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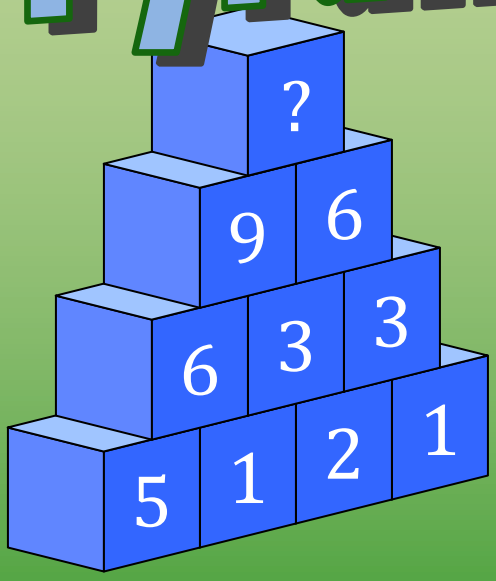
Pyramid Math



- ☑ Builds number sense
- ☑ Provides practice in addition of whole numbers, decimals, fractions, integers, and algebraic expressions
- ☑ Promotes creative thinking
- ☑ Fosters critical thinking
- ☑ Encourages valuable discussion
- ☑ Self assessing

By Brad Fulton
Educator of the Year, 2005
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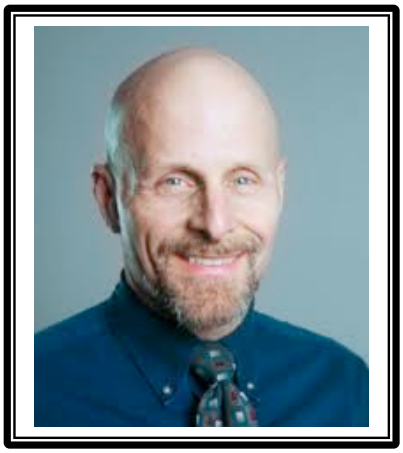
Brad Fulton

Educator of the Year



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- ♦ Consultant
- ♦ Educator
- ♦ Author
- ♦ Keynote presenter
- ♦ Teacher trainer
- ♦ Conference speaker

Known throughout the country for motivating and engaging teachers and students, Brad has co-authored over a dozen books that provide easy-to-teach yet mathematically rich activities for busy teachers while teaching full time for over 30 years. In addition, he has co-authored over 40 teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

Seminar leader and trainer of mathematics teachers

- ♦ 2005 California League of Middle Schools Educator of the Year
- ♦ California Math Council and NCTM national featured presenter
- ♦ Lead trainer for summer teacher training institutes
- ♦ Trainer/consultant for district, county, regional, and national workshops

Author and co-author of mathematics curriculum

- ♦ Simply Great Math Activities series: six books covering all major strands
- ♦ Angle On Geometry Program: over 400 pages of research-based geometry instruction
- ♦ Math Discoveries series: bringing math alive for students in middle schools
- ♦ Teacher training seminar materials handbooks for elementary, middle, and secondary school

Available for workshops, keynote addresses, and conferences

All workshops provide participants with complete, ready-to-use activities that require minimal preparation and give clear and specific directions. Participants also receive journal prompts, homework suggestions, and ideas for extensions and assessment.

Brad's math activities are the best I've seen in 38 years of teaching!
Wayne Dequer, 7th grade math teacher, Arcadia, CA

"I can't begin to tell you how much you have inspired me!"
Sue Bonesteel, Math Dept. Chair, Phoenix, AZ

"Your <u>entire</u> audience was fully involved in math!! When they chatted, they chatted math. Real thinking!"
Brenda McGaffigan, principal, Santa Ana, CA

"Absolutely engaging. I can teach algebra to second graders!"
Lisa Fellers, teacher

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References available upon request

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Like me even more? Then please don't make copies for your colleagues. I know it's tempting when they say, "Wow! Groovy activity! Can I have a copy?" But this is how I make my money, and why are they still saying "groovy" anyway?

If we make copies for our friends, can we honestly tell our students not to copy or take things that don't belong to them? (Ouch!)

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Thanks and happy teaching,

Brad

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Pyramid Math

Where Skill Practice and Number Sense Combine

Overview:

This creative activity facilitates discovery of number patterns and develops number sense while providing critical skills practice. The activity works great with both positive and negative numbers and decimals, fractions, and even variable expressions in algebra. Because it can be designed to be self-checking, it is easy for the teacher and engaging for the students.

Required Materials:

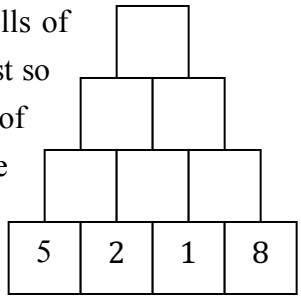
- ☒ Paper
- ☒ Blank master

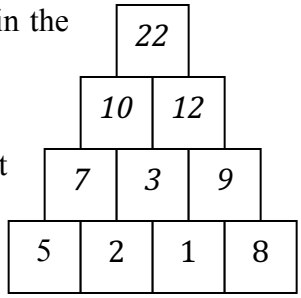
Optional Materials:

- ☐ activity master
- ☐ calculators

Procedure:

1 Display the transparency master and enter four numbers in the cells of the bottom row as shown. (Use single-digit whole numbers at first so students can focus on the structure of the problem instead of struggling with the computation at this point.) To solve the pyramid, an adjacent number pair is added. The sum is written in the box above the number pair. This is repeated for the other number pairs in the bottom row. Then this process is repeated for the second row to fill the third row. Finally the number pair in the third row is added to get the final top number as shown.





2 Since each sum is based on the sums below, all students should get the same answer in the top cell. **Thus they only need to check the top answer.** If that is correct, all other cells are likely correct too.

3 Now try another pyramid using new numbers. Students will catch on to the process quickly and will be eager to check their answer with those of their classmates. (No more correcting papers!)

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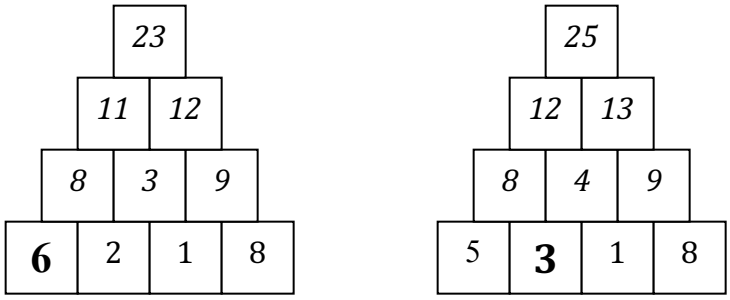
4 As students understand how the problems work, introduce appropriate numbers. If you are studying decimals, throw in a few decimal points. If you have covered integers, use some negative numbers. Fractions make these problems much more difficult. Try one yourself before asking the students to do so. I suggest beginning with like denominators. Or you could use fractions that have a fairly small common denominator. For example, ¼, ½, ½, and ¾ can all use fourths for a common denominator.

5 Try to make slight variations in the arrangement and values of the numbers to help children focus on the number sense involved. For example, in the first problem if we increase the five by one, making it a six, the top number also increases by one, but if we change the two to a three, the top number increases by three.



Is this always true when we add one to a cell? What would happen if we added two to the first or second cell of the bottom row? What happens when we do this to a five-row pyramid? As students answer these questions they will develop number sense.

6 Ask students to change the order of the numbers in the bottom row of a pyramid. How does this affect the top cell? Is the result always the same? How does the commutative property affect this result?

7 If everyone puts the same number in the top cell of a blank pyramid, will everyone get the same bottom row by working backwards? Why or why not?

8 Introduce subtraction by using Pyramid Math 6 in which other cells are filled in. You can create one of your own easily, or have students create them for their classmates to solve.

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9 Explore what happens when all odd numbers or all even numbers are used. What patterns occur when all four cells in the bottom row contain the same number?

Journal Prompts:

If you rearrange the numbers on the bottom row of a pyramid, will you always get the same numbers on top? Why or why not?

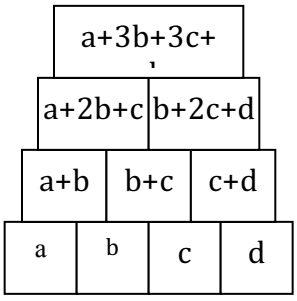
What can you predict about the number on the top of a four-row pyramid if all the starting numbers are equal? Does the number of rows in the pyramid affect this? In what way?

Homework:

Use one of the accompanying activity masters or tailor one to your students' needs using one of the blank masters.

Taking a Closer Look:

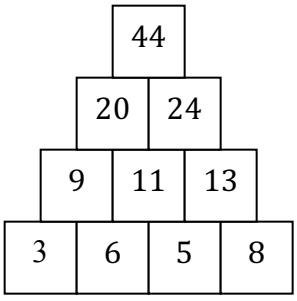
There is a way of predicting the top of the pyramid without solving all the rows. This leads students into the algebra involved in the process. For example, let's assume that we are going to solve a four-row pyramid. The bottom cells contain four numbers called a, b, c, and d as shown. It follows that the second row contains three sums which are a + b, b + c, and c + d respectively. The third row contains these two sums:



(a + b) + (b + c) and (b + c) and (c + d)

These simplify into a + 2b + c and b + 2b + c. Adding these to get the top row gives a + 3b + 3c + d.

Now let's start with four numbers: a = 3, b = 6, c = 5, and d = 8. Using the formula, the top answer should be:



a + 3b + 3c + d =
3 + 3(6) + 3(5) + 8 =
3 + 18 + 15 + 8 = 44

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Assessment: ☑

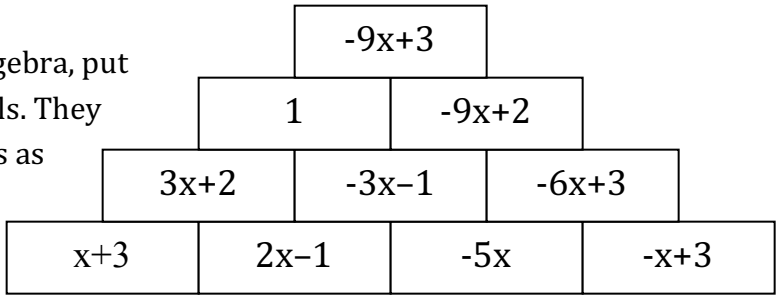
These activities can be made self-assessing by writing the answers at the bottom of the page. As students solve each pyramid, they can cross off the answers. If they get an answer that is not listed, they know they have made a mistake and can try again.

