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The Sunshine Citrus Company

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The Sunshine Citrus Company

As our plane leaves San Antonio and the 81st annual NCTM conference, we put the finishing touches on May's newsletter. We were pleased to meet so many of you at our presentations and strolling along the meandering San Antonio River. We see that the mathematical community is a tightly woven fabric of teachers working together to support one another for the sake of our students. With this in mind, we offer you an intriguing activity to follow up last month's newsletter on teaching ratios. This problem will not only promote the diversity of thinking in your classroom, but the solutions we offer are just as unique.

Required Materials:
- ☒ Activity Master
- ☒ Super Citrus Solution

Optional Materials:
- ☐ Transparency of Answer Key

Procedure:

1 Give each student a copy of the Activity Master and explain the problem. As an example, ask them how many lemons can be shipped in a crate that contains 9 grapefruit. Though the problem is simple enough, the solution is challenging for students. If they have learned to solve proportions they may approach the problem this way:

$$\frac{\text{Grapefruit}}{\text{Lemons}} \quad \frac{12}{20} = \frac{9}{x} \qquad 12x = 180$$

They might think this suggests that 15 lemons can be put in the case. However this solution cannot be possible since that puts a total of 24 mixed fruit in a box that can hold only 20 lemons at the most. Although the problem can be solved using proportions, it is worth exploring first using other means.

2 Ask the students to use the first t-table to explore possible combinations. Are there any combinations of grapefruit and lemon we know will fit in the box? In fact there are two obvious ones: 12 grapefruit with zero lemons and 20 lemons with zero grapefruit. Once these two solutions are put on the t-table, a third solution begins to become apparent. You could fill half the box with grapefruit (6) and the other half with lemons (10). Midway between these solutions, we have a box that is one-quarter grapefruit (3) and three-fourths lemons (15). There is also a solution that fills the box three-fourths full of grapefruit (9) and one-fourth full of lemons (5). These combinations are shown in the t-table.

3 Students may now notice the unit rate for this ratio. As the number of grapefruits increases from zero to three, the number of lemons goes down five. This is true throughout the table. That is, three grapefruit equal five lemons. Thus one grapefruit is equal to $^5/_3$ or $1^2/_3$ lemons.

$$\frac{\text{Grapefruit}}{\text{Lemons}} \quad \frac{3}{5} = \frac{1}{x} \qquad 3x = 5$$

4 Now we know how many lemons to deduct for each grapefruit added to the case. We must keep in mind though that when dealing with a fraction of a lemon the answer should be rounded down to the nearest whole number.

G	L
0	20
1	
2	
3	15
4	
5	
6	10
7	
8	
9	5
10	
11	
12	0

G	L
0	20
1	$18^1/_3 \approx 18$
2	$16^2/_3 \approx 16$
3	15
4	$13^1/_3 \approx 13$
5	$11^2/_3 \approx 11$
6	10
7	$8^1/_3 \approx 8$
8	$6^2/_3 \approx 6$
9	5
10	$3^1/_3 \approx 3$
11	$1^2/_3 \approx 1$
12	0

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The company would never ship a fraction of a lemon, nor would they try to overstuff a box and smash the fruit. In reality the remaining space would be filled with packing material. This results in the table shown here.

5 It seems that this table solves the problem. However this chart is dependent upon a customer ordering a specific number of grapefruit. But what if the customer orders twelve lemons? This is not shown in the table. For this situation, we need a second t-table that converts lemons to grapefruit. The same processes can be used to make this table as we used in the previous one. The unit rate is now 1 lemon = $^{3}/_{5}$ grapefruit. The table is shown below to the right. Notice that in both tables there is a pattern to the numbers of the rounded data.

L	G	
0	12	
1	$11^{2}/_{5}$	≈ 11
2	$10^{4}/_{5}$	≈ 10
3	$10^{1}/_{5}$	≈ 10
4	$9^{3}/_{5}$	≈ 9
5	9	
6	$8^{2}/_{5}$	≈ 8
7	$7^{4}/_{5}$	≈ 7
8	$7^{1}/_{5}$	≈ 7
9	$6^{3}/_{5}$	≈ 6
10	6	
11	$5^{2}/_{5}$	≈ 5
12	$4^{4}/_{5}$	≈ 4
13	$4^{1}/_{5}$	≈ 4
14	$3^{3}/_{5}$	≈ 3
15	3	
16	$2^{2}/_{5}$	≈ 2
17	$1^{4}/_{5}$	≈ 1
18	$1^{1}/_{5}$	≈ 1
19	$^{3}/_{5}$	≈ 0
20	0	

