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Sum Thing Interesting

1+2+3+4+5

Finding Amazing Patterns in Addition

By Brad Fulton
Educator of the Year, 2005
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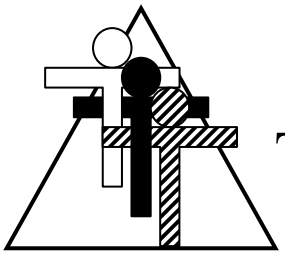

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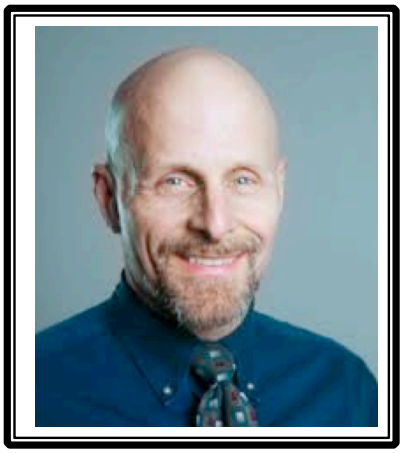
Brad Fulton

Educator of the Year



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- ♦ Consultant
- ♦ Educator
- ♦ Author
- ♦ Keynote presenter
- ♦ Teacher trainer
- ♦ Conference speaker

Known throughout the country for motivating and engaging teachers and students, Brad has co-authored over a dozen books that provide easy-to-teach yet mathematically rich activities for busy teachers while teaching full time for over 30 years. In addition, he has co-authored over 40 teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

Seminar leader and trainer of mathematics teachers

- ♦ 2005 California League of Middle Schools Educator of the Year
- ♦ California Math Council and NCTM national featured presenter
- ♦ Lead trainer for summer teacher training institutes
- ♦ Trainer/consultant for district, county, regional, and national workshops

Author and co-author of mathematics curriculum

- ♦ Simply Great Math Activities series: six books covering all major strands
- ♦ Angle On Geometry Program: over 400 pages of research-based geometry instruction
- ♦ Math Discoveries series: bringing math alive for students in middle schools
- ♦ Teacher training seminar materials handbooks for elementary, middle, and secondary school

Available for workshops, keynote addresses, and conferences

All workshops provide participants with complete, ready-to-use activities that require minimal preparation and give clear and specific directions. Participants also receive journal prompts, homework suggestions, and ideas for extensions and assessment.

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Brenda McGaffigan, principal, Santa Ana, CA

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Brad 

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Sum Thing Interesting

Finding Amazing Patterns in Addition

Overview:

This simple exploration in addition can lead to very advanced discoveries. Because the entrance point involves basic addition, any student can get on board. Then you can take them as far down the math track as you want as you explore sums of consecutive numbers, exponents, and quadratic patterns.

Required Materials:

☒ Activity master (included)

Optional Materials:

☐ Calculators

☐ Graph paper (included)

Procedure:

1. Pass out the activity master to the students. Explain that they can write most numbers as a sum of ***consecutive*** positive numbers. They will not need to use negative numbers or even zero. Explain that the word consecutive means *next to* or *adjacent*. For example, 5, 6, and 7 are consecutive numbers.
2. Give them some examples and allow them to begin working.
    a. $3 = 1 + 2$
    b. $7 = 3 + 4$
3. It will seem that they cannot solve some problems. They may have to use more than two numbers to create their target number. For example, $6 = 1 + 2 + 3$.
4. Soon they will realize that some numbers cannot be solved no matter how many consecutive positive numbers are used. For example, there is no way to solve problem 1. ($0 + 1$ is not a solution, since they were restricted to use only *positive* numbers, and 0 is not positive.)
5. Encourage the students to seek multiple solutions if possible. $9 = 4 + 5$ and can also be solved by $2 + 3 + 4$.
6. When they finish, ask them to offer their solutions. You may wish to specifically focus on numbers that can be solved multiple ways.
7. Now ask them to list the numbers that have no solution. They should find this set of numbers:

    $$\{1, 2, 4, 8, 16, 32\}$$

8. Ask them if they see a pattern. Can they predict the next few numbers in the pattern? These numbers are the ***powers of 2***. Each number is twice the preceding number.

    $2^0 = 1$

    $2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

The next three numbers in this pattern are $2^6 = 64$, $2^7 = 128$, and $2^8 = 256$.

The formula for the powers of 2 is $y = 2^n$.

9. Ask them what they notice about the solutions to 3, 6, 10, 15, 21, 28 and 36. They may see that they can be found by adding all the counting numbers.