Answer Key:

	Pyramid Math Worksheet Number				
	1	2	3	4	5
Problem					
1	33	16.5	21	267	$4\,^{2}/_{5}$
2	24	10.2	-21	224	$3\,^{3}/_{7}$
3	40	4	11	377	
4	32	3.2	-20	264	
5	60	2.05	-8	312	
6	68	23.86	0	332	
7	80	6.84	24	222	
8	23	16	-34	308	
9	23	21.81	0	332	
10	100	3.08	100	363	
11	106	1	1	528	
12	188	1.76	9	2000	

Great Tip!

For students who are preparing for algebra, put variable expressions in the bottom cells. They can then practice combining like terms as shown. Or put them in some of the upper cells so they can subtract binomials by working downward.



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For sets six through eight, these are the answers to the bottom row of each pyramid.

Pyramid Math Worksheet Number

Problem	6	7	8
1	8, 3, 2, 2	.3, .1, .7, .6	6, -4, 5, -2
2	3, 4, 4, 2	.5, .6, .2, .5	8, -12, 6, -8
3	11, 3, 2, 10	.6, .2, .5, .5	8, 6, -12, -8
4	1, 0, 11, 23	3, .6, .9, 1	-3, -6, -9, -14
5	7, 12, 10, 11	.7, .9, .9, 2	-8, 0, 19, -20
6	19, 0, 6, 26	.9, 0, .5, .9	-7, -6, 16, -16
7	27, 0, 0, 27	1.2, 6.3, 2.7, 3.1	21, -9, -19, -8
8	8, 19, 10, 9	17, 1.9, .8, 6.4	11, 30, -16, -14
9	31, 26, 19, 17	8.7, 7.6, 6.5, 5.4	11, -30, -16, 14
10	18, 2, 0, 27	.29, .16, .5, 1.2	-17, -3, 3, 17
11	5, 38, 19, 31	7.1, .99, .09, .5	-15, -5, 6, 13
12	13, 6, 34, 9	7.7, .77, .55, 5	18, -9, 6, -3

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The Common Core Connection

In addition to providing skills practice in addition of all number types, these pyramid problems cab also help students think mathematically and develop number sense. They are a great way to develop the **eight mathematical practices** of the Common Core Math Standards:

1. Make sense of problems and persevere in solving them

This practice applies when students are developing an understanding of the problem and its underlying mathematics. Students also practice this when they make conjectures about what will happen in a problem. That is why it is important to pose questions to the students such as, "What do you think will happen if we rearrange the numbers?" "If the bottom numbers in a pyramid are all odd, will the top number be even or odd?"

2. Reason abstractly and quantitatively

As students move past the arithmetic of a problem and begin to think about the fact that the numbers can vary and what happens when they do, they are thinking abstractly. That is why it is important to help your students to move toward exploring what happens when the bottom row contains variables instead of specific numbers.

3. Construct viable arguments and critique the reasoning of others

When students are asked to make conjectures, we should also ask them to explain their reasoning. "Why do you think that the top number will be even?" Students will need to learn the skills of explaining their thinking and of disagreeing with the conjectures of others in an appropriate manner.

4. Model with mathematics

This practice involves using mathematics to represent the problems they see in the real world. At first it might seem not to apply to the pyramid problems because they are skills practice. However, using a formula as a model to explain why the pyramids behave the way they do is an example of this practice.

5. Use appropriate tools strategically

In this activity students might employ pencil and paper as their primary tools. However, the teacher may wish to allow calculators on some of the more challenging problems. A good guide in knowing when to switch from paper to calculator is this:

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when the arithmetic is impeding the mathematical thinking, students are only getting skills practice and are not likely to develop number sense.

Students in a computer class could also create a spreadsheet that would solve pyramid problems. Designing such a spreadsheet would require the students to know the *how and why* behind the mathematics that governs the pyramid problems.

6. Attend to precision

When students self-assess either by comparing their answers to others or by checking their solution against an answer bank, they are more likely to amend their errors (Marcy). Working separately, they often lack the number sense to know if the result of their calculation is right or even reasonable.

7. Look for and make use of structure

As students explore and come to understand how the structure of these problems affects the number at the top of the pyramid, they are developing this skill. That is why it is important not only to provide them with random numbers in practice problems, but also with examples that illustrate what happens when the same numbers are rearranged or changed slightly.

8. Look for and express regularity in repeated reasoning

As students notice the effects of these changes in the bottom numbers, they begin to notice shortcuts. For example, adding one to either of the two squares on the outside of the bottom row *always* increases the top box by one, but adding one to one of the interior cells of the bottom row *always* increases the top number by three.

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Activity master

Pyramid Math

Add pairs of adjacent numbers and write their sums above them. Keep going until you reach the top of the pyramid.

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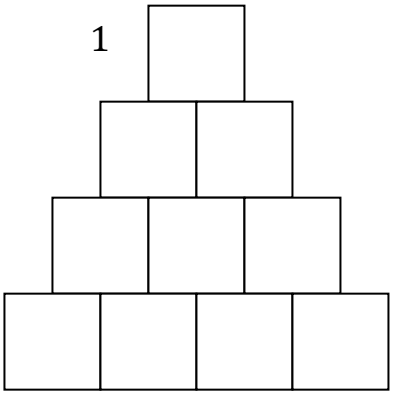

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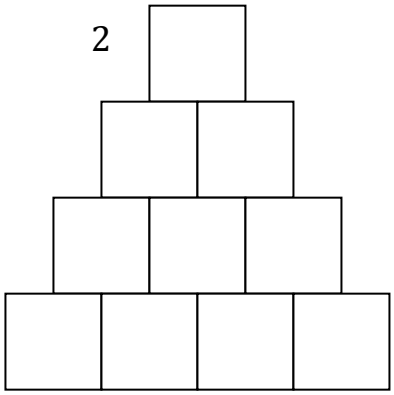

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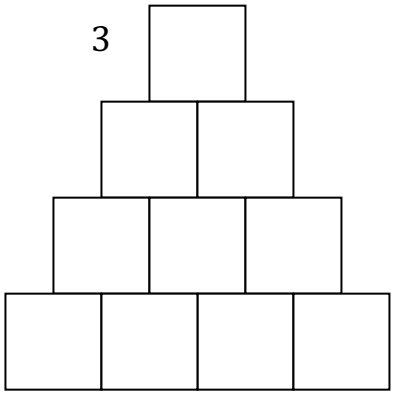

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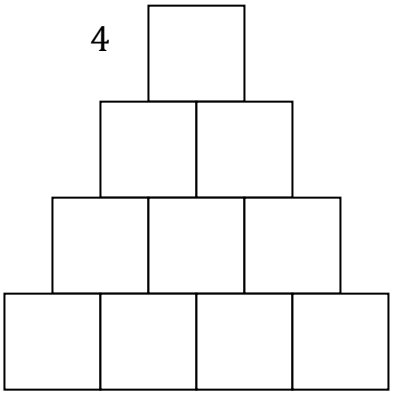

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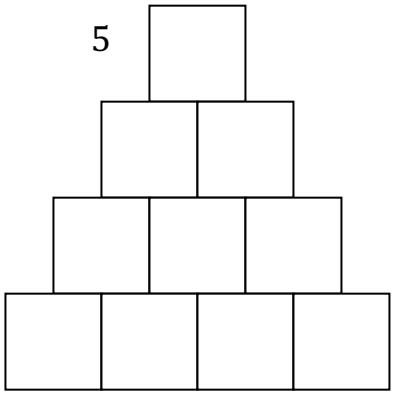

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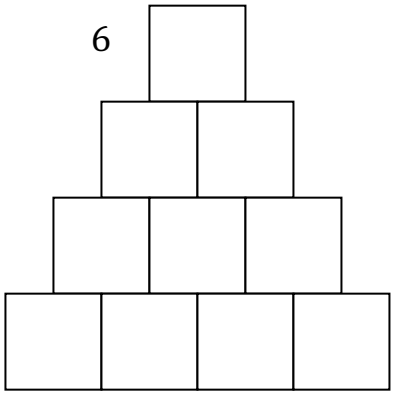

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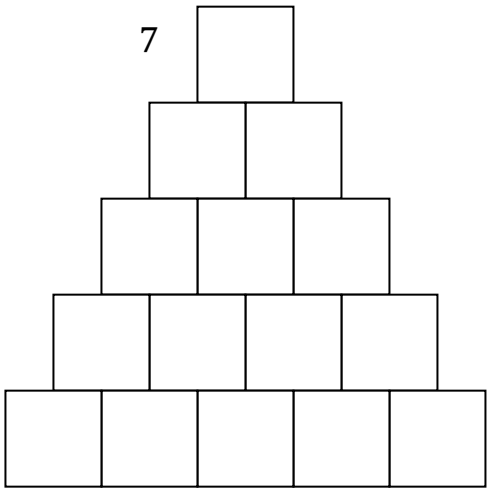

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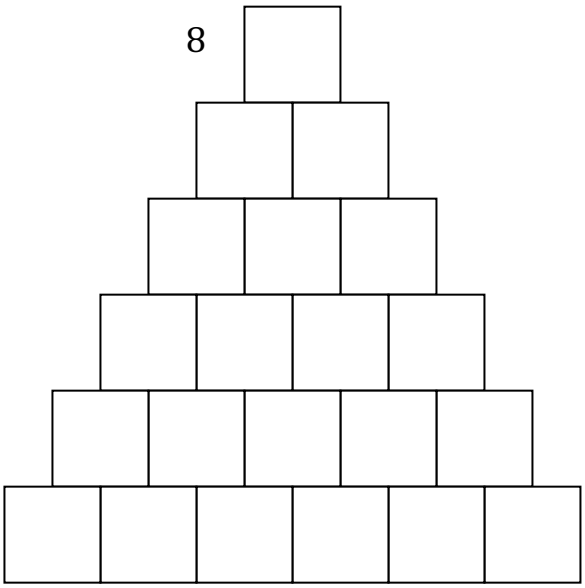

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Activity Master 1

Pyramid Math

Name_____

Add pairs of adjacent numbers and write their sums in the box above them. Keep going until you reach the top of the pyramid.