6 Another approach is to graph the problem. For this purpose we will consider the graph of the first t-table. If we graph the first point, (0, 20), and last point, (12, 0), on the table and connect them with a straight line, the line also passes through the points (3, 15), (6, 10), and (9, 5). The points that have been rounded off can also be shown on the graph, but they will not be on the line; they will be below it. On the answer key, these points have been connected with a dotted line and the area between it and the solid line has been shaded. You might consider the graph the following way. The solid line represents what the box is capable of holding. The dotted line represents the fruit that can actually be placed in the crate, and the shaded area represents packing material. The graph is a wonderful representation of the real-world aspects of the problem.

7 An even more visual approach to the problem is the Super Citrus Solution. Here a model of the 12 grapefruit and 20 lemons has been scaled so they are the same lengths. You might think of the space between the two dotted lines as a long box that can hold the 12 grapefruit, the 20 lemons, or any combination of the two. By sliding the joined strips between the dotted slits, students can see the different combinations that are possible.

Lastly, we can use proportions to solve the problem. If we consider the original problem posed in step one, it seems to suggest an impossible solution of 9 grapefruit and 15 lemons. However, what the problem tells us is that the 9 grapefruit *are equivalent to* 15 lemons. This means there is only room for five more lemons. This type of proportion is called an *inverse proportion*. This is demonstrated by the fact that the graph slopes downward instead of upward. Although this is a perfectly appropriate way to solve the problem, it is the least visual of the methods and should be saved for last.

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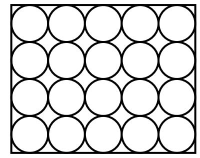
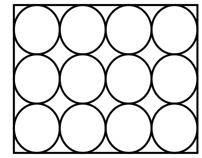
Activity Master

Name______________________



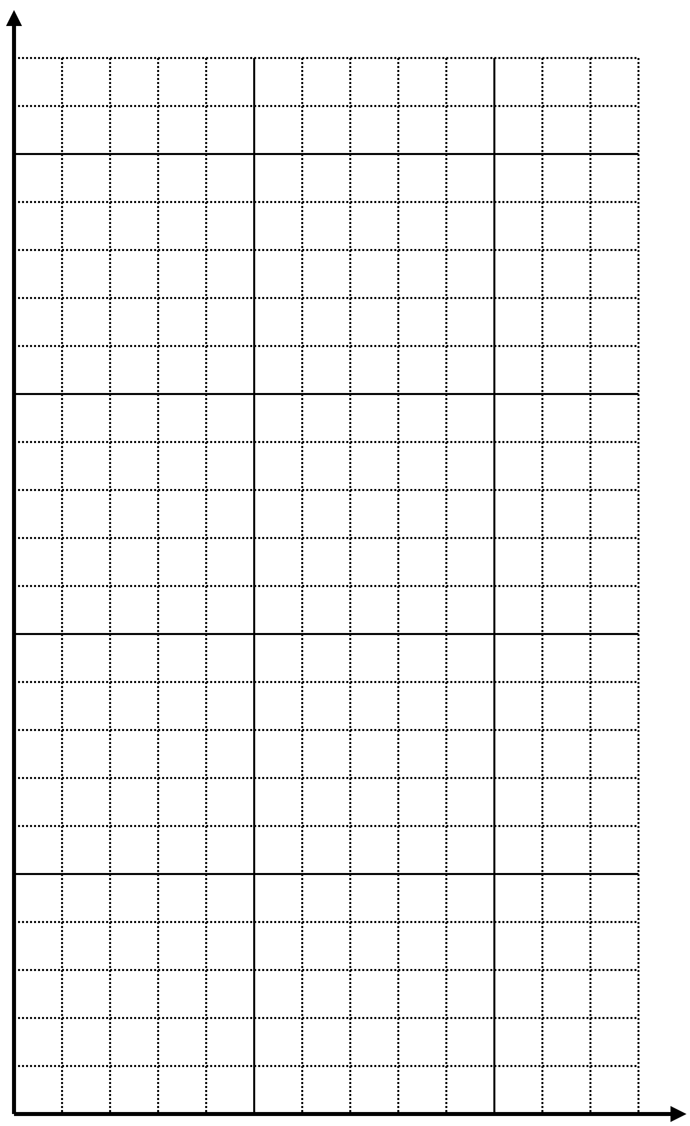


The Sunshine Citrus Company sells cases of lemons or grapefruits. Each case holds 20 lemons or 12 grapefruit. To increase sales, the company decides to ship cases of mixed fruit. They have hired you to help them decide how many of each type of fruit can be shipped in a case.