$3 = 1 + 2$

$6 = 1 + 2 + 3$

$10 = 1 + 2 + 3 + 4$

$15 = 1 + 2 + 3 + 4 + 5$

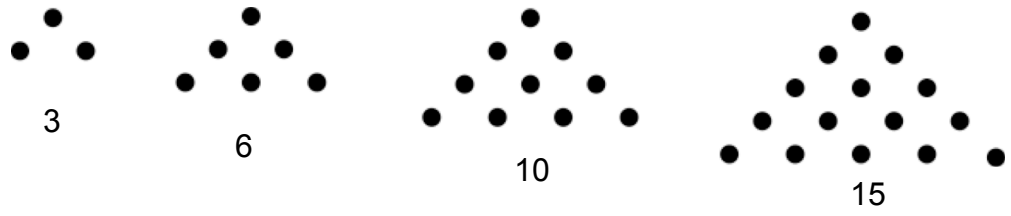
$21 = 1 + 2 + 3 + 4 + 5 + 6$

$28 = 1 + 2 + 3 + 4 + 5 + 6 + 7$

$36 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$

Ask them to find the next three numbers in this pattern. They should see that they would be 45, 55, and 66.

10. These are called ***triangular numbers*** as you can form triangles with these sets of numbers:



(One is the first triangular number; it just makes a very small triangle! Three is the second triangular number, and so on.)

The formula for the triangular numbers is $y = \frac{x^2 + x}{2}$.

For example, $\frac{4^2 + 4}{2} = \frac{16 + 4}{2} = \frac{20}{2} = 10$

11. You may wish to have students graph these patterns. They are ***nonlinear functions***. Two graphs are provided for this purpose. If you wish, you can have students graph only the positive x values of the functions. If you have more advanced students, you can have them input negative values into the formulas and graph in the first and second quadrants.

12. Other explorations can be used to extend this activity. For example, if ***consecutive odd numbers*** are added, we get the square numbers:

$$1 = 1^2$$

$$4 = 1 + 3 = 2^2$$

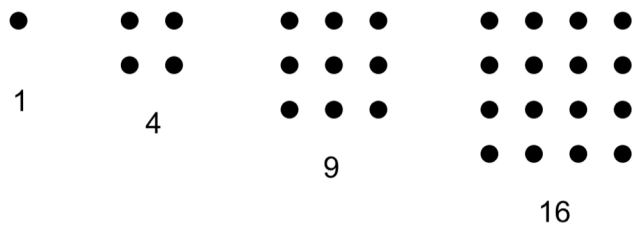
$$9 = 1 + 3 + 5 = 3^2$$

$$16 = 1 + 3 + 5 + 7 = 4^2$$

$$25 = 1 + 3 + 5 + 7 + 9 = 5^2$$

$$36 = 1 + 3 + 5 + 7 + 9 + 11 = 6^2$$

Just as triangular numbers can be made into triangles, these numbers can be arranged to build squares.



13. Advanced students may wish to explore what happens if negative numbers and zero are allowed. In that case, we can begin to solve the previously unsolved powers of 2:

$1 = \mathbf{0} + 1$

$2 = \mathbf{-1 + 0 + 1} + 2$

$4 = \mathbf{-3 + -2 + -1 + 0 + 1 + 2 + 3} + 4$

$8 = \mathbf{-7 + -6 + -5 + -4 + -3 + -2 + -1 + 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7} + 8$

Notice that the boldfaced addends cancel out (add up to zero) leaving only the power of 2.

Answer Key

1	no solution		
2	no solution		
3	1 + 2		
4	no solution		
5	2 + 3		
6	1 + 2 + 3		
7	3 + 4		
8	no solution		
9	4 + 5	2 + 3 + 5	
10	1 + 2 + 3 + 4		
11	5 + 6		
12	3 + 4 + 5		
13	6 + 7		
14	2 + 3 + 4 + 5		
15	7 + 8	4 + 5 + 6	1 + 2 + 3 + 4 + 5
16	no solution		
17	8 + 9		
18	5 + 6 + 7		
19	9 + 10		
20	2 + 3 + 4 + 5 + 6		
21	10 + 11	6 + 7 + 8	1 + 2 + 3 + 4 + 5 + 6
22	4 + 5 + 6 + 7		
23	11 + 12		
24	7 + 8 + 9		
25	12 + 13	3 + 4 + 5 + 6 + 7	
26	5 + 6 + 7 + 8		
27	13 + 14	8 + 9 + 10	2 + 3 + 4 + 5 + 6 + 7
28	1 + 2 + 3 + 4 + 5 + 6 + 7		
29	14 + 15		
30	9 + 10 + 11	4 + 5 + 6 + 7 + 8	
31	15 + 16		
32	no solution		
33	16 + 17	10 + 11 + 12	
34	7 + 8 + 9 + 10		
35	17 + 18	5 + 6 + 7 + 8 + 9	2 + 3 + 4 + 5 + 6 + 7 + 8
36	11 + 12 + 13	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	

Sum Thing Interesting

Name_____

Write each number as the sum of ***consecutive positive numbers***. In some cases, you may need to use more than two addends. Some numbers can be solved in more than one way, while others cannot be solved at all. Do you see any patterns?