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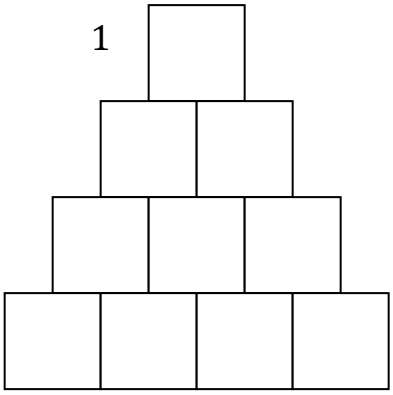

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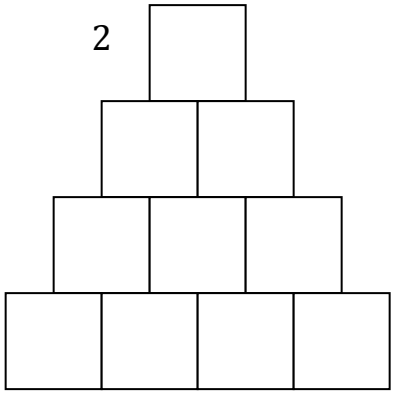

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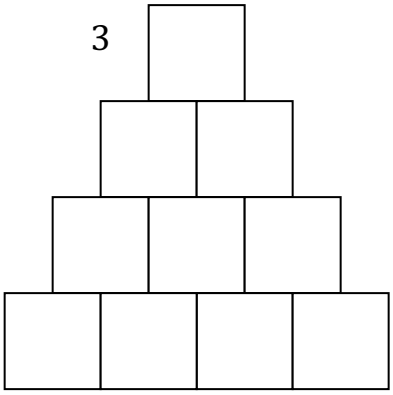

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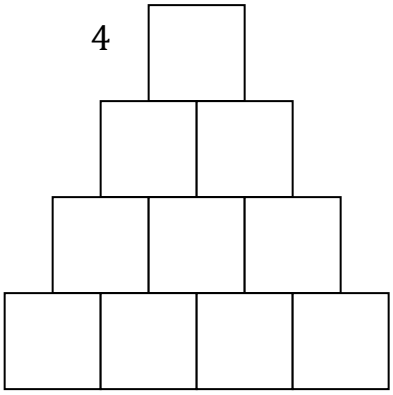

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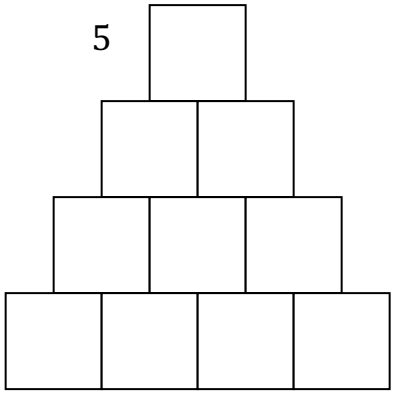

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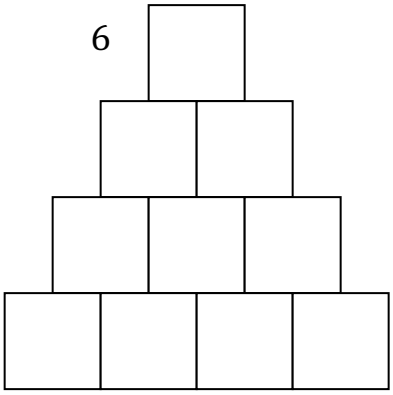

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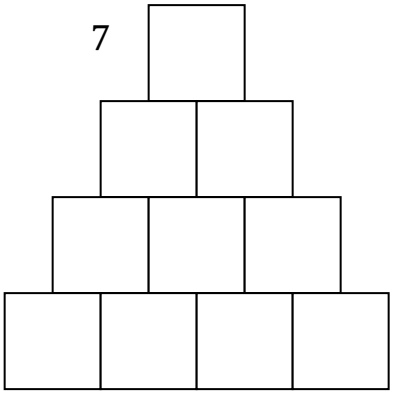

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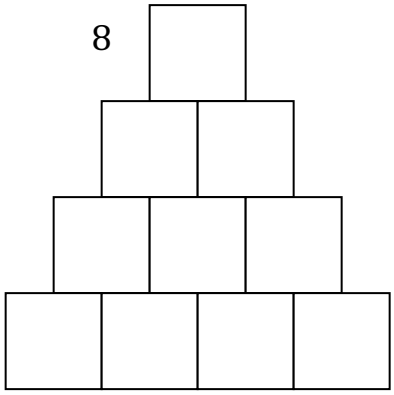

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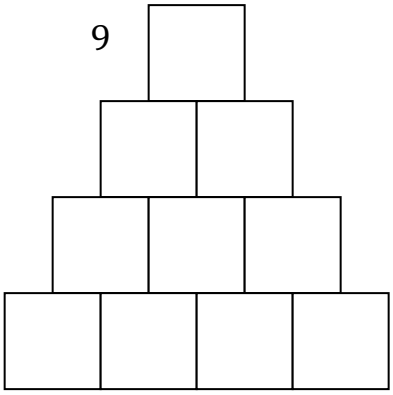

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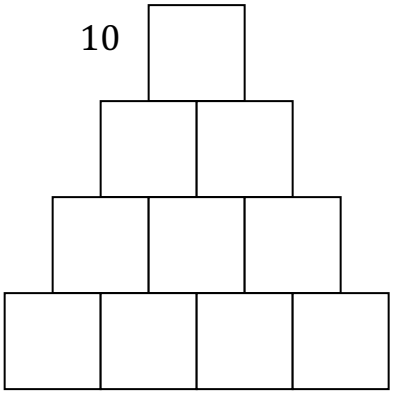

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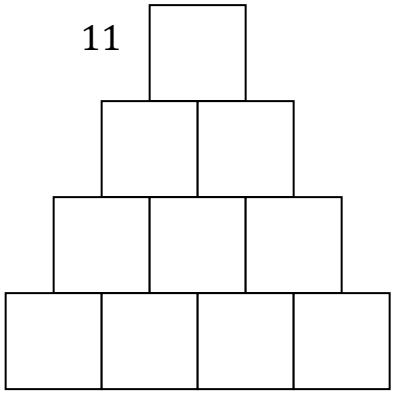

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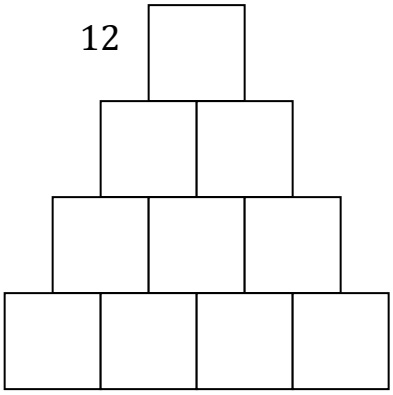

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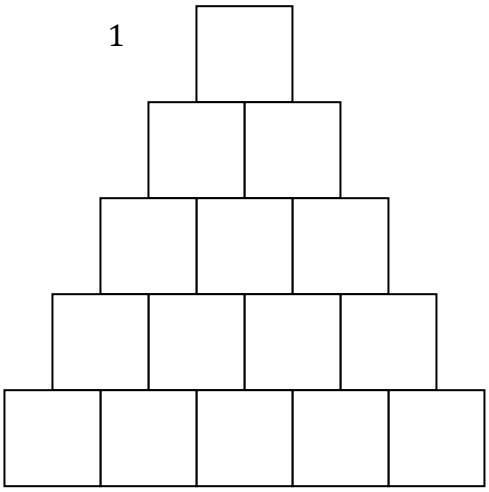
Activity Master 2

Pyramid Math

Name_____

Add pairs of adjacent numbers and write their sums in the box above them. Keep going until you reach the top of the pyramid.

1



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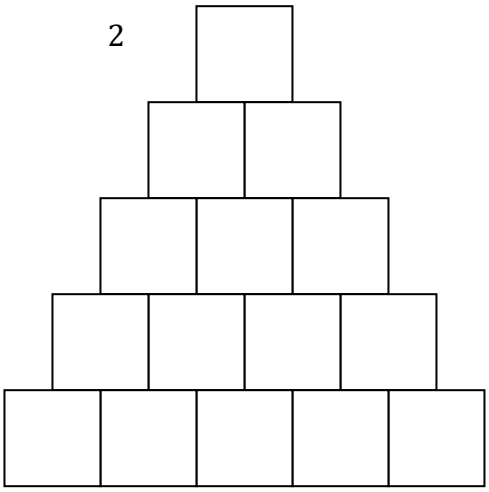

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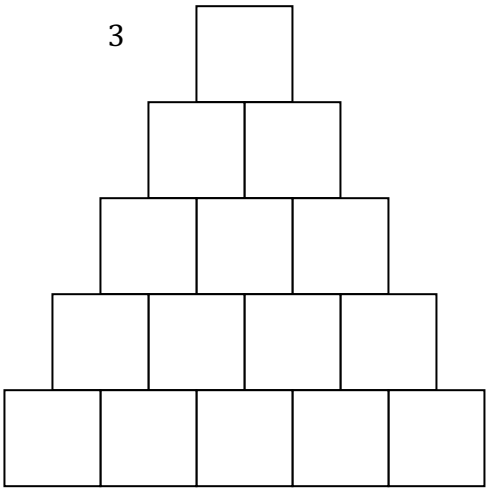