G	L
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	

L	G
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

Lemons



Grapefruit

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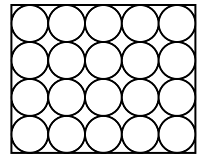
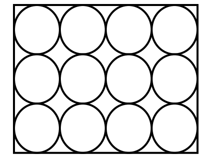
Answer Key

Name______________________





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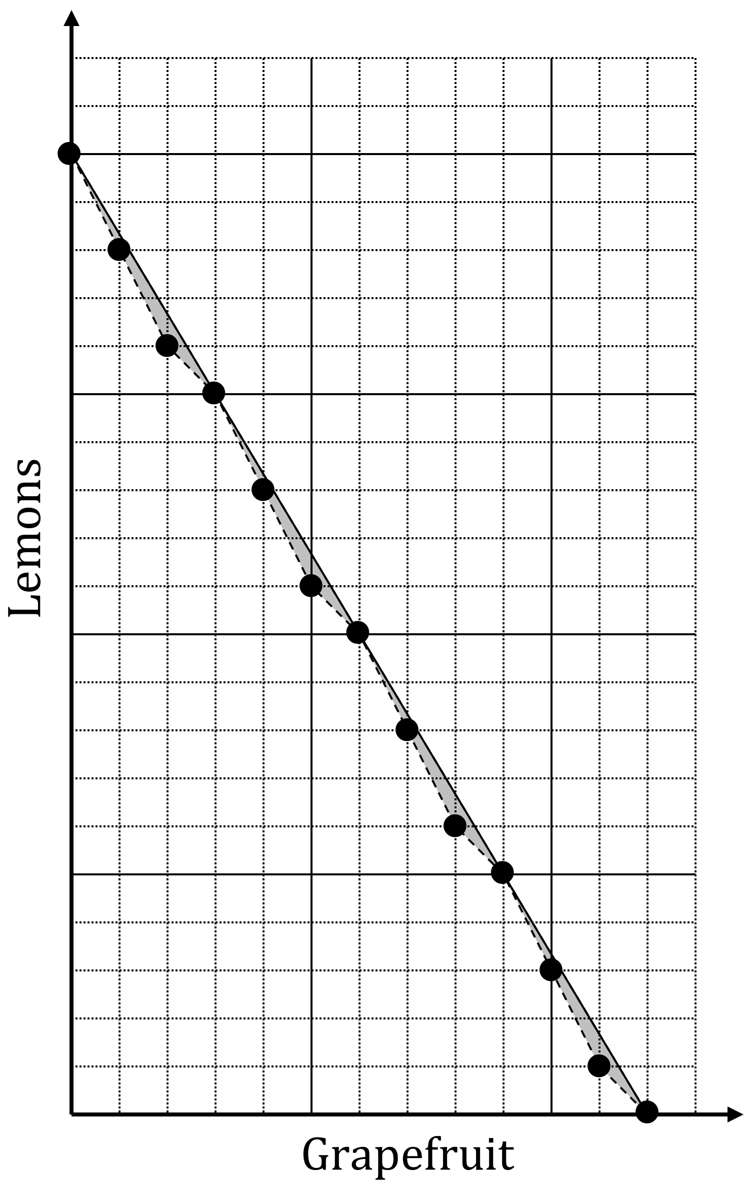
G	L
0	**20**
1	18
2	16
3	**15**
4	13
5	11
6	**10**
7	8
8	6
9	**5**
10	3
11	1
12	**0**

L	G
0	**12**
1	11
2	10
3	10
4	9
5	**9**
6	8
7	7
8	7
9	6
10	**6**
11	5
12	4
13	4
14	3
15	**3**
16	2
17	1
18	1
19	0
20	**0**



