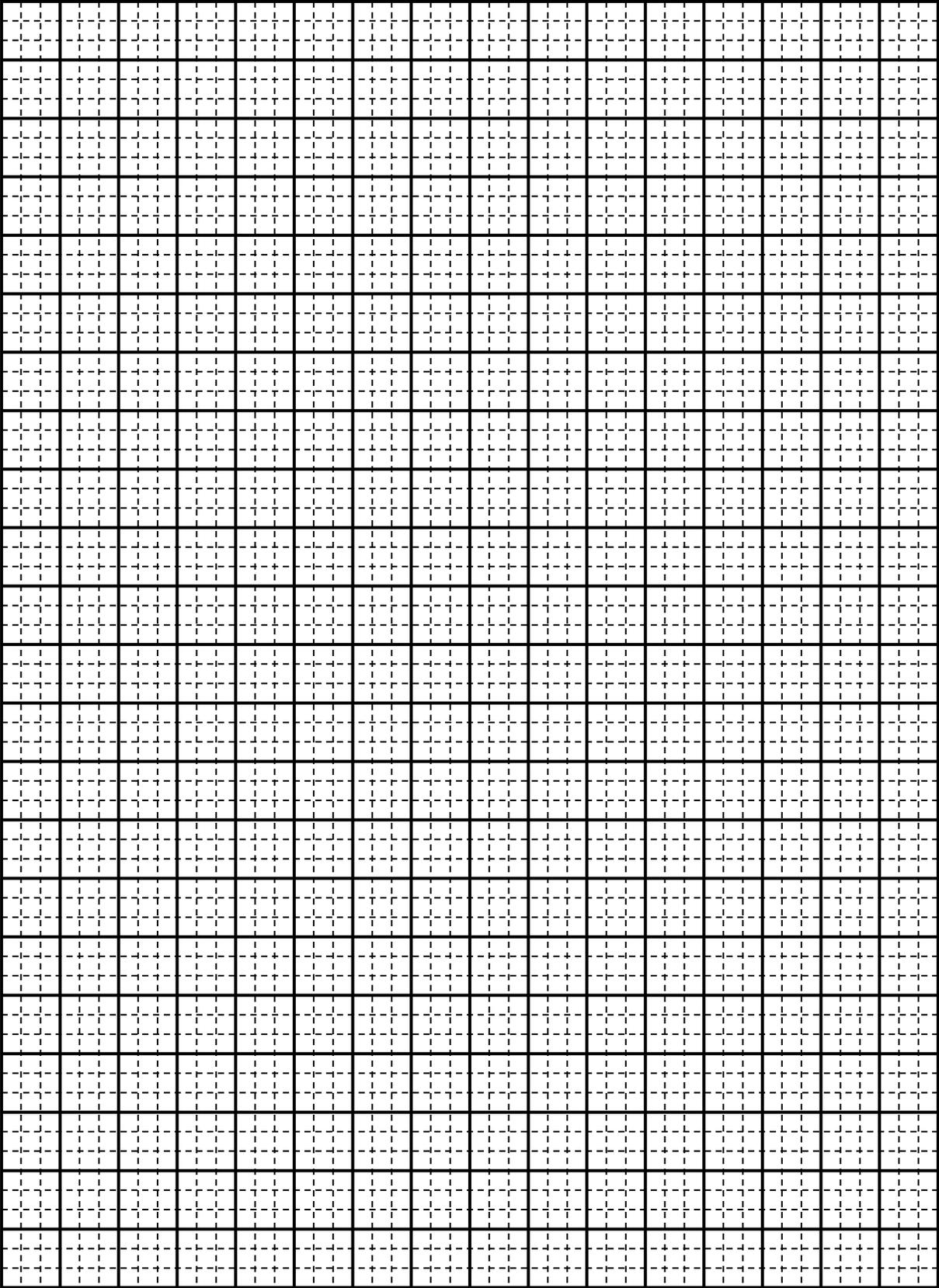
Improper Fractions

- | | | | |
|----|------------------------|----|-----------------------------|
| 1 | $15/4 = 3\frac{3}{4}$ | 2 | $18/4 = 9/2 = 4\frac{1}{2}$ |
| 3 | $49/4 = 12\frac{1}{4}$ | 4 | $27/4 = 6\frac{3}{4}$ |
| 5 | $40/4 = 10/1 = 10$ | 6 | $8/4 = 2/1 = 2$ |
| 7 | $56/9 = 6\frac{2}{9}$ | 8 | $25/9 = 2\frac{7}{9}$ |
| 9 | $28/9 = 3\frac{1}{9}$ | 10 | $48/9 = 5\frac{3}{9}$ |
| 11 | $22/9 = 2\frac{4}{9}$ | 12 | $100/9 = 11\frac{1}{9}$ |