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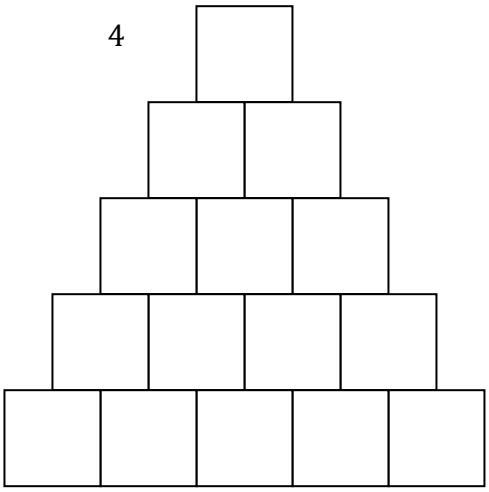

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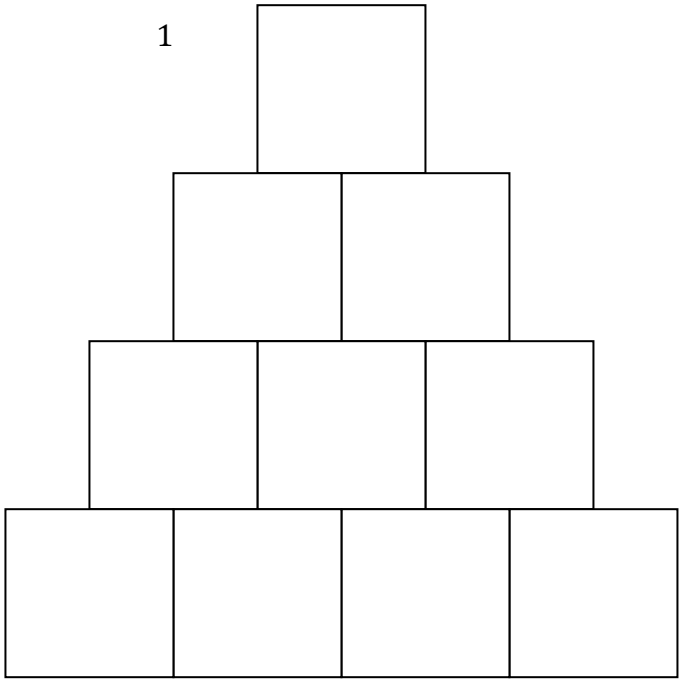
Activity Master 3

Pyramid Math

Name_____

Add pairs of adjacent numbers and write their sums in the box above them. Keep going until you reach the top of the pyramid.

1



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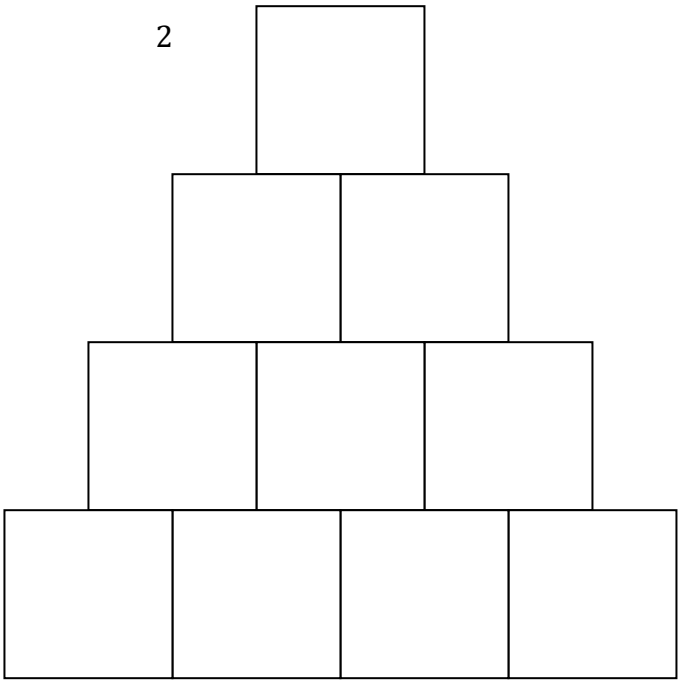

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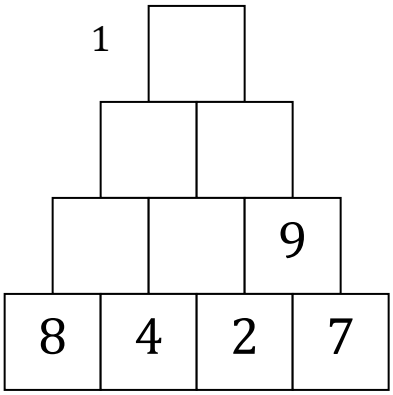
Activity Master

Pyramid Math 1

Name_____

Add pairs of adjacent numbers and write their sums in the box above them as in the first example. Keep going until you reach the top of the pyramid.

1



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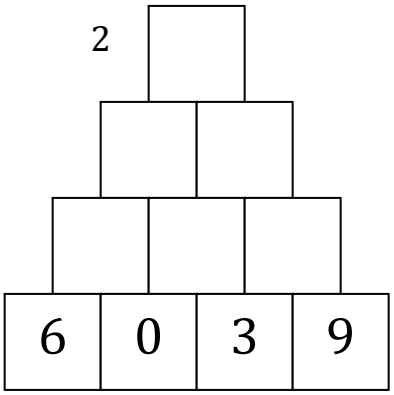

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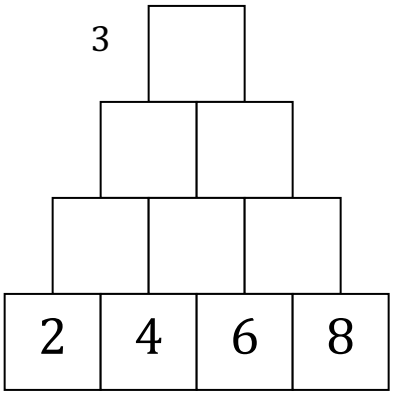

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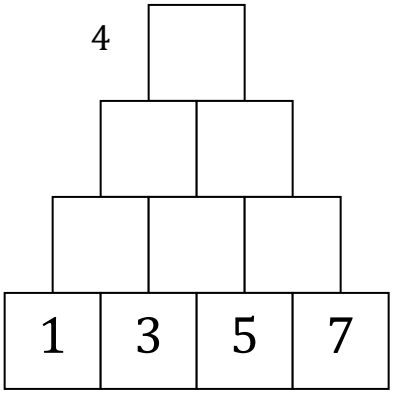

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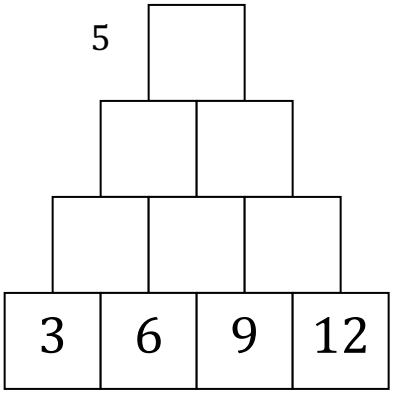

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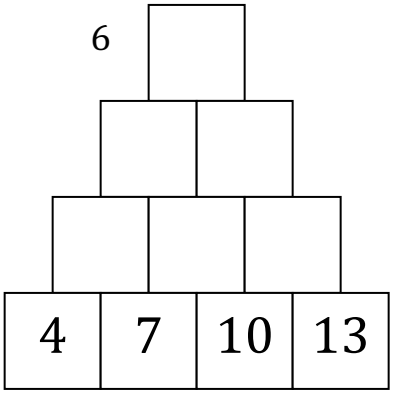

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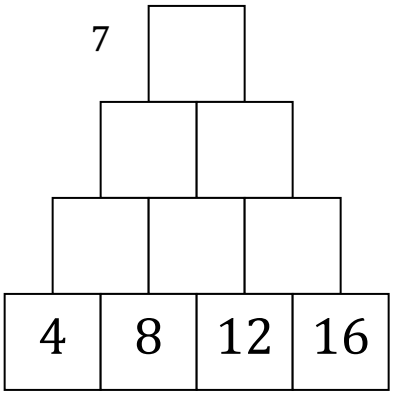

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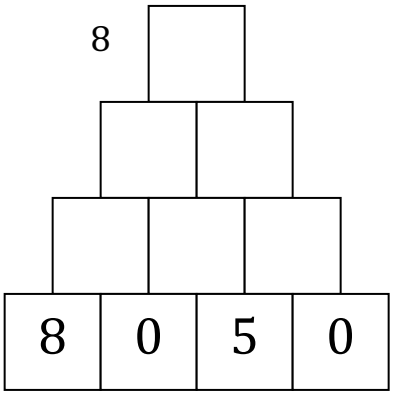

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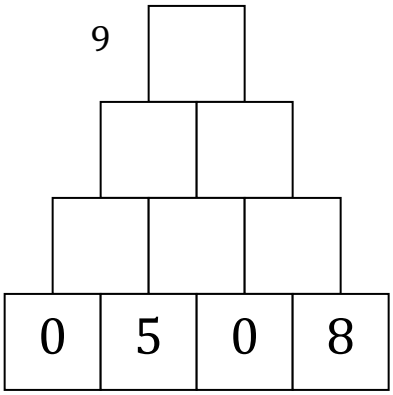

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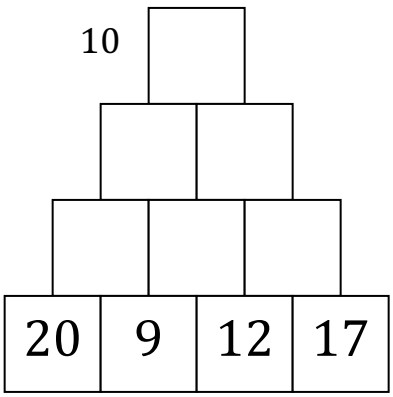