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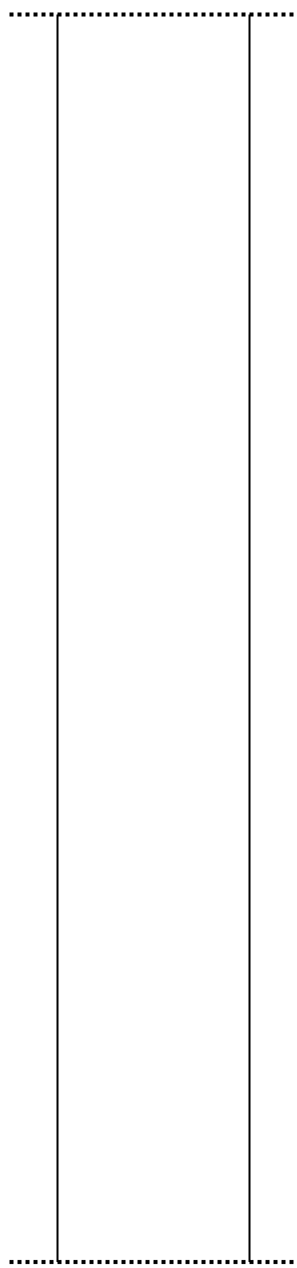
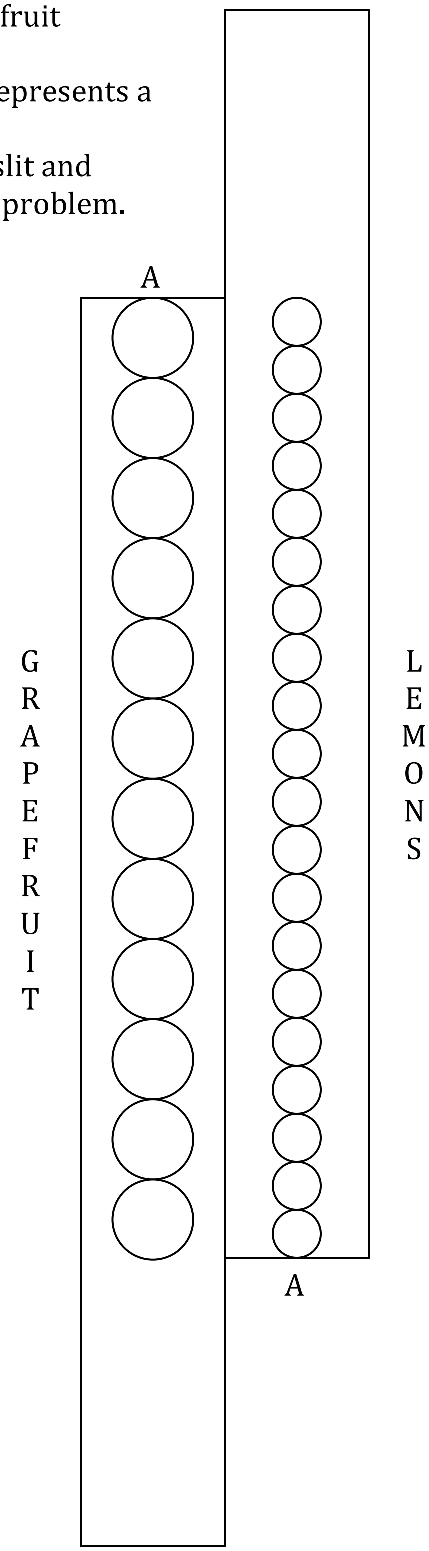
The Super Citrus Solution

1 Color the lemons yellow and the grapefruit pink.
2 Cut out the two rectangles and tape them together so end A of the lemon rectangle is joined with end A of the grapefruit rectangle.
3 Slit the two dotted lines on the left rectangle which represents a long packing case.
4 Slide the lemon and grapefruit strip up through one slit and down through the other to solve the Sunshine Citrus problem.





