MODERATOR'S GUIDE

This document serves as a guide for the educator who wishes to conduct a Teacher to Teacher Press workshop featuring Brad Fulton. While you can't replace the interactive nature of his live workshop, this DVD format provides some distinct advantages for your school or district. By purchasing these DVD's you own this workshop and can conduct it over and over. As you hire new staff, they can benefit from the same training your veteran teachers have enjoyed. These DVD's are also inexpensive when you compare them to the cost of hiring a consultant. You can also take as much time as you want to proceed through the series. You may wish to view five or so of them in one day of staff development. On the other hand, some districts find these to be great assets for department meetings and collaboration time. It's your choice, and we encourage you to adapt these to fit the needs of your site and your staff.

Here is the format we suggest for viewing and using the materials.

1. First, we suggest that you, the moderator, view the DVD before showing it to your staff. This will help you direct discussions and planning time in the most effective manner possible
2. Prior to the workshop, print out any handouts on the CD so these will be ready for your teachers. You have purchased the rights to make as many copies as needed to distribute to the educators at your site. (Making copies of the handout or DVD in any manner for personnel beyond your site is a violation of copyright law.)
3. When your staff gathers to view the DVD, begin with a discussion. List concerns and issues that your students face in dealing with the mathematical topic that will be addressed.
4. Watch the DVD, and then discuss the accompanying questions. This can be done informally or formally through oral or written formats.
5. If you wish, allow planning time for your staff so they can discuss the implementation of these activities and teaching strategies.

If you follow this model, you will probably get through about four or five DVD's in a day of staff development. Each viewing, discussion, and planning cycle will take approximately one hour. Some will take longer and some will be shorter.

And remember, I am always available if you have questions. You can reach me through the contact information below:

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Happy teaching,

Brad

FAST FACTS AND FRACTIONS

Initial discussion:

1. What are the issues that our students face with their multiplication facts?
2. What factors cause them to struggle?
3. What have we tried so far?
4. What are the issues that confront them at each grade level regarding fraction, decimal, and percent representations?
5. How have we taught this in the past?
6. What worked well, and what didn't work?

Post-viewing discussion:

1. Which patterns in the multiplication table were new to you?
2. Are there other patterns you can suggest that will help students learn these facts?
3. How can we best implement this strategy for teaching the multiplication facts?
4. Can we let our students use grid paper as scratch paper for the state test so they can make their own multiplication table? (Check with your state department of education. In California, grid paper can be used.)
5. Which fraction, decimal, and percent strategies will work best with our students?
6. How will these be implemented at different grade levels? What are the standards for these concepts at each grade level? (Consult your state framework for this answer.)
7. How do we balance procedural teaching with conceptual approaches on fractions, decimals, and percents?

If desired, allow educators collaboration time to schedule the implementation strategies.

HUNDREDS MAGIC

Initial discussion:

1. What algebra skills do students possess at each grade level currently?
2. What obstacles hinder their algebraic thinking?
3. How is algebra currently introduced?
4. What precursor skills are needed for later algebraic success?
5. At what grade level are the precursor algebraic skills introduced?

Post-viewing discussion:

1. Hundreds Magic has three levels of implementation:

    a. Discovery of number patterns

    b. Linguistic approach to rationalizing why these patterns exist.

    c. Algebraic proofs showing why these patterns work.

    How should this *conceptual layering* be distributed across the grade levels? ("Conceptual layering" refers to the incremental development of a concept so that all students can get enter the lesson and be taken up to an appropriate level of mathematical rigor. You might think of it as slowing down the train so all passengers can board safely and gradually taking them up to speed without losing passengers along the way.)

2. What arithmetic skills can be reinforced by exploring patterns in the hundreds chart?
3. Which chart is most appropriate for your students?
4. What trick or pattern most intrigues you? Which one will be most appropriate for your students?
5. Students will often be so engaged in this exploration that they discover tricks and patterns of their own. What algebra tricks can you find in the chart?
6. Can you explain linguistically and prove algebraically why your trick works?

