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## - Consultant

## - Educator

- Keynote presenter
- Teacher trainer
- Conference speaker

Known throughout the country for motivating and engaging teachers and students, Brad has coauthored over a dozen books that provide easy-to-teach yet mathematically rich activities for busy teachers while teaching full time for over 30 years. In addition, he has co-authored over 40 teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

## Seminar leader and trainer of mathematics teachers

- 2005 California League of Middle Schools Educator of the Year
- California Math Council and NCTM national featured presenter
- Lead trainer for summer teacher training institutes
- Trainer/consultant for district, county, regional, and national workshops


## Author and co-author of mathematics curriculum

- Simply Great Math Activities series: six books covering all major strands
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# Tangram Math <br> Integrating Fractions, Decimals, Percent, Geometry, and Algebra 

## Overview:

In this powerfully engaging activity students of all skill levels will study standard and nonstandard tangrams to determine the values of the pieces. Students will compare the pieces to see how they relate to one another as fractions, decimals, percent, and areas. They will also develop geometric vocabulary and form an understanding of congruence and similarity. The best part is that the entire process will lead them seamlessly into algebraic thinking as they navigate among the pieces and their representations. The activity can also be used to help students learn probability, solve proportions and equations and understand algebraic

## Required Materials:

® Student copies of the tangrams
Optional Materials:
Rulers
Scissors
Calculators
Document camera or projection device properties!

## Procedure:

1 This activity works best when students work in groups of two to four. This fosters important dialogue that facilitates understanding. Display a copy of tangram 1. Students should have individual copies. You may wish to distribute scissors for this activity as some students find it helpful to cut the pieces for comparison.

2 Ask the students, "If the entire square tile has a value of 1 , what is the value of the region $a$ ?" They can see that it is $1 / 4$ since four of the large triangles can fit in the square.

3 Next ask them to evaluate the medium-sized triangle, region b. Since two b 's will fit into one $\mathrm{a}, \mathrm{b}$ is $\frac{1}{2}$ of a, or $1 / 8$ of the whole. Another way to see this is by showing that the entire tile can be cut into eight b's.

$2 \mathrm{c}=\mathrm{b}$

$2 \mathrm{c}=\mathrm{d}$

4 Ask the students to find the values of the other regions. When they find answers, ask them to justify them. They may do this verbally, by

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rearranging pieces on their desks or on your projection device. They will see that the small triangle c is ${ }^{1} / 16$ of the tile since two of them can fit into b . Since two small triangles also fit into the square, $d$ is equal to $b$. Two c's also fit into the parallelogram e, so $\mathrm{b}, \mathrm{d}$, and e are all $1 / 8$. This is shown in the margin on the previous page.

5 Another way to show the relationships among the polygons is by drawing lines to subdivide the tangram into the smallest unit (in this case, triangle $c$ ) as shown here in the margin. It is then easy to see that c is $1 / 16$ of the tile. Regions b, d, and e are each $2 / 16$ or $1 / 8$ of the tile, and region a is $4 / 16$ or $1 / 4$ of the tile. This cut-up method will not work on all of the other tangram patterns however.


6 After the students have found the fractional values of each piece, they can add them together to check their work. Remember that there are two of shape a and two of shape c. Adding all of these pieces gives a total of ${ }^{16} / 16$ or 1 (whole tangram).

7 If students are familiar with decimals, you can ask them to find the decimal values of each piece. They will find that $\mathrm{a}=0.25$. Ask them to explain their reasoning. They may say that in money a quarter is $\$ .25$.

8 When they try to find the value of b , students may give different answers. Beginning students fail to see that c , which is $1 / 8$, has a decimal representation of 0.125 . They may think that 0.125 is greater than 0.25 since it has more places. I have seen students suggest that $\mathrm{b}=0.12^{1 / 2}$. This may confuse some as it incorporates both decimal and common fractions, but essentially it is correct. We see these sorts of representations on gasoline prices: $\$ 2.99^{9} / 10$. By annexing a zero and writing the value of a as 0.250 instead of 0.25 , many students are able to halve the 0.250 and get 0.125 .

