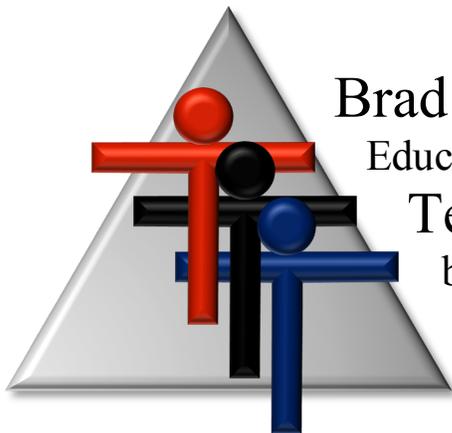
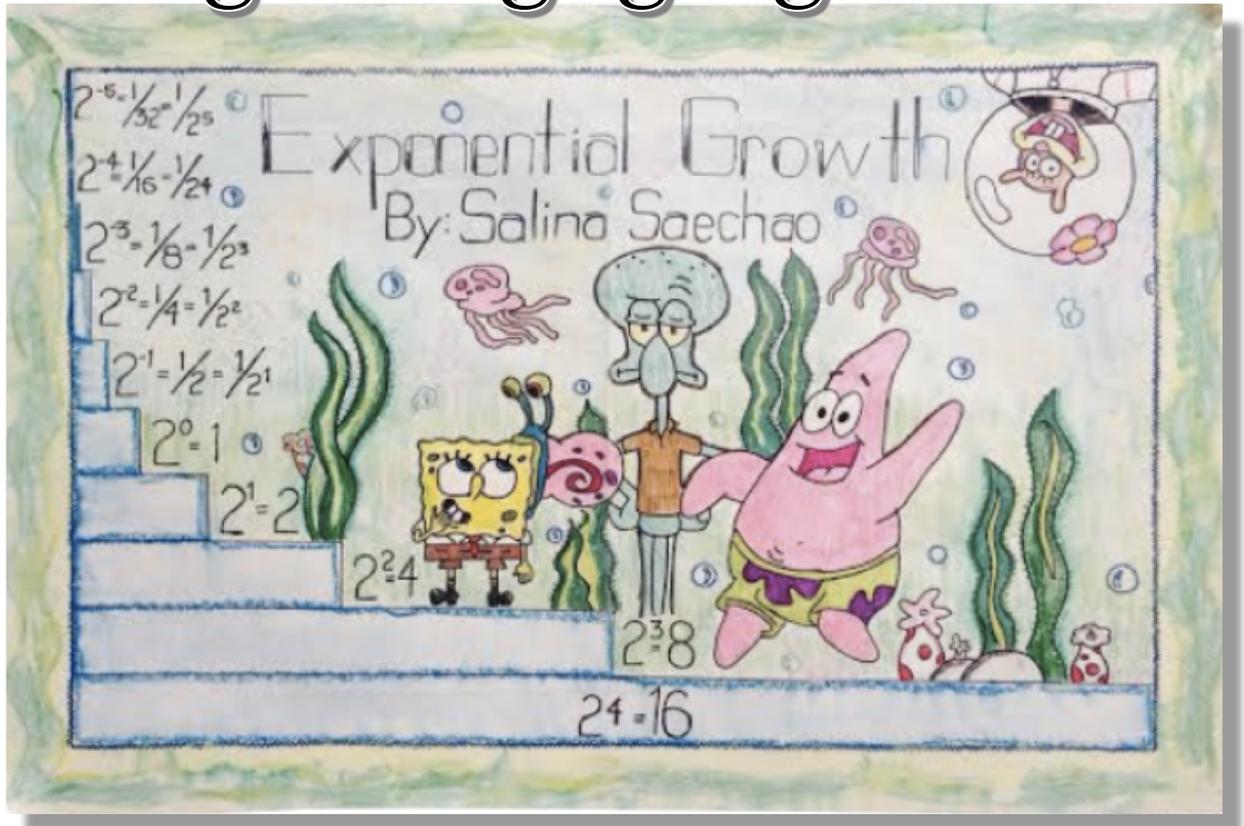


Understanding Exponents Through Engaging Activities



Brad Fulton

Educator of the Year, 2005

Teacher to Teacher Press

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Brad

Teaching Exponents for Greater Understanding

The activities in this handout are from a publication titled *More Power² You*. That book was born out of my masters research on the teaching of exponents, exponential growth, and how students conceptualize these topics.