If desired, allow educators collaboration time to schedule the implementation strategies. They may also want time to discover patterns that are appropriate the their grade level assignment.

INTEGER STRATEGIES

Initial discussion:

1. What integer standards are taught at each grade level?
2. What percent of the state test requires proficiency with integers?
3. What struggles do students face with each of these concepts?
4. How are integers presented conceptually in our school?
5. What procedures have we tried?
6. What successes have we had with those procedures?
7. How do the separate integer procedures overlap in a student's thinking to cause problems? For example, many students apply the "two negatives make a positive" rule to all integer operations.

Post-viewing discussion:

1. What are the pitfalls of the chips approach to teaching integer operations?
2. Which strategies will be most beneficial to your students?
3. What are the strengths and weaknesses of each approach?
4. Which are most conceptual?
5. Which strategies are more procedural?
6. How will these be distributed among grade levels at your site?
7. Will you present one of these strategies to your students, or will you lean toward an eclectic blend that allows students to choose their preferred strategy?
8. Some of the strategies will appeal to visual and kinesthetic learners. Other will appeal more to students who tend to recognize patterns and favor rules and procedures. Can you identify students in your classes who will likely choose one strategy over another?

If desired, allow educators collaboration time to schedule the implementation strategies.

LEO'S PATTERNS

Initial discussion:

1. What knowledge do your students currently have about the Fibonacci sequence?
2. What types of numbers or expressions are added and subtracted at your grade level: positive whole numbers, decimals, fractions, integers, monomials and binomials, or something else?
3. What struggles do students face with these addition and subtraction concepts?
4. What is your state or districts belief about teaching the "Guess and Check" strategy of problem solving?
5. How do students encounter practice in solving equations at your school or district?

Post-viewing discussion:

1. This activity is another example of *conceptual layering*. (Refer to "Hundreds Magic" for a definition of this term.) How are students gradually led from the simple task of adding 1 + 1 to solving equations in "Leo's Patterns"?
2. Where would you begin this activity with your students? What types of numbers or expressions would you use?
3. Where would you exit this lesson for your students? How far would you take them?
4. How might your students view subtraction in the model where the sequence is begun on the right and developed toward the left? Would they think of this as subtraction or as a missing addend model?
5. Which of those approaches might be more intuitive for them?
6. What are the advantages and disadvantages of these two approaches?
7. What are your thoughts about using an answer bank with the worksheets? Do you think this would help or hinder your students' learning?

If desired, allow educators collaboration time to schedule the implementation strategies.

<u>MATH PROJECTS</u>

Initial discussion:

1. What projects are students currently doing in their math classes?

2. What role does art and visual representation play in our math curriculum?

3. What would be the advantages of such an approach.

Post-viewing discussion:

1. How do these projects illustrate high-level mathematical thinking?

2. How do they function as a teaching tool?

3. How do they function as an assessment tool?

4. How can these be incorporated into your teaching schedule?

5. Which projects are best suited to your students?

6. How will you address the issue of finding time to do them?

7. How will you address materials management?

8. Many of the projects shown to you in this two DVD set were made by students who had serious obstacles in their math education. Some were in the process of learning English; some didn't receive an 8^{th} grade diploma; some had struggles at home or other issues that affected their education. On the other hand, some of these projects were made by our top students and our class president. The beautiful thing is that you can't tell one student from another by their work. What does this mean from an educational perspective?

9. It could be argued that you may not have the time to do projects. It could also be argued that we can't afford not to do them, given the quality work students do when allowed to express their learning this way. How was this issue addressed in the DVD?

10. Math projects allow students to demonstrate understanding in new and advanced ways. We call this *the emancipation of learning*. How do projects set students free to truly show what they know?