9 I have also seen students suggest that c has a value of 0.625 . They annex a zero on b to get 0.1250 and then cut it in half without regard to the place value. These discrepancies will be discovered when students check their answers by adding all the decimals (keeping in mind that there are two a's and two c's). They should get a total value of 1.0000 .

10 Next you can ask students to write these as percent representations. This should be much easier than it typically is since students are beginning to see the connections among the shapes and their other representations. In fact, they will see that half of $25 \%$ (a) is $12 \frac{1}{2} \%$ (b). Similarly, if they had written that c had a value of 0.625 , they will think it has a percent value of $62.5 \%$. This is a contradiction since they can see that it is not over half the total shape. However, by writing the correct answer of 0.0625 , they can then convert that to $61 / 4 \%$. Adding the percent values of the pieces yields $100 \%$.

11 Students can also calculate the areas of the pieces. Have the students measure the base and height of large triangle $a$. They will see that the base is four inches long and the height is equal to two inches. Using the area formula for a triangle leads to:

$$
A=\frac{4 \cdot 2}{2}=4 \mathrm{sq} \mathrm{in}
$$

The students may not need to use these formulas if they make connections with the values of the pieces. For example, since a has an area of $4 \mathrm{in}^{2}$, and $b=1 / 2$ of $a$, it must have an area of $2 \mathrm{in}^{2}$.

12 Ask them to find the areas of the other triangular regions using the same formula. This will show that medium-sized triangle $b$ has a base and height of two inches and an area of two square inches.

$$
A=\frac{2 \cdot 2}{2}=2 \text { sq.in. }
$$

The small triangle has a base of two inches, a height of one inch, and an area of one square inch.

$$
A=\frac{2 \cdot 1}{2}=1 \text { sq.in. }
$$

13 The students should now find the area of parallelogram e. They can see that it is composed of two small triangles (c), so its area must be two square inches. Measuring its base and height shows they are equal to two inches and one inch respectively. Multiplying these gives us the area:

$$
A=2 \cdot 1=2 \text { sq. in. }
$$

14 The square will be more difficult to solve. Measuring the side shows it to be approximately $1 / 8$ inches. Squaring this shows that the area is $15 / 64$ square inches. This is almost two square inches. The Pythagorean theorem is more accurate in this
case. The sides of the square will represent legs $a$ and $b$ of a right triangle. Since the sides are equal, we will use only the letter a. The diagonal of the square is the hypotenuse, c. Since measuring the hypotenuse shows that it is two inches long, the Pythagorean Theorem is written:

$$
\begin{aligned}
a^{2}+b^{2} & =c^{2} \\
a^{2}+a^{2} & =2^{2} \\
2 a^{2} & =2^{2} \\
2 a^{2} & =4 \\
a^{2} & =2 \text { (the area of the square) } \\
a=\sqrt{2} & \approx 1.414 \mathrm{in} .
\end{aligned}
$$

15 Students should also write the names of each shape. Younger students may want to label shape a with the name triangle. I have my middle school students use the full name: isosceles right triangle. Similarly, some students will say that shape $d$ is a diamond. Others may opt for the more formal term, rhombus. However, the most specific name is square. The fact that it has been rotated from a familiar orientation causes many students to assume it is no longer a square. This is because most squares they have seen have a base parallel to

## Good Tip

This activity even can be used as a formative assessment tool! When students talk about their thinking, their vocabulary will show the level of sophistication in their thinking as explained here.
 the edges of their paper. When students change the name of a shape when it is rotated, it shows that they are not functioning at a high level of geometric thinking. They assume that shapes are defined by orientation instead of by their properties. Since the formal definition of a square is a quadrilateral with four congruent sides and four congruent angles, d is a square.
Remind them that triangles are always described by their angles and their sides.
There are three classifications of each:

## angles

acute - all three angles are less than $90^{\circ}$
right - one angle is exactly $90^{\circ}$
obtuse - one angle is more than $90^{\circ}$
sides
equilateral - all three sides are equal
isosceles - two sides are equal
scalene - no sides are equal
16 Notice that as students discuss their thinking in their group, they will be talking algebraically! You will hear statements such as, "Two c's is a d," and, "D is equal to b." These can then be written as algebraic equations: $2 \mathrm{c}=\mathrm{d}$ and $\mathrm{d}=\mathrm{b}$. I require my students to write an equation for each shape. The equation must be correct and must contain the letter for that region. For example, these five equations could be used for Tangram 1:

| Region |  | Equation |
| :--- | :--- | :--- |
|  |  | $4 \mathrm{c}=\mathbf{a}$ |
| $b$ | $\mathbf{b}=2 \mathrm{~d}$ |  |
| c | $\mathrm{a} / 4=\mathbf{c}$ |  |
| d | $\mathbf{d}=\mathrm{e}$ |  |
| e | $2 \mathrm{a}=\mathrm{b}+2 \mathrm{c}+\mathrm{d}+\mathbf{e}$ |  |

17 If you want to explore proportions, assign a new value to the tangram. For example, if the entire tangram has a value of $\$ 3.00$, what is the value of $a$ ? Since $a=1 / 4$ of the total, it has a value of \$.75.
If $b$ has a value of $1 / 2$, what is the value of the entire tangram? Since 8 b's can fit in the tangram, the total value of the tangram is $8 \cdot 1 / 2=4$.

18 Advanced students can calculate the perimeters of the pieces. However, this will require the use of the Pythagorean theorem or very accurate measurement. Each tangram has a side length of 4 inches. Therefore the legs of triangle $b$ are each 2 " long. The Pythagorean theorem gives the length of the hypotenuse as:

$$
\begin{gathered}
\mathrm{a}^{2}+\mathrm{b}^{2}=\mathrm{c}^{2} \\
2^{2}+2^{2}=\mathrm{c}^{2} \\
4+4=\mathrm{c}^{2} \\
8=\mathrm{c}^{2} \\
2.83 \approx \mathrm{c} \\
\text { Perimeter }=\mathrm{a}+\mathrm{b}+\mathrm{c} \approx 2+2+2.83=6.83
\end{gathered}
$$

19 Have students explore the other tangram patterns. The first six involve fractions that are based on halves such as $1 / 4,1 / 8$, and $1 / 16$. Halves are easier to work with conceptually than thirds or fifths. For this reason, only the last two tangrams involve thirds or fifths. Also, in the first three tangrams, all areas use unit fractions; that is,
their numerators are one. This changes in tangrams four, five, and six where we start to see fractions such as $3 / 8$. Again this represents another conceptual step in part/whole thinking.


## Journal Prompts:



Explain how polygons $\mathrm{b}, \mathrm{d}$, and e are alike and how they are different. Why does the cut-up method only work on some tiles but not on others?

## Homework:



You may wish to assign an unfinished tangram as homework.
Another option is to have students make a tile of their own and write fractions for each region. It should be made of at least seven sections using at least five different shapes. This should be drawn on a 4" or a 6 " square. You may wish to pass out the included grid paper for this task.

## Taking a Closer Look: <br> 

Assign a cost or value to various regions. If the medium-sized triangle of tangram one sells for $\$ 1.37$, what is the cost of each region and the whole tile? If tangram two costs $\$ 5.64$, what would each piece cost?

Explore probability. What is the probability of a dart randomly landing on c ?
$P(c)=1 / 16$
What is the probability of a dart randomly hitting a quadrilateral?
$\mathrm{P}($ quadrilateral $)=\mathrm{d}+\mathrm{e}=1 / 8+1 / 8=1 / 4$

## Assessment:

By allowing students to work in groups and by asking them to rationalize their answers, you will be able to assess their levels of understanding. Listen to their
discussions as mentioned previously. Are they using sophisticated reasoning and accurate vocabulary?

Homework can be assessed after collecting it using the enclosed answer keys, or you may wish to have students trade papers and solve each other's puzzles.