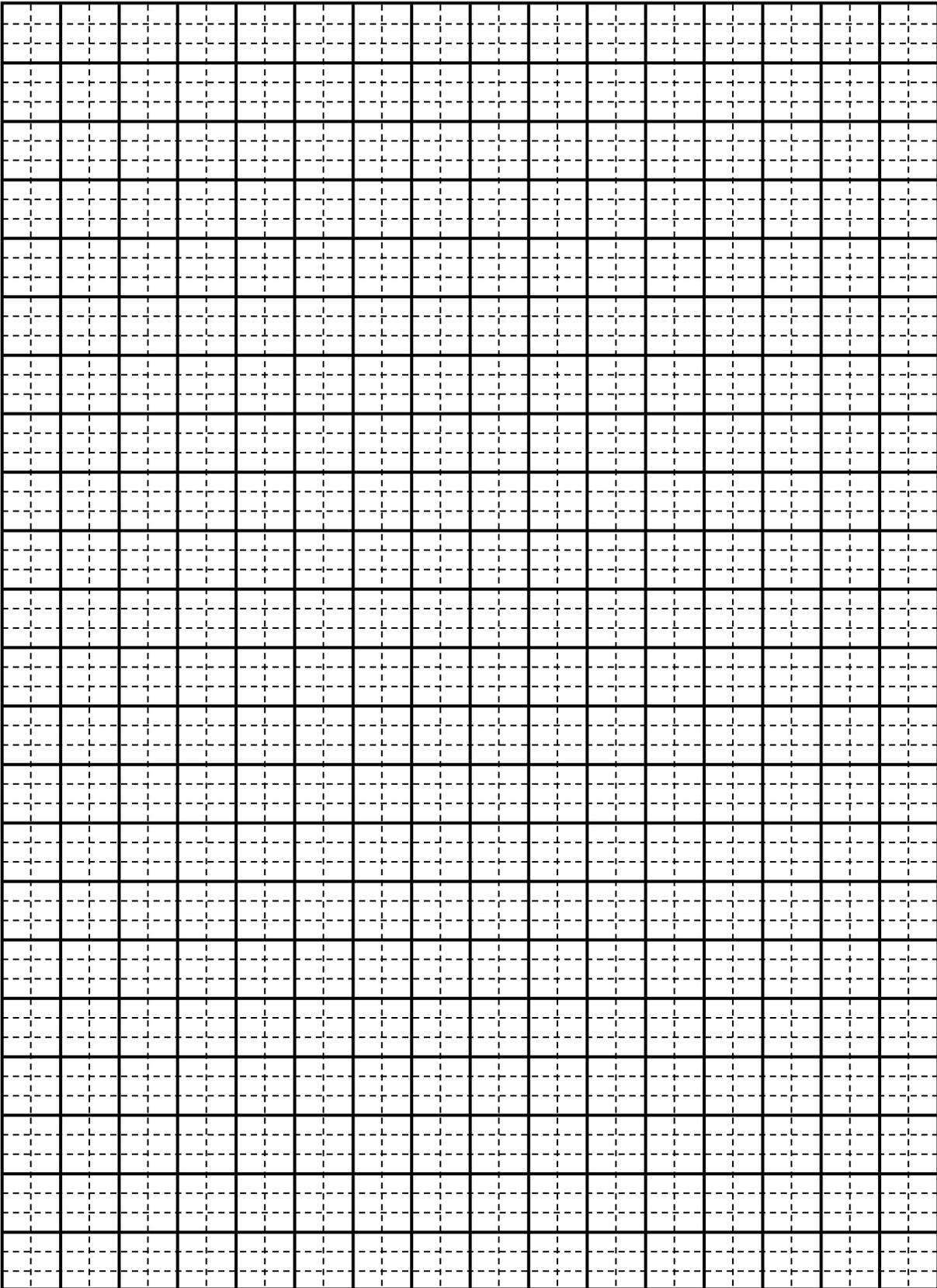
Centimeter grid paper: $\frac{1}{2}$ cm



Centimeter grid paper: $\frac{1}{3}$ cm



Centimeter grid paper: $\frac{1}{2}$ by $\frac{1}{3}$



Fraction Arrays: Multiplying Halves Name_____

Use the $\frac{1}{2}$ cm grid paper to sketch each problem. Then write the answer next to the problem. Simplify answers when possible.