If desired, allow educators collaboration time to schedule the implementation strategies. Teachers can also use this time to think of ways to create new projects geared to the standards they teach at their grade level.

MENU MATH

Initial discussion:

1. What algebraic concepts are addressed at the various grade levels?
2. What concepts are mandated by the state at each grade level?
3. How have we taught these concepts in the past?
4. What successes and failures have students experienced?
5. What struggles do they face in learning these algebraic concepts.

Post-viewing discussion:

1. Which of these activities is best suited to the students you teach?
2. How can these activities be adapted for upper grade students?
3. How can they be adapted for lower grade students? For example, the "Al Jibber's Pet Store" approach has been used successfully with second graders. The prices vary, but the concepts taught are identical.
4. Once you introduce these lessons, where will you go next with your students?
5. How will you transition students from this intuitive approach to the way these concepts are presented in your textbook or on your state test?
6. Think of these activities as a foundation upon which we build a structure of algebraic thinking. As we compare this teaching model to the building of a structure, what role does the foundation play?
7. What extensions can you suggest for each of the seven activities modeled in this DVD?
8. Can you think of other algebra standards that can be introduced and taught with this menu model? For example, if we want to study linear functions, we can plot the cost of one hamburger and a medium soda, two hamburgers and a medium soda, three hamburgers and a medium soda, and so on. The slope of this function is the cost of the hamburger and the y-intercept is the cost of the medium soda. If we substitute cheeseburgers for the hamburgers, the slope increases. If we substitute a large soda for the medium one, the y-intercept increases.

If desired, allow educators collaboration time to schedule the implementation strategies.

MULTIPLYING AND FACTORING POLYNOMIALS

Initial discussion:

1. What models do you use to teach multiplication of polynomials?

2. What models do you use to teach factoring of polynomials?

3. What are the advantages and shortcomings of these models?

4. What errors do your students commonly make with distribution?

5. What errors do they commonly make when factoring?

6. What connections do your students make between models of multiplication used in arithmetic and those use with polynomials?

7. To what degree do you think your students understand factoring of polynomials?

Post-viewing discussion:

1. How does the flow of the models that were demonstrated here help to build new concepts upon previous ones?

2. What advantages do you see in introducing lattice multiplication?

3. Do you think that different students might better connect with different methods?

4. Should all of these methods be taught? Why or why not?

5. At what grade level would you begin this process of instruction?

6. Do you see a jumping in point and a jumping off point for your students in this process?

7. What connections might your students discover between multiplication of polynomials and factoring?

8. Which models that were demonstrated in this video would be of most use to your students?

If desired, allow educators collaboration time to schedule the implementation strategies.

NUMBER LINE

Initial discussion:

1. How well do your students understand the magnitude of fractions as they compare them?
2. What strategies do they use to compare fractions?
3. What errors do they make when comparing fractions?
4. What strategies do they use to compare decimals and what errors do they commonly make?
5. How well do they transition between the three part/whole models: common fractions, decimals, and percents?
6. How well do your students find equivalencies between the three part whole relationships? For example, do they understand that $^{1}/_{5}$ = .2 = 20%?
7. Other than finding common denominators, what other strategies can be used to compare and order fractions?
8. As you view this video, avoid using a calculator, pencil and paper, or finding common denominators for comparing these fractions, as this will open up the opportunity to discover other effective strategies.

Post-viewing discussion:

1. What strategies did you see in the video that you had not previously considered?
2. Had you ever thought about finding common numerators?
3. Had you thought about how much a fraction lacks from being a whole? For example did you consider that $^{7}/_{10}$ and $^{8}/_{11}$ are both three parts away from a whole?
4. Did you find it difficult watch when students made errors and these were not corrected immediately? Why was it important to present the lesson this way and allow the errors to be corrected at the end when calculators were used?
5. How does this activity develop number sense and model the Mathematical Practices?
    a. Make sense of problems and persevere in solving them.
    b. Reason abstractly and quantitatively.
    c. Construct viable arguments and critique the reasoning of others.
    d. Model with mathematics.
    e. Use appropriate tools strategically.
    f. Attend to precision.
    g. Look for and make use of structure.
    h. Look for and express regularity in repeated reasoning.