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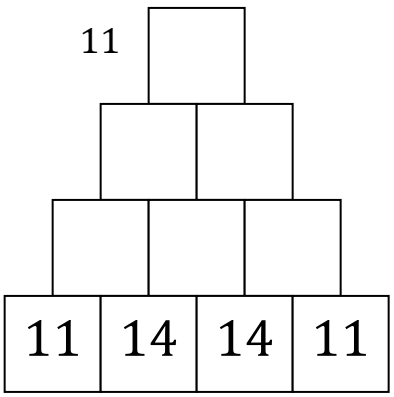

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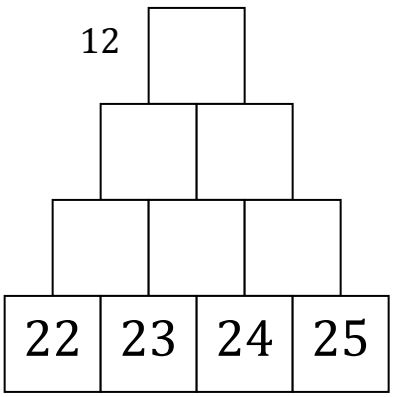

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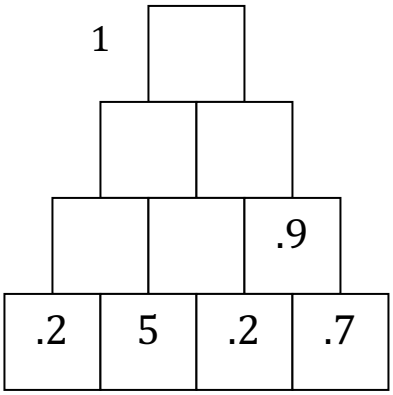
Activity Master

Pyramid Math 2

Name_____

Add pairs of adjacent numbers and write their sums in the box above them as in the first example. Keep going until you reach the top of the pyramid.

1



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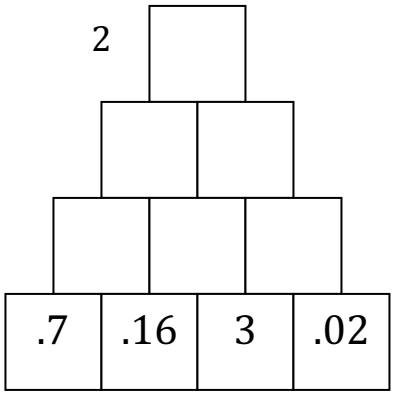

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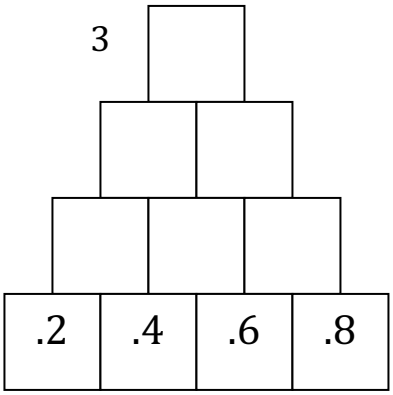

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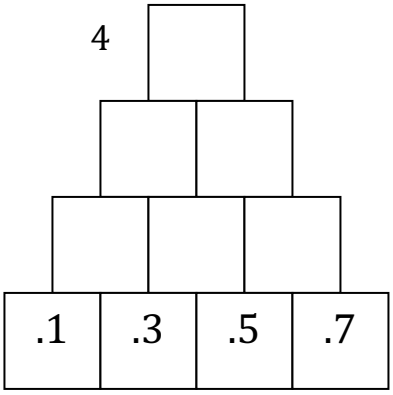

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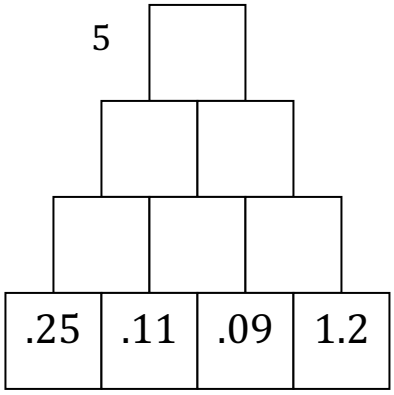

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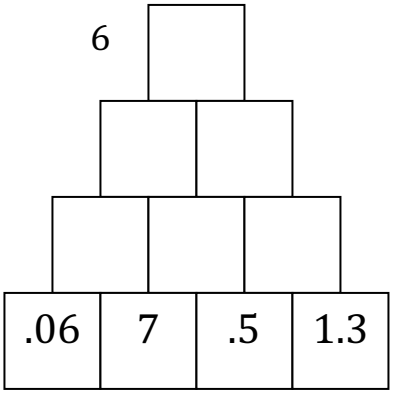

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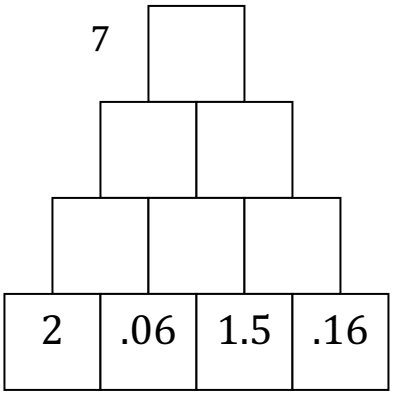

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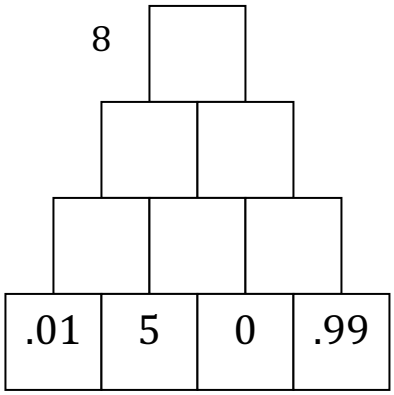

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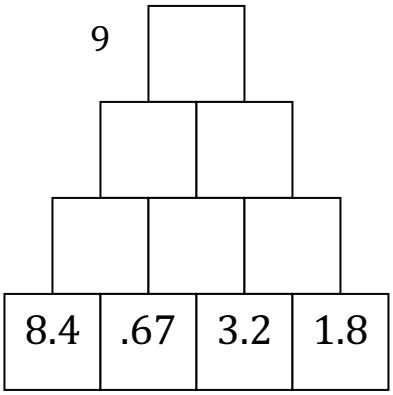

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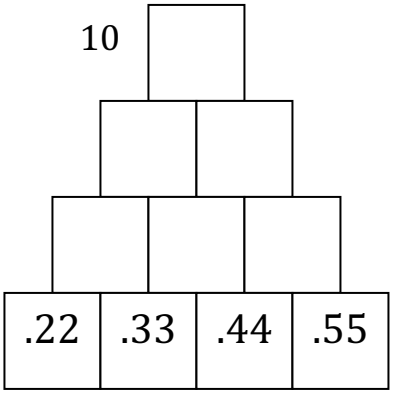

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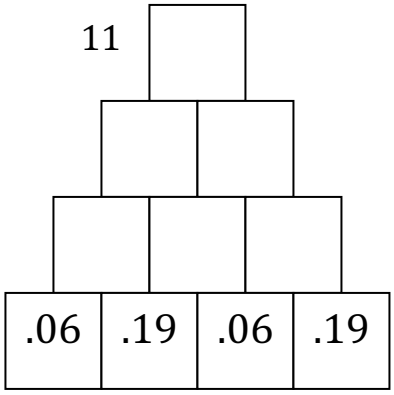

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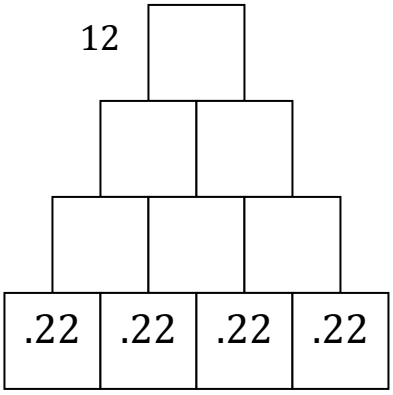

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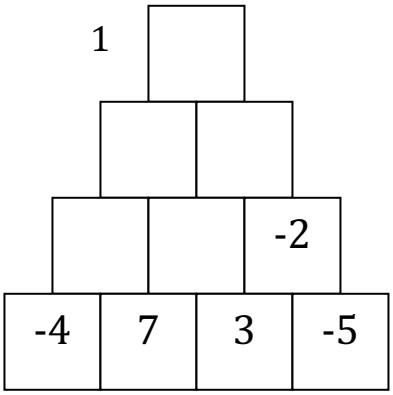
Activity Master

Pyramid Math 3

Name_____

Add pairs of adjacent numbers and write their sums in the box above them as in the first example. Keep going until you reach the top of the pyramid.