1) $\frac{1}{2} \times \frac{1}{2} =$

2) $2\frac{1}{2} \times 3 =$

3) $2\frac{1}{2} \times 1\frac{1}{2} =$

4) $4\frac{1}{2} \times 2\frac{1}{2} =$

5) $2\frac{1}{2} \times 3\frac{1}{2} =$

6) $3\frac{1}{2} \times 3\frac{1}{2} =$

7) $3\frac{1}{2} \times 3 =$

8) $1\frac{1}{2} \times 1\frac{1}{2} =$

9) $4 \times 2\frac{1}{2} =$

10) $\frac{1}{2} \times 5 =$

11) $\frac{1}{2} \times 5\frac{1}{2} =$

12) $3\frac{1}{2} \times \frac{1}{2} =$

Fraction Arrays: Multiplying Thirds Name_____

Use the $\frac{1}{3}$ cm grid paper to sketch each problem. Then write the answer next to the problem. Simplify answers when possible.

1) $\frac{1}{3} \times \frac{1}{3} =$

2) $2\frac{1}{3} \times 2\frac{1}{3} =$

3) $1\frac{2}{3} \times 3 =$

4) $2\frac{2}{3} \times 2\frac{1}{3} =$

5) $4\frac{1}{3} \times 2 =$

6) $3\frac{1}{3} \times 2\frac{2}{3} =$

7) $1\frac{2}{3} \times 1\frac{2}{3} =$

8) $\frac{1}{3} \times 4\frac{1}{3} =$

9) $3\frac{1}{3} \times 3 =$

10) $6 \times \frac{2}{3} =$

11) $5\frac{1}{3} \times \frac{2}{3} =$

12) $\frac{1}{3} \times \frac{2}{3} =$

Fraction Arrays: Improper Fractions Name_____

Use the $\frac{1}{2}$ cm and $\frac{1}{3}$ cm grid paper to sketch each problem. Then write the answer next to the problem. Simplify answers when possible.

1) $\frac{3}{2} \times \frac{5}{2} =$

2) $\frac{3}{2} \times \frac{6}{2} =$

3) $\frac{7}{2} \times \frac{7}{2} =$

4) $\frac{3}{2} \times \frac{9}{2} =$

5) $\frac{8}{2} \times \frac{5}{2} =$

6) $\frac{8}{2} \times \frac{1}{2} =$

7) $\frac{8}{3} \times \frac{7}{3} =$

8) $\frac{5}{3} \times \frac{5}{3} =$

9) $\frac{7}{3} \times \frac{4}{3} =$

10) $\frac{6}{3} \times \frac{8}{3} =$

11) $\frac{2}{3} \times \frac{11}{3} =$

12) $\frac{10}{3} \times \frac{10}{3} =$

If you liked this activity, you might also like some of the other lessons available in my TeachersPayTeachers store. Simply search for "**Teacher to Teacher Press**".

You can also find many free and inexpensive resources on my personal website, www.tttpress.com. **Be sure to subscribe to receive monthly newsletters, blogs, and FREE activities.**

Similar activities include:

- *Number Line: Elementary version* – An engaging strategy to help students develop number sense with fractions, decimals, and percents.
- *Fast Facts and Fractions* – My most popular handout shows how I helped my struggling students master their multiplication facts and all four fraction operations in only 5 minutes a day!
- *Fraction House Plans* – A great way to connect all three fraction representations: common, decimal, and percent, to a geometry foundation.
- *Fraction Strips* – Develop key conceptual connections with common fraction representations as students see how to simplify and find equivalent fractions in this hands-on activity.

Feel free to contact me if you have questions or comments or would like to discuss a staff development training or keynote address at your site.

Happy teaching,
Brad