If desired, allow educators collaboration time to schedule the implementation strategies.

THE POWER OF TWO

Initial discussion:

1. What roadblocks prevent students from understanding exponents and exponential notation?

2. How is exponential growth modeled in the world of a child?

3. How have we taught exponents in the past with our students?

4. What procedural models have we used?

5. What conceptual models have been implemented in their learning?

6. What concepts about exponents are taught at each grade level?

7. Why do students think the zero power of a number is zero?

8. Often we say that exponents are simply repeated multiplication. Using this model, what struggles would a student encounter in thinking about 2^0? How might they view 2^{-3}? How would they explain $2^{3.76}$?

Post-viewing discussion:

1. Often students fail to see how math applies to the real world. How does this activity address this problem?

2. Having seen the DVD, how would you explain *why* $2^0 = 1$?

3. Would this model work for other folding patterns such as tripling or quadrupling folds?

4. Does this suggest that any nonzero number raised to the zero power is one?

5. How could a student explain negative exponents using the exponential graph project?

6. Having seen this experiment, student Brittney Gallivan set out to exceed the known limits on paper folding. See her world record fold on the web at:
   www.osb.net/pomona/12times.html

7. The authors have often said that no matter what we need to teach, there is probably some object or experience that will allow us to connect that to the real world. Do you agree or disagree with this? Why or why not?

If desired, allow educators collaboration time to schedule the implementation strategies.

SAFELY NAVIGATING SOCIAL NETWORKS

Initial discussion:

1. How involved are your students in social networking?
2. What dangers do you think they face? Do you think the dangers are exaggerated or not?
3. What are the benefits that social networking brings to our society, to our schools, and to our students?
4. What steps can your school take or has it taken to protect your students?
5. To what degree do you feel this is a part of your responsibility?
6. You may wish to take the survey in the video prior to seeing the results that are presented afterwards.

Post-viewing discussion:

1. How has this video changed your thinking about the potential dangers of social networking?
2. Which of the survey responses surprised you the most? Why is that?
3. Are there survey results that didn't surprise you?
4. Do you think your students would respond similarly to these in the video?
5. Do you feel this subject is something we should address as educators?
6. How do you think your students would respond to this message?
7. How would you present it so that they didn't see it as meddlesome on your part?
8. How can you share this message with students or parents?

If desired, allow educators collaboration time to schedule the implementation strategies.

SOLVING LINEAR EQUATIONS

Initial discussion:

1. What struggles do your students face in solving equations?
2. Do your students understand the concepts of balance and equality?
3. What techniques do you use to help them solve equations more successfully?
4. What techniques do you use to help them *understand* the solving of equations?
5. Do you use any physical or visual models in teaching equation solving
6. How do you transition students from these models into more traditional and abstract representations?
7. What issues come up in multi-step equations that require distribution or the combining of like terms?

Post-viewing discussion:

1. How did Bill and Brad build the difficulty incrementally while maintaining a fluid development of the concepts?
2. Where would your students need to enter this lesson sequence? Would they exit it before they got to the final stages?
3. How could your staff build a continuum of instruction throughout the grades to maximize the effects of this approach?
4. Why is this important?
5. Explain the idea of zeroes and ones in solving equations. Why are these important?
6. When would you introduce the terms *additive inverse* and *multiplicative inverse* or *reciprocal* in your instruction?
7. Vocabulary terms are typically introduced at the onset of a lesson so that students will understand them when they encounter the terms as they arise. What advantages do you see in introducing vocabulary *after* students encounter the concept instead of before the lesson?

If desired, allow educators collaboration time to schedule the implementation strategies.