## Answer Key

Tangram 1

| Region | Name |
| :--- | :--- |
| a | Isosceles right triangle |
| b | Isosceles right triangle |
| c | Isosceles right triangle |
| d | Square |
| e | Parallelogram |

Tangram 2

| Region | Name | Frac. | Dec. | $\%$ | Area |
| :--- | :--- | :--- | :--- | :--- | :--- |
| a | Isosceles right triangle | $1 / 4$ | 0.25 | 25 | $4 \mathrm{in}^{2}$ |
| b | Isosceles right triangle | $1 / 16$ | 0.0625 | 6.25 | $1 \mathrm{in}^{2}$ |
| c | Isosceles right triangle | $1 / 8$ | 0.125 | 12.5 | $2 \mathrm{in}^{2}$ |
| d | Rectangle | $1 / 4$ | 0.25 | 25 | $4 \mathrm{in}^{2}$ |
| e | Scalene right triangle | $1 / 16$ | 0.0625 | 6.25 | $1 \mathrm{in}^{2}$ |
| f | Obtuse isosceles triangle | $1 / 8$ | 0.125 | 12.5 | $2 \mathrm{in}^{2}$ |

Tangram 3

| Region | Name |
| :--- | :--- |
| a | Rectangle |
| b | Scalene right triangle |
| c | Square |
| d | Rectangle |
| e | Acute isosceles triangle |
| f | Rhombus |
| g | Obtuse isosceles triangle |
| h | Acute isosceles triangle |

## Tangram 4

Region Name
a Isosceles right triangle
b Isosceles right triangle
c Right trapezoid
d Isosceles right triangle
e Isosceles trapezoid

| Frac. | Dec. | \% | Area |
| :--- | :--- | :--- | :--- |
| $1 / 4$ | 0.25 | 25 | 4 in $^{2}$ |
| $1 / 8$ | 0.125 | 12.5 | 2 in $^{2}$ |
| $1 / 16$ | 0.0625 | 6.25 | 1 in $^{2}$ |
| $1 / 8$ | 0.125 | 12.5 | 2 in $^{2}$ |
| $1 / 8$ | 0.125 | 12.5 | 2 in $^{2}$ |


| Frac. | Dec. | \% | Area |
| :--- | :--- | :--- | :--- |
| $1 / 8$ | 0.125 | 12.5 | 2 in $^{2}$ |
| $1 / 16$ | 0.0625 | 6.25 | $1 \mathrm{in}^{2}$ |
| $1 / 16$ | 0.0625 | 6.25 | $1 \mathrm{in}^{2}$ |
| $1 / 32$ | 0.03125 | 3.125 | $.5 \mathrm{in}^{2}$ |
| $1 / 8$ | 0.125 | 12.5 | $2 \mathrm{in}^{2}$ |
| $1 / 4$ | 0.25 | 25 | $4 \mathrm{in}^{2}$ |
| $1 / 32$ | 0.03125 | 3.125 | $.5 \mathrm{in}^{2}$ |
| $1 / 32$ | 0.03125 | 3.125 | $.5 \mathrm{in}^{2}$ |


| Frac. | Dec. | \% | Area |
| :--- | :--- | :--- | :--- |
| $1 / 4$ | 0.25 | 25 | 4 in $^{2}$ |
| $1 / 8$ | 0.125 | 12.5 | 2 in $^{2}$ |
| $1 / 8$ | 0.125 | 12.5 | 2 in $^{2}$ |
| $1 / 16$ | 0.0625 | 6.25 | 1 in $^{2}$ |
| $3 / 16$ | 0.1875 | 18.75 | 3 in $^{2}$ |

Tangram 5

| Region | Name |
| :--- | :--- |
| a | Rectangle |
| b | Scalene right triangle |
| c | Isosceles right triangle |
| d | Isosceles right triangle |
| e | Isosceles trapezoid |
| f | Right trapezoid |

Tangram 6

| Region | Name |
| :--- | :--- |
| a | Rectangle |
| b | Acute isosceles triangle |
| c | Acute right triangle |
| d | Rectangle |
| e | Isosceles trapezoid |
| f | Scalene right triangle |
| g | Acute isosceles triangle |
| h | Parallelogram |

Tangram 7

| Region | Name |
| :--- | :--- |
| a | Rectangle |
| b | Obtuse isosceles triangle |
| c | Scalene right triangle |
| d | Rectangle |
| e | Rectangle |
| f | Scalene right triangle |