Too often students encounter exponents in middle grades without any conceptual understanding of them. They have no number sense for exponential growth. Textbooks often add to this problem by oversimplifying the concepts. Many textbooks include statements like, “Exponents are simply repeated multiplication.” Not only do students not see them as simple, this definition is faulty and leads to faulty conclusions in our students.

Such definitions are often accompanied by examples such as $2^3=2\cdot 2\cdot 2=8$. This works fine for positive whole number exponents. However, it leads students to conclude that 2^0 means no twos exist and that 2^0 must be equal to zero. That is a logical conclusion based on that definition. Moreover, by that definition and example, a student would be fair to mistakenly conclude that $2^{-3}=2\div 2\div 2=1/2$.

One textbook tried to prove that $2^0=1$ by this process:

$$\frac{7^3}{7^3} = 1$$

$$\frac{7^3}{7^3} = 7^{3-3} = 7^0$$

$$\textit{Therefore, } 7^0 = 1$$

This hardly constitutes a “proof” as it is only demonstrating that $7^0=1$. Worse, it requires students to understand the transitive property of equality, a mathematical proof, *and* the laws of exponents!

One publisher went further by stating that mathematicians had *invented* the idea that negative exponents are fractions, so there is “nothing to understand.”

What exponents really represent are a multiplicative growth as opposed to an additive growth. For example, a base of two means that we are *doubling* something, and this doubling usually happens repeatedly. The idea that doubling something three times gets us to 8 is only true if we begin with something. Doubling zero never gets us anywhere. So, what do we start with in our growth? It turns out, that though we don’t write it this way, we always begin with the *multiplicative identity*.

Let’s back up and consider addition. If we model $3+2$ on a number line, we could start at zero, *the additive identity*. Then we take three steps to the right followed by two more steps to the right to land on five. However, we don’t write the problem as $0+3+2$. We omit the implied identity of addition.

Similarly, in multiplication, the identity is implied. $3 \cdot 2$ could be written as $1 \cdot 3 \cdot 2$ meaning, start with 1, triple it, then double that. This lands us on six.

What this suggests in regard to exponential notation is this: “Two to the third power” means “Start at the multiplicative identity of one, and double it three times: $1 \cdot 2 \cdot 2 \cdot 2 = 8$.” This means that 2^2 implies $1 \cdot 2 \cdot 2 = 4$. Similarly, 2^1 is $1 \cdot 2$, and thus 2^0 is 1 with no doubling. Likewise, 2^{-3} would imply we are considering $1 \div 2 \div 2 \div 2$ which is $\frac{1}{8}$.

The activities in this handout lead students to discover these concepts themselves through kinesthetic, visual, and conceptual applications. They are high-interest, engaging activities that have been proven to provide students with a deep understanding of exponential growth and a fluency with the exponential notation we use to represent it.

16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31

8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31

4	5	6	8
13	14	15	16
21	22	23	24
28	29	30	31

2	3	6	7
10	11	14	15
18	19	22	23
26	27	30	31

1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31

Birthday Magic!

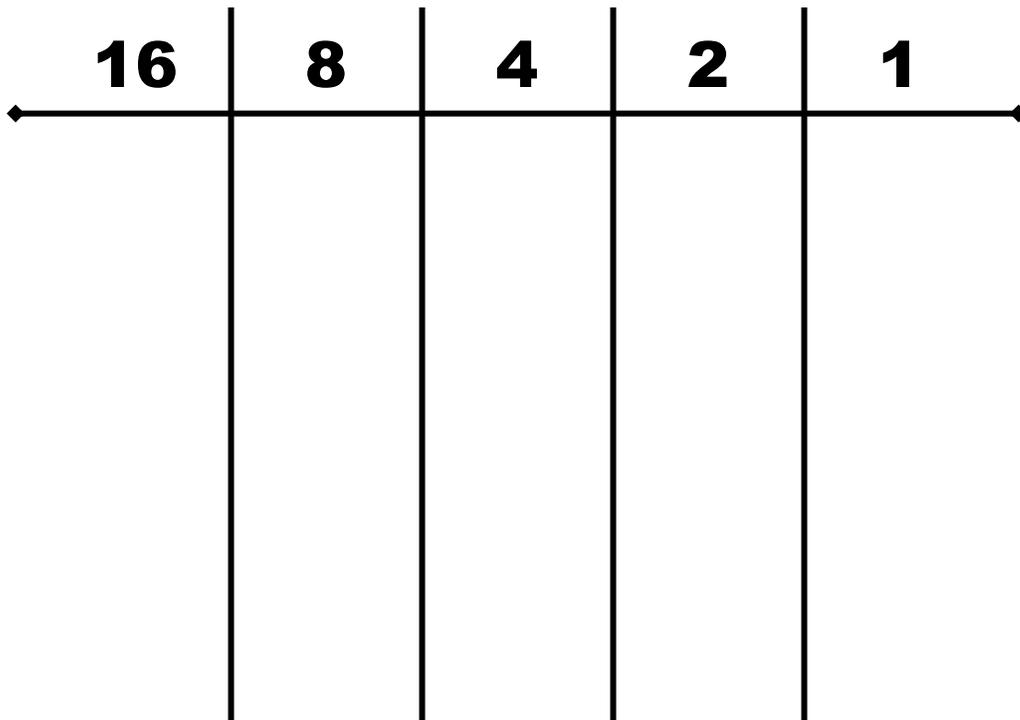
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31