1



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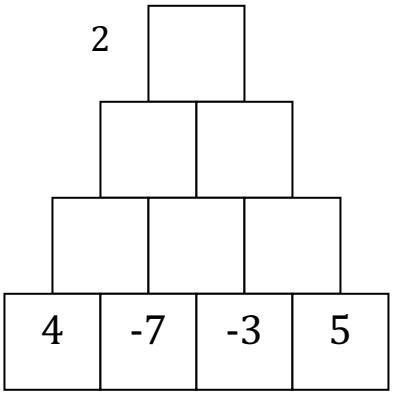

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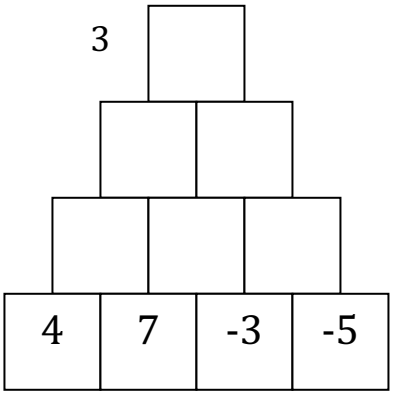

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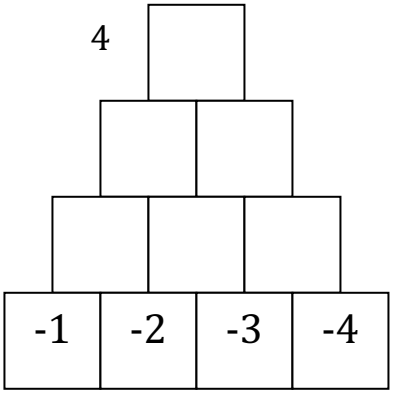

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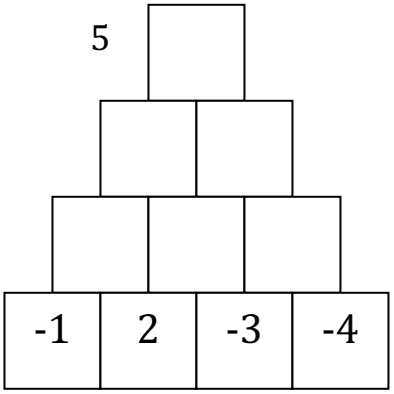

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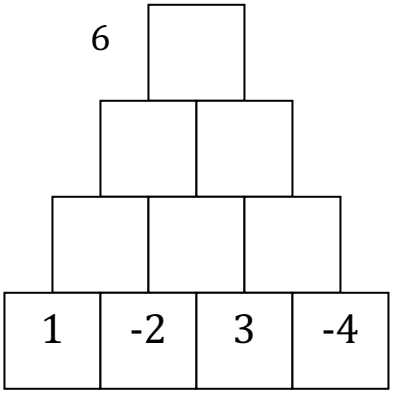

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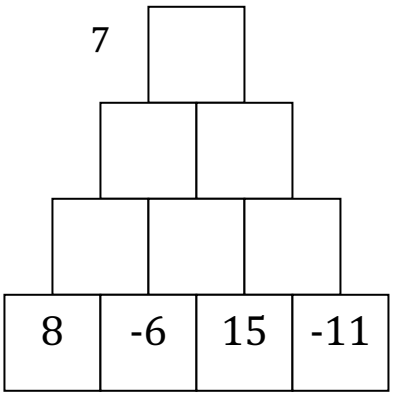

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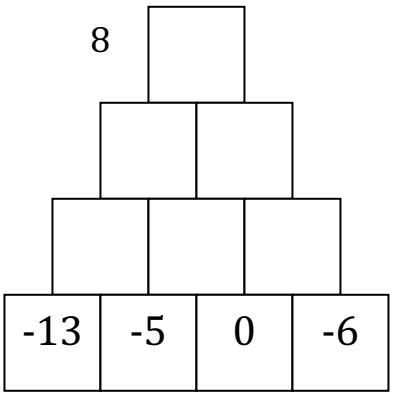

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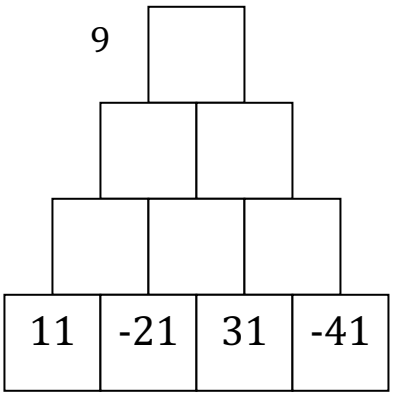

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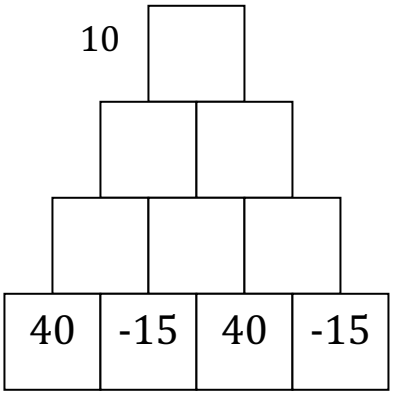

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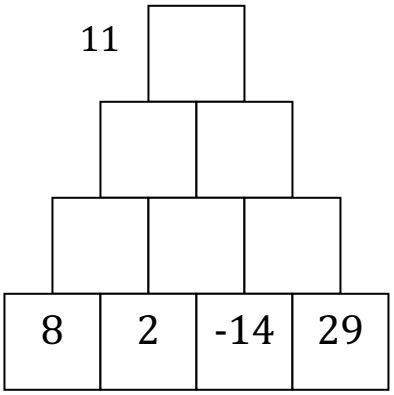

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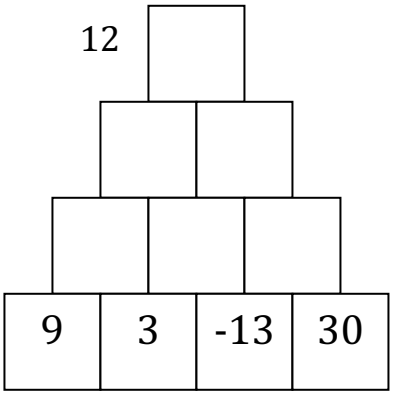

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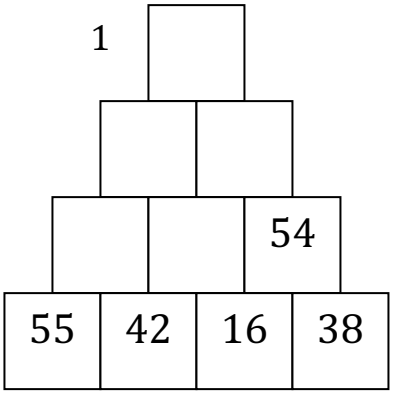
Activity Master

Pyramid Math 4

Name_____

Add pairs of adjacent numbers and write their sums in the box above them as in the first example. Keep going until you reach the top of the pyramid.

1



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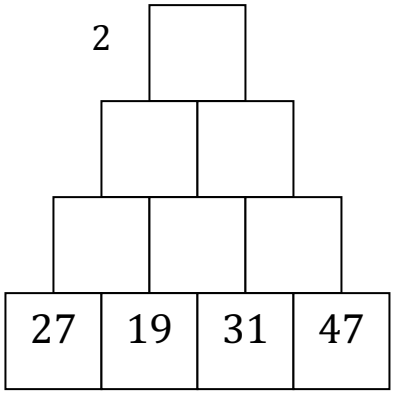

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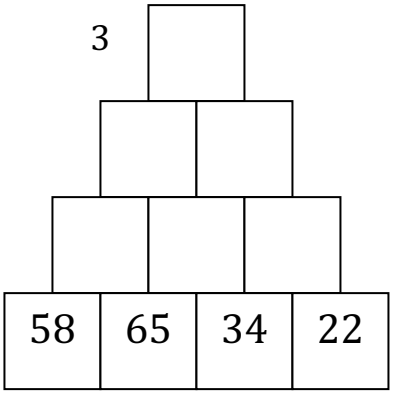

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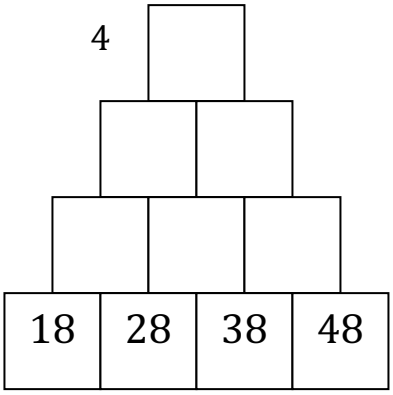

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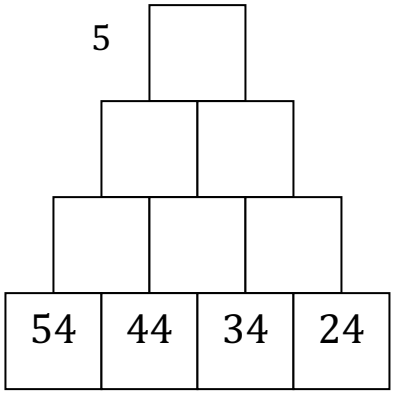

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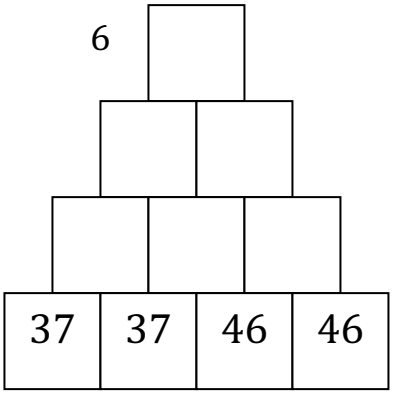

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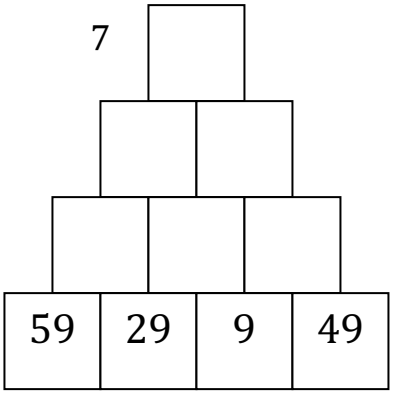

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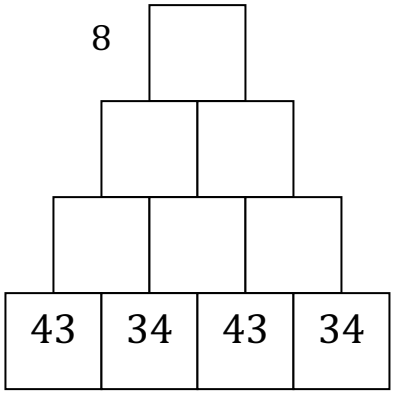

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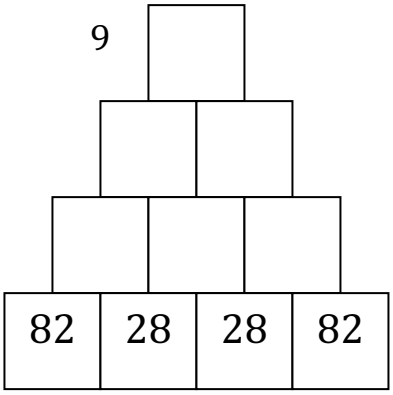