Tangram 8

| Region | Name |
| :--- | :--- |
| a | Rectangle |
| b | Scalene right triangle |
| c | Obtuse isosceles triangle |
| d | Acute isosceles triangle |
| e | Scalene right triangle |
| f | Parallelogram |
| g | Rectangle |
| h | Acute isosceles triangle |
| i | Concave pentagon |


| Frac. | Dec. | \% | Area |
| :--- | :--- | :--- | :--- |
| $1 / 8$ | 0.125 | 12.5 | 2 in $^{2}$ |
| $1 / 16$ | 0.0625 | 6.25 | $1 \mathrm{in}^{2}$ |
| $1 / 16$ | 0.0625 | 6.25 | $1 \mathrm{in}^{2}$ |
| $1 / 32$ | 0.03125 | 3.125 | $.5 \mathrm{in}^{2}$ |
| $3 / 32$ | 0.09375 | 9.375 | $1.5 \mathrm{in}^{2}$ |
| $3 / 8$ | 0.375 | 37.5 | 6 in $^{2}$ |


| Frac. | Dec. | 0 | Area |
| :--- | :--- | :--- | :--- |
| $1 / 8$ | 0.125 | 12.5 | 2 in $^{2}$ |
| $1 / 16$ | 0.0625 | 6.25 | $1 \mathrm{in}^{2}$ |
| $1 / 32$ | 0.03125 | 3.125 | $.5 \mathrm{in}^{2}$ |
| $1 / 8$ | 0.125 | 12.5 | 2 in $^{2}$ |
| $3 / 16$ | 0.1875 | 18.75 | 3 in $^{2}$ |
| $1 / 16$ | 0.0625 | 6.25 | 1 in $^{2}$ |
| $1 / 8$ | 0.125 | 12.5 | 2 in $^{2}$ |
| $3 / 16$ | 0.1875 | 18.75 | 3 in $^{2}$ |


| Frac. | Dec. | $\%$ | Area |
| :--- | :--- | :--- | :--- |
| $1 / 3$ | $0 . \overline{3}$ | $33^{1 / 3}$ | $5^{1 / 3}$ in $^{2}$ |
| $1 / 6$ | $0.1 \overline{6}$ | $16^{2} / 3$ | $2^{2} / 3$ in $^{2}$ |
| $1 / 12$ | $0.08 \overline{3}$ | $8^{1 / 3}$ | $11 / 3$ in $^{2}$ |
| $1 / 6$ | $0.1 \overline{6}$ | $16^{2} / 3$ | $2^{2} / 3$ in $^{2}$ |
| $1 / 12$ | $0.08 \overline{3}$ | $8^{1 / 3}$ | $11 / 3$ in $^{2}$ |
| $1 / 24$ | $0.041 \overline{6}$ | $4^{1 / 6}$ | $2 / 3$ in $^{2}$ |


| Frac. | Dec. | \% | Area |
| :--- | :--- | :--- | :--- |
| $1 / 5$ | 0.2 | 20 | $3.2 \mathrm{in}^{2}$ |
| $1 / 10$ | 0.1 | 10 | $1.6 \mathrm{in}^{2}$ |
| $1 / 20$ | 0.05 | 5 | $0.8 \mathrm{in}^{2}$ |
| $1 / 20$ | 0.05 | 5 | $0.8 \mathrm{in}^{2}$ |
| $1 / 20$ | 0.05 | 5 | $0.8 \mathrm{in}^{2}$ |
| $1 / 10$ | 0.1 | 10 | $1.6 \mathrm{in}^{2}$ |
| $1 / 20$ | 0.05 | 5 | $0.8 \mathrm{in}^{2}$ |
| $1 / 10$ | 0.1 | 10 | $1.6 \mathrm{in}^{2}$ |
| $1 / 5$ | 0.2 | 20 | $3.2 \mathrm{in}^{2}$ |

Look at all the Common Core Standards you can teach with tangrams!

## $4^{\text {th }}$ grade:

CCSS.MATH.CONTENT.4.NF.A. 1
Explain why a fraction $a / b$ is equivalent to a fraction $(n \times a) /(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
CCSS.MATH.CONTENT.4.NF.A. 2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1 / 2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>,=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

CCSS.MATH.CONTENT.4.NF.B. 3
Understand a fraction $a / b$ with $a>1$ as a sum of fractions $1 / b$.
CCSS.MATH.CONTENT.4.NF.B.3.A
Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

CCSS.MATH.CONTENT.4.NF.B.3.B
Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: $3 / 8=1 / 8+1 / 8+1 / 8 ; 3 / 8=1 / 8+2 / 8 ; 21 / 8$ $=1+1+1 / 8=8 / 8+8 / 8+1 / 8$.