TAKE YOUR PLACES

Initial discussion:

1. How would you rate your students' level of understanding of the grade level concepts you teach?
2. How important is it to have a foundational understanding of a mathematical concept before learning a procedure or skill?
3. To what degree do your students *know* math and to what degree can they *perform* math?
4. How important is it to know math and to perform math?
5. Do you often integrate mathematical concepts or would you rather teach them in isolation?
6. How would you interrelate arithmetic, probability, and mathematical reasoning in a single lesson?
7. What is number sense and how is it taught?

Post-viewing discussion:

6. Even though the topic covered in this video involved elementary multiplication, were you at times challenge in your thinking?
7. If so, how would a lesson like this challenge the thinking of your students?
8. How is number sense developed in this lesson?
9. How are separate mathematical domains interwoven in this lesson?
10. How does this lesson help students develop the Mathematical Practices?
    - i. Make sense of problems and persevere in solving them.
    - j. Reason abstractly and quantitatively.
    - k. Construct viable arguments and critique the reasoning of others.
    - l. Model with mathematics.
    - m. Use appropriate tools strategically.
    - n. Attend to precision.
    - o. Look for and make use of structure.
    - p. Look for and express regularity in repeated reasoning.
11. When would you be most likely to incorporate a lesson like this in your classroom?
12. How would you adapt this lesson to target the grade level of your students?

If desired, allow educators collaboration time to schedule the implementation strategies.

TEACHING TWO-DIGIT MULTIPLICATION THROUGH CONCEPTUAL LAYERING

Initial Discussion:

1. Do you think your students "get it" when they are learning mathematics?

2. What portion of your class would you say is mimicking steps in solving a problem versus actually understanding those steps?

3. To what degree do you find yourself backtracking or re-teaching concepts already taught?

4. What different strategies do you employ when you re-teach? How would you measure the success of those strategies?

5. Do you have a typical plan for leading into a lesson or a new concept?

6. To what degree do you feel it is important to start at a pre-abstract level when introducing new concepts at higher grade levels?

7. Do you sometimes suspect that some of your students have maxed out and learned about as much math as they ever will?

Post-viewing discussion

1. How would you define *Conceptual Layering* for someone who had not seen today's video?

2. What are the characteristics unique to *Conceptual Layering*?

3. Are you currently employing any of these techniques, and how have they worked?

4. What do you see as the strengths and advantages of the *Conceptual Layering* approach?

5. Do you foresee problems in implementing *Conceptual Layering* in your instruction?

6. Does the fact that this technique was used to help 4^{th} and 5^{th} graders understand and perform high-level algebraic thinking lend credence to this instructional strategy?

7. How might *Conceptual Layering* be integrated into other math concepts?

8. How might you implement some of the components of *Conceptual Layering* into your instruction?

If desired, allow educators collaboration time to schedule the implementation strategies.

X MARKS THE SPOT

Initial discussion:

1. How do students currently practice the four operations at each grade level?

2. What types of numbers are used: positive whole numbers, decimals, fractions, integers, or something else?

3. What has worked well with your current format for drill and practice?

4. How much is number sense and mathematical reasoning fostered during this practice?

5. What struggles do students currently face with the four operations?

Post-viewing discussion:

1. To what degree will this drill and practice model supplant, replace, or supplement your current approach?

2. Which of these many versions of X Marks the Spot is most appropriate for your students?

3. How will you divide these versions among the grade levels at your site?

4. How does the simultaneous practice of two operations, such as addition and multiplication, foster mathematical thinking?

5. In what other ways does this approach foster number sense and mathematical reasoning?

6. How do you see the algebraic applications of X Marks the Spot fitting into your teaching assignment? For example, do you plan to use the activity to help students factor quadratics (of the form $ax^2 + bx + c$ when $a = 1$)?

If desired, allow educators collaboration time to schedule the implementation strategies.