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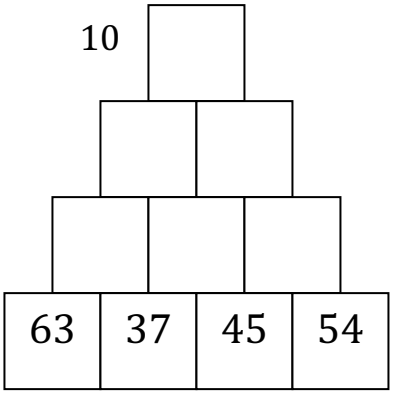

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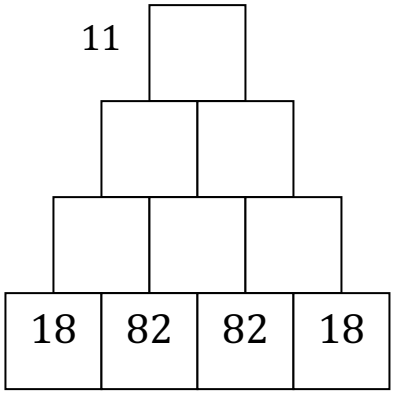

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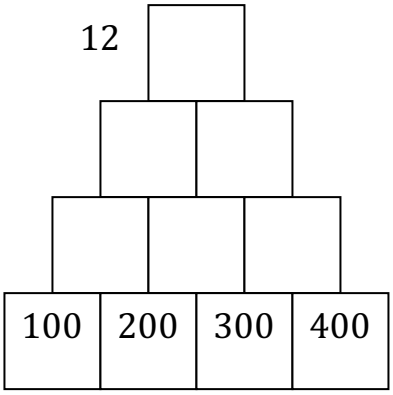

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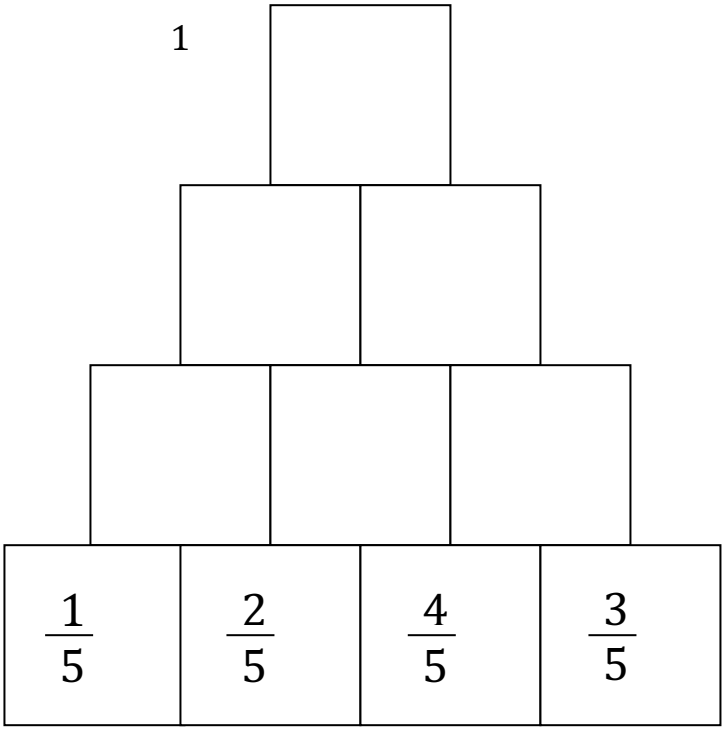
Activity Master

Pyramid Math 5

Name_____

Add pairs of adjacent numbers and write their sums in the box above them. Keep going until you reach the top of the pyramid.

1



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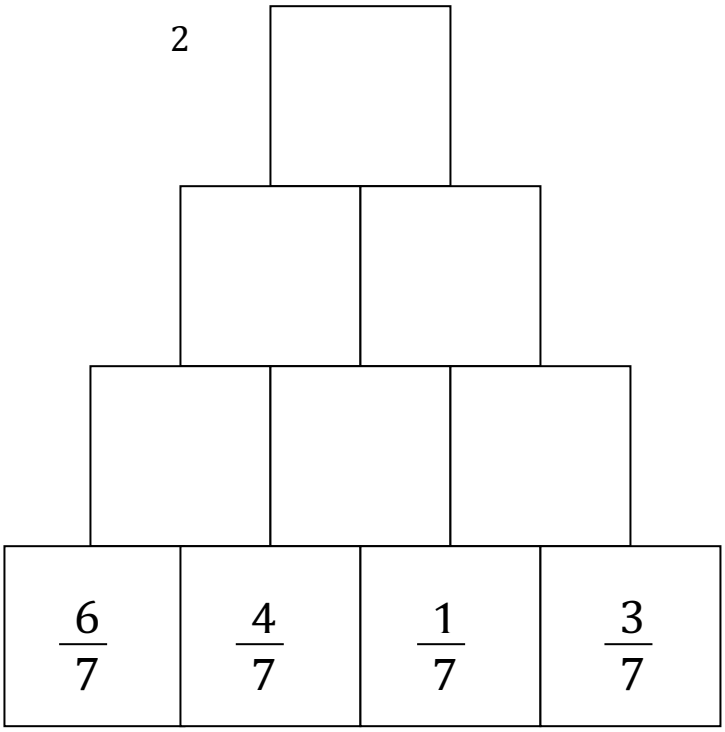

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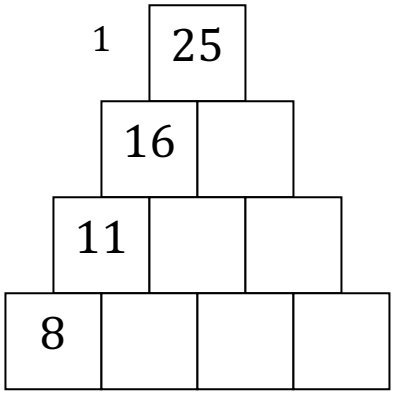
Activity Master

Pyramid Math 6

Name_____

Each number is the sum of the two numbers below it. Work backward to fill in the bottom row.

1



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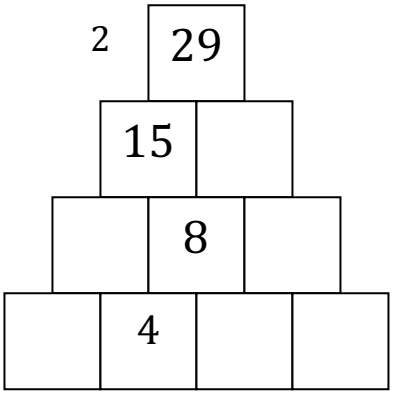

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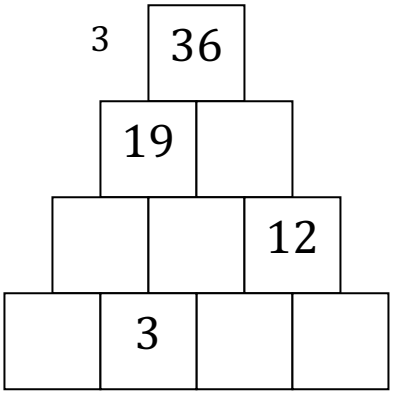

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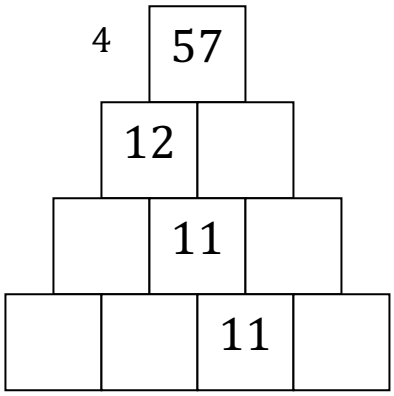

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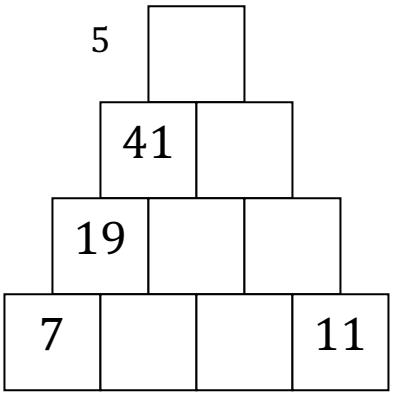

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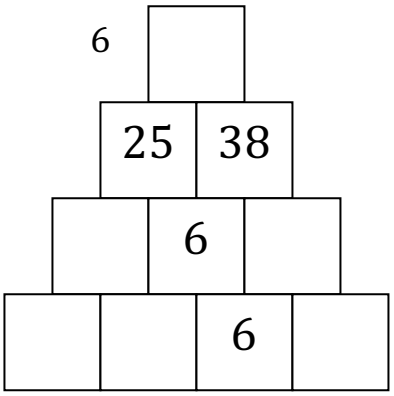

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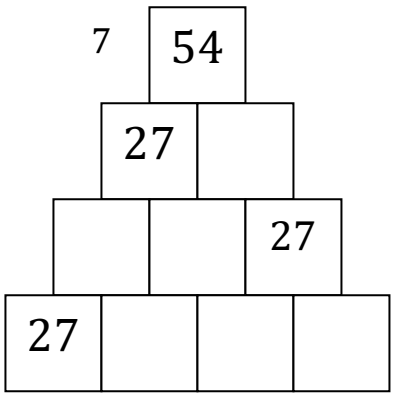

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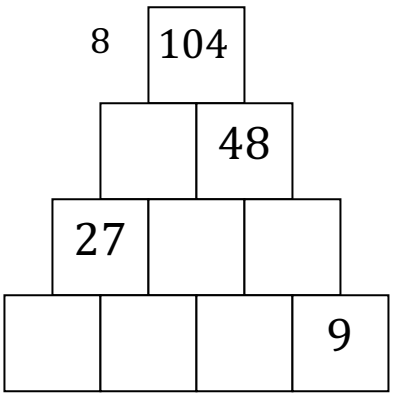