1 = _____

2 = _____

3 = _____

4 = _____

5 = _____

6 = _____

7 = _____

8 = _____

9 = _____

10 = _____

11 = _____

12 = _____

13 = _____

14 = _____

15 = _____

16 = _____

17 = _____

18 = _____

19 = _____

20 = _____

21 = _____

22 = _____

23 = _____

24 = _____

25 = _____

26 = _____

27 = _____

28 = _____

29 = _____

30 = _____

31 = _____

32 = _____

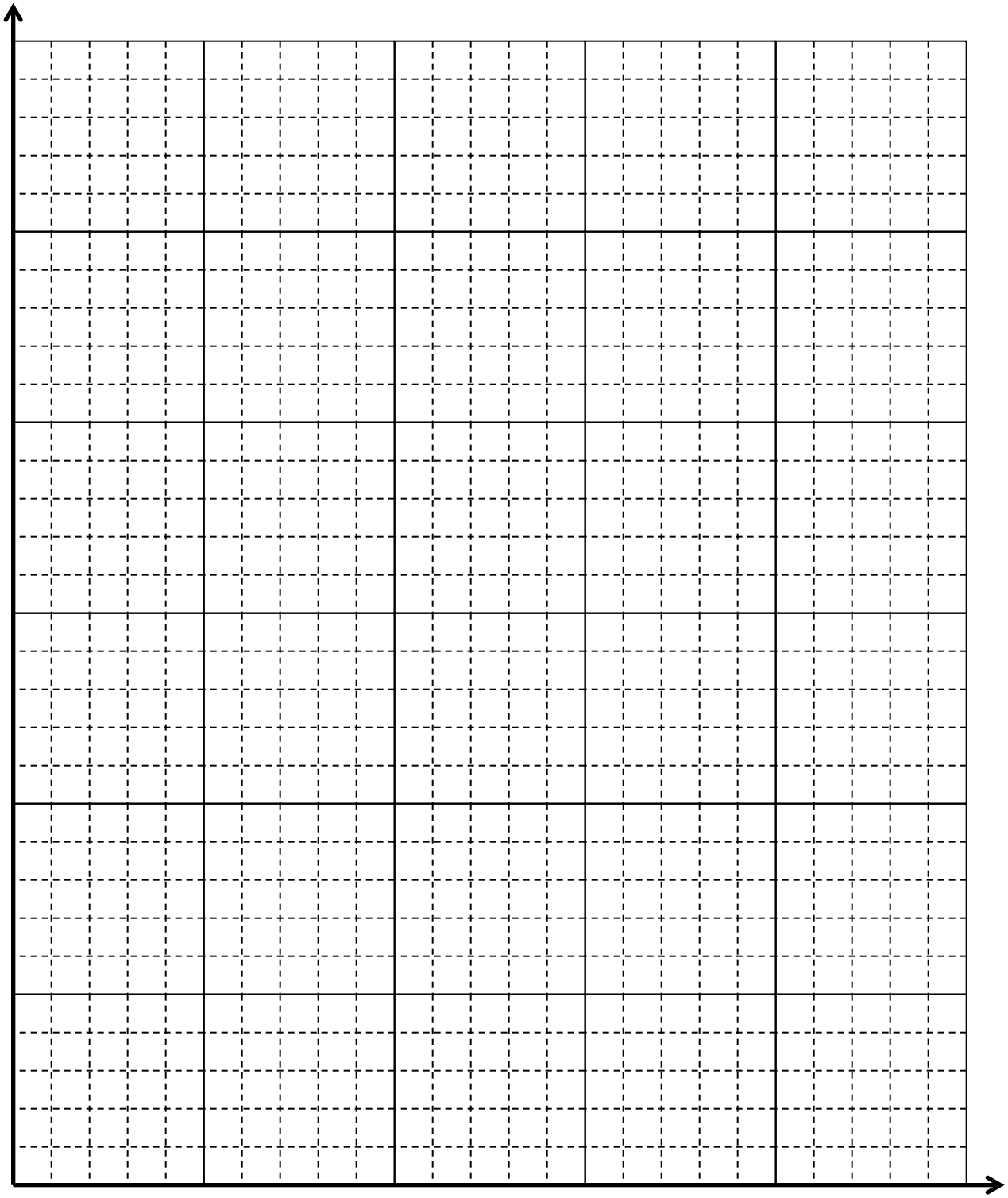
33 = _____

34 = _____

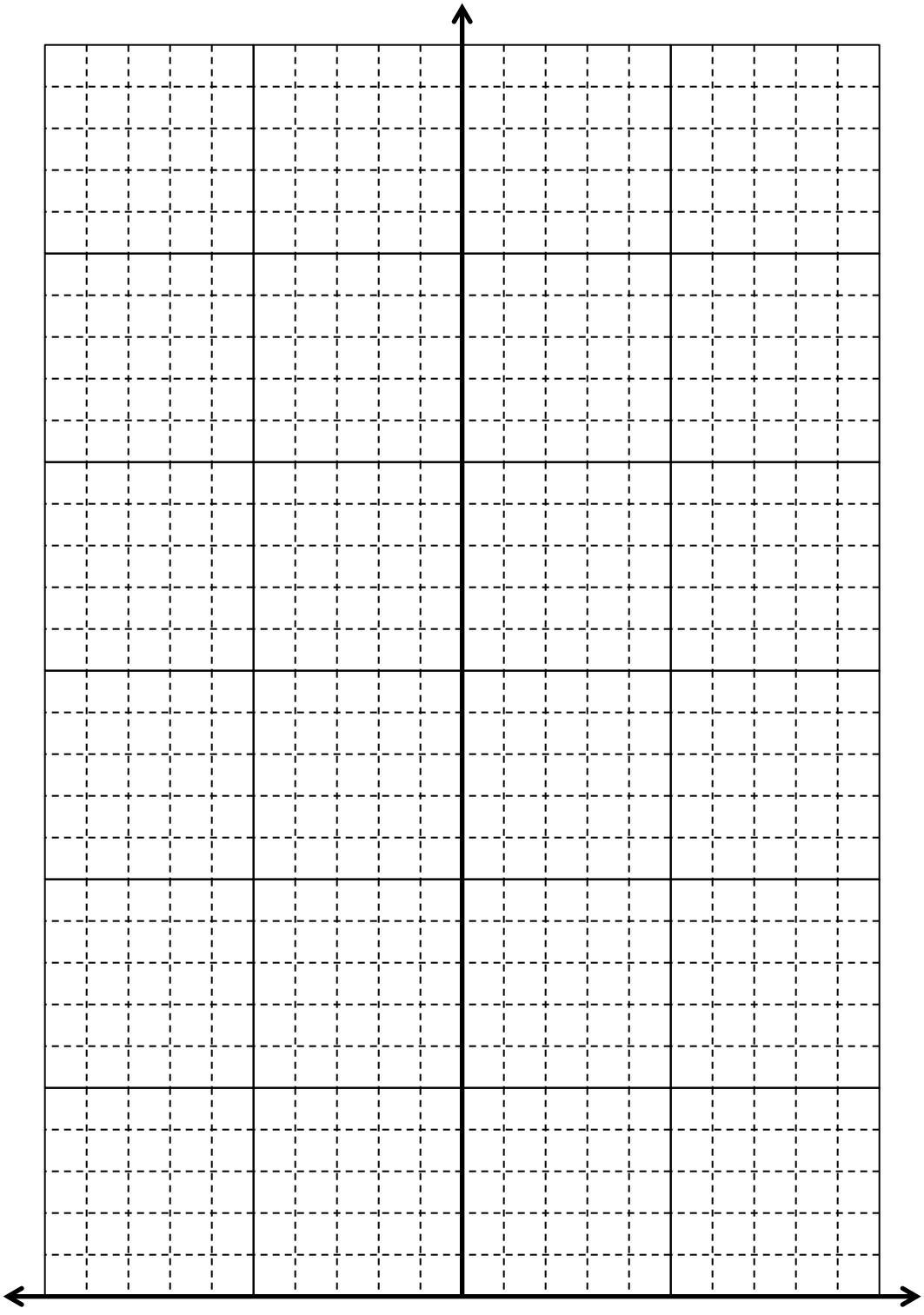
35 = _____

36 = _____

Name_____



Name_____



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Happy teaching,

Brad