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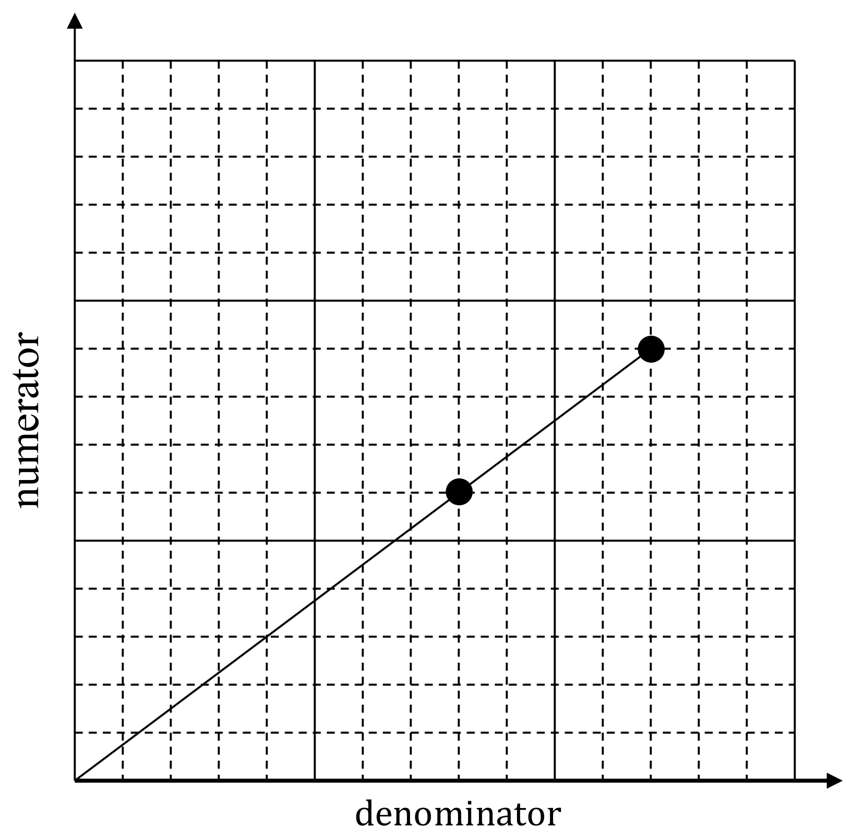
What is a proportion? The answer to this question can be somewhat technical. While a proportion can be defined simply as an equality of two ratios, how can we tell which of the following are proportions and which are not?

$$\frac{8}{18} = \frac{20}{45} \qquad \frac{9}{15} = \frac{8}{12} \qquad \frac{6}{8} = \frac{9}{12}$$

One way is by comparing cross products. Proportions have equal cross products. While this is a reliable method, it relies on a student's ability to multiply numbers correctly…twice. A graph however offers us a visual way to identify what is a proportion and what is not.
Let's graph the third proportion by plotting the denominator on a horizontal axis, and the numerator on the vertical axis. When we do this to both of the ratios in the example, we see that a line passing through them will also pass through the origin. *This is true for all proportions.*



Proportional ratios

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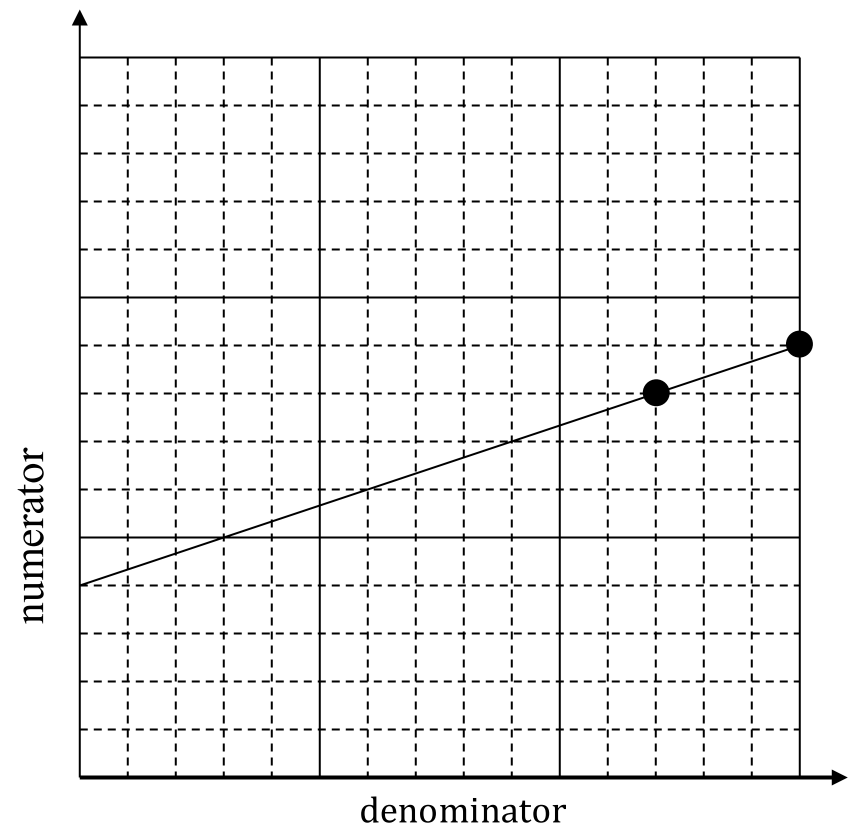
However, When we try this for the second example, we see that the two ratio points are not contiguous with the origin, proving that they are not proportional.



Nonproportional ratios

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