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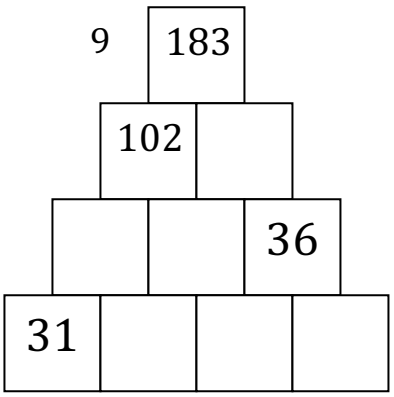

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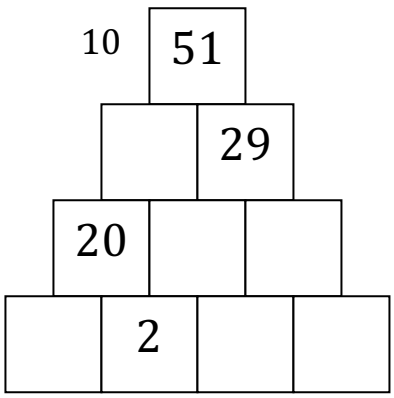

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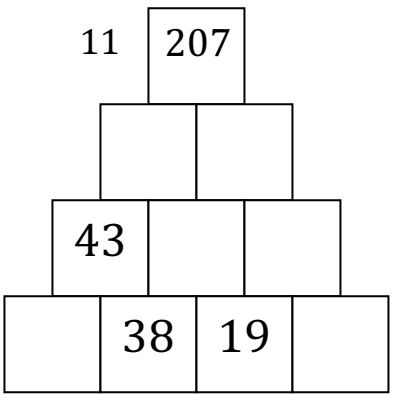

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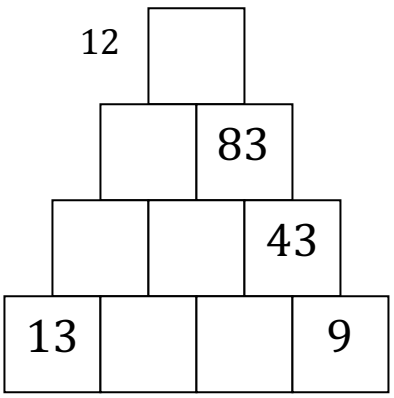

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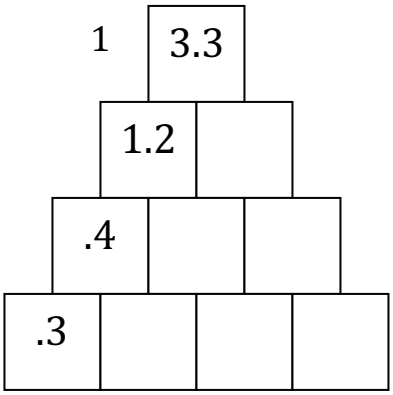
Activity Master

Pyramid Math 7

Name_____

Each number is the sum of the two numbers below it. Work backward to fill in the bottom row.

1



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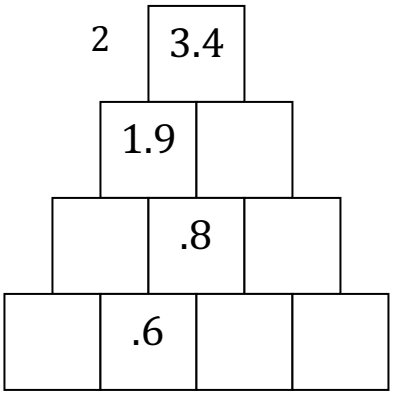

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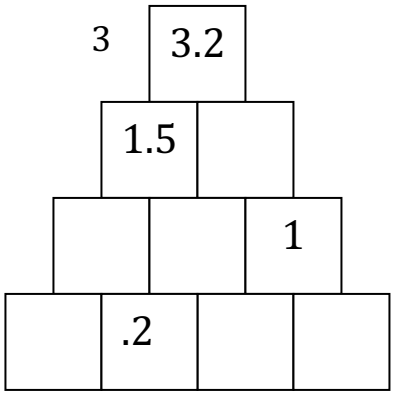

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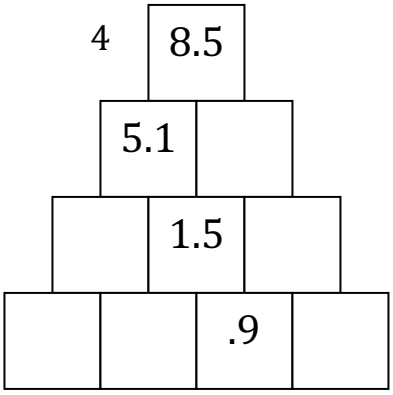

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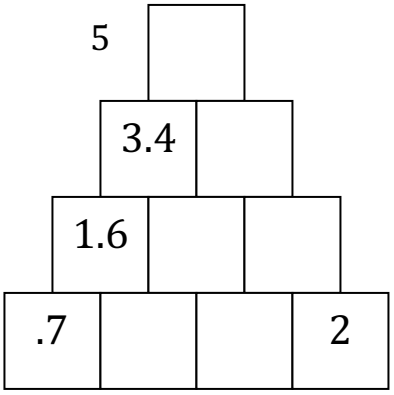

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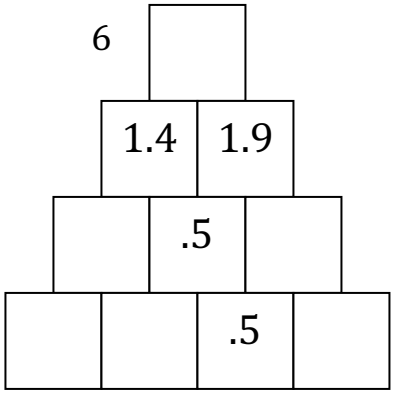

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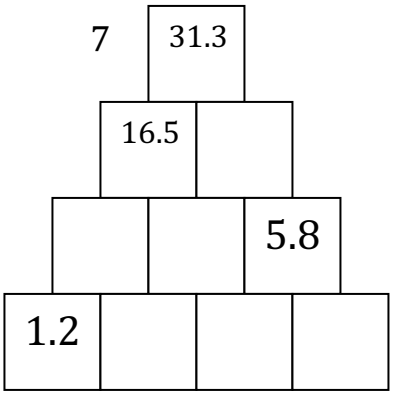

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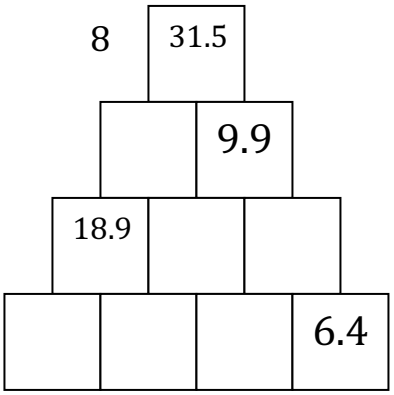

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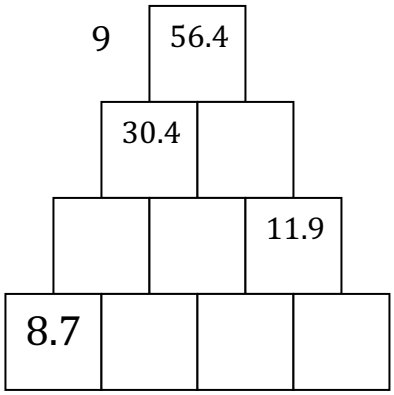

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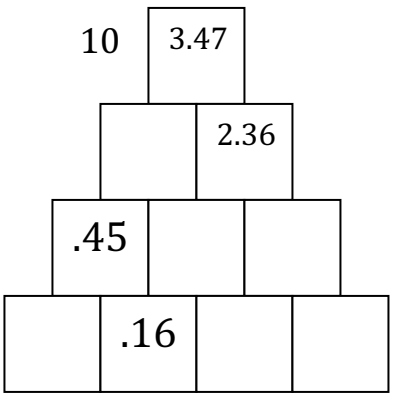

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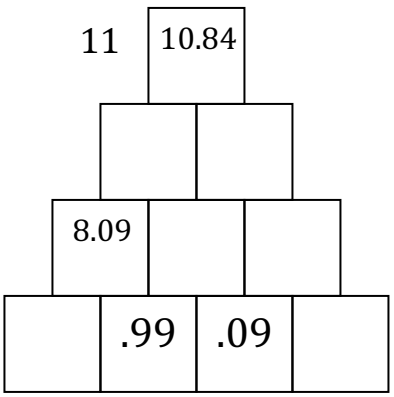

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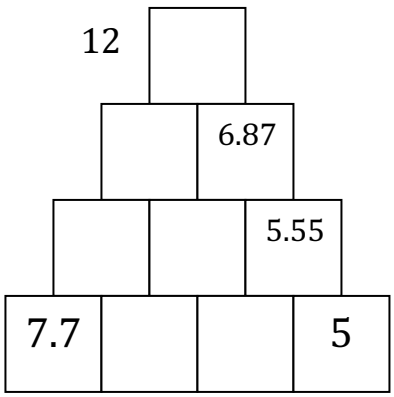

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Activity Master

Pyramid Math 8

Name_____

Each number is the sum of the two numbers below it. Work backward to fill in the bottom row.

1

	7		
	3		
2			
6			

2

	-18		
	-10		
		-6	
	-12		

3

	-18		
	8		
			-20
	6		

4

	-62		
	-24		
		-15	
			-9

5

	11		
		-8	
-8			-20

6

		-3	10
		10	
			16

7

	-71		
	-16		
			-27
21			

8

	39		
		-16	
	41		
			-14

9

	-113		
	-65		
			-2
11			

10

	0		
		20	
-20			
	-3		

11

	1		
-20			
	-5	6	

12

		0	
			3
18			-3

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- *Math Maps: Developing the Mathematical Practices*
- *Sum Thing Interesting: Finding Amazing Patterns in Addition*
- *Array We Go: Building: An Engaging and Visual Representation of Factors, Multiples, Primes, and Composites, and More*

Feel free to contact me if you have questions or comments or would like to discuss a staff development training or keynote address at your site.

Happy teaching,

Brad

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