CCSS.MATH.CONTENT.4.G.A. 2
Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

## $5^{\text {th }}$ grade:

## CCSS.MATH.CONTENT.5.NBT.A. 1

Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $1 / 10$ of what it represents in the place to its left.

## CCSS.MATH.CONTENT.5.NBT.A. 3

Read, write, and compare decimals to thousandths.

## CCSS.MATH.CONTENT.5.NBT.A.3.A

Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392=3 \times 100+4 \times 10+7 \times 1+3 \times(1 / 10)+9 \times(1 / 100)+2 \times$ (1/1000).

CCSS.MATH.CONTENT.5.NBT.A.3.B
Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.

## CCSS.MATH.CONTENT.5.NF.A. 1

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 $+5 / 4$ $=8 / 12+15 / 12=23 / 12$. (In general, $a / b+c / d=(a d+b c) / b d$.)

CCSS.MATH.CONTENT.5.NF.A. 2
Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2 / 5+1 / 2=3 / 7$, by observing that $3 / 7<1 / 2$.

CCSS.MATH.CONTENT.5.G.B. 3
Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.

## $6^{\text {th }}$ grade:

CCSS.MATH.CONTENT.6.RP.A. 1
Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate $A$ received, candidate $C$ received nearly three votes."
CCSS.MATH.CONTENT.6.RP.A. 3
Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

CCSS.MATH.CONTENT.6.RP.A.3.C
Find a percent of a quantity as a rate per 100 (e.g., $30 \%$ of a quantity means $30 / 100$ times the quantity); solve problems involving finding the whole, given a part and the percent.

## CCSS.MATH.CONTENT.6.RP.A.3.D

Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
CCSS.MATH.CONTENT.6.NS.C. 6
Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
CCSS.MATH.CONTENT.6.EE.A. 3
Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2+x)$ to produce the equivalent
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expression $6+3 x$; apply the distributive property to the expression $24 x+18 y$ to produce the equivalent expression $6(4 x+3 y)$; apply properties of operations to $y+y+y$ to produce the equivalent expression $3 y$.
CCSS.MATH.CONTENT.6.EE.A. 4
Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y+y+y$ and $3 y$ are equivalent because they name the same number regardless of which number y stands for.
CCSS.MATH.CONTENT.6.EE.B. 6
Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

## CCSS.MATH.CONTENT.6.G.A. 1

Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

## $7^{\text {th }}$ grade:

CCSS.MATH.CONTENT.7.RP.A. 2
Recognize and represent proportional relationships between quantities.
CCSS.MATH.CONTENT.7.RP.A.2.C
Represent proportional relationships by equations. For example, if total cost tis
proportional to the number $n$ of items purchased at a constant price $p$, the relationship between the total cost and the number of items can be expressed as $t=p n$.
CCSS.MATH.CONTENT.7.NS.A. 2
Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
CCSS.MATH.CONTENT.7.NS.A.2.D
Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

CCSS.MATH.CONTENT.7.EE.B. 4
Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
CCSS.MATH.CONTENT.7.G.B. 6
Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

## $8^{\text {th }}$ grade:

CCSS.MATH.CONTENT.8.NS.A. 2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\pi^{2}$ ). For example, by truncating the decimal expansion of $\sqrt{ } 2$, show that $\sqrt{ } 2$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

## CCSS.MATH.CONTENT.8.NS.A. 1

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually

## CCSS.MATH.CONTENT.8.EE.A. 2

Use square root and cube root symbols to represent solutions to equations of the form $x^{2}=p$ and $x^{3}=\mathrm{p}$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{ } 2$ is irrational.

CCSS.MATH.CONTENT.8.G.A. 2
Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
CCSS.MATH.CONTENT.8.G.A. 4
Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

CCSS.MATH.CONTENT.8.G.A. 5
Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

## CCSS.MATH.CONTENT.8.G.B. 7

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Tangram 1
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Date $\qquad$ Class $\qquad$


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Tangram 2
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Tangram 3
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## Tangram 6

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## Tangram 7

Name $\qquad$

Date $\qquad$ Class $\qquad$





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Happy teaching,
Brad
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