8	9	10	11
12	13	14	15
24	25	26	27
28	29	30	31

4	5	6	8
13	14	15	16
21	22	23	24
28	29	30	31

2	3	6	7
10	11	14	15
18	19	22	23
26	27	30	31

1	3	5	7
9	11	13	15
17	19	21	23
25	27	29	31



ACTIVITY 2

Materials:

- Any type of paper
- Napkins, tissue, newspaper
- Calculators

The Power of Two

Overview:

At last: a truly tangible way to introduce exponents! Astonish your students with a model of exponential growth. They will see the value of exponential notation as they put it to use. They will even be able to tell you why $2^0 = 1$!

Vocabulary: exponent, exponential growth, base, power

PROCEDURE

Skills:

- Modeling exponential growth
- Writing exponential notation

1. Ask students to get out any piece of paper. This can be lined or blank, new or used, standard-sized or less. In fact, it is best if students use different types and sizes of paper.
2. Ask them to predict how many times they could fold the piece of paper in half. Take time to get the students to announce their predictions. You will likely hear many estimates as high as fifteen or twenty. Record them on the board. You may also wish to analyze the range, mean, median, and mode of their answers.
3. Now ask them to try the experiment. Soon they will be trying to stomp and bite their paper to make it live up to their estimates! You can tell them that if you find footprints or teeth marks, it's not a fold, it's a stomp or a chomp. All students will end up with six or seven folds if they have followed the instructions correctly. You may have occasional students who claim to have dozen or more folds. By unfolding their papers, you can show these students that they have not truly folded the paper in half each time.
4. Record their results on the board next to the original estimates. The students will be intrigued by the differences. Ask them why most students didn't get as many layers as they predicted. Some student will usually say that the paper got too thick. Ask for an explanation. Someone will suggest that this occurs because the number of layers is doubling each time the paper is folded. This is a critical observation and should be brought to the attention of the entire class.
5. This is the time to introduce exponential notation. Ask the students to make a chart recording the number of folds compared to the number of layers. The chart should look like the one in the margin at the top of the next page. Explain that the large "2" (the base) represents that the number of layers is doubling each time. The small number (the exponent) represents the number of folds. The number on the right of the equation is the number of layers.

each fold is doubling the number of layers

number of layers

number of folds

$$2^1 = 2$$

6. Ask them to complete their charts to show how many layers were achieved on the sixth and seventh folds. Record the chart on the board. The chart will then look like the one in the margin.
7. Ask them to take their chart into the “theoretical world” and explore some of the early predictions. How many layers of paper would you get if you folded a paper in half ten times or twenty times? How far can their calculator go? A good way to find out is to type “1 x 2 =” in the calculator. Then repeatedly type the equal key. This will repeat the “times two” operation on most calculators.
8. Lastly, ask the students to complete the first line of the chart on the right.

$$\begin{array}{l}
 2^1 = 2 \\
 2^2 = 4 \\
 2^3 = 8 \\
 2^4 = 16 \\
 2^5 = 32 \\
 2^6 = 64 \\
 2^7 = 128 \\
 \\
 2^0 = ? \\
 2^1 = 2 \\
 2^2 = 4 \\
 2^3 = 8 \\
 2^4 = 16 \\
 2^5 = 32 \\
 2^6 = 64 \\
 2^7 = 128
 \end{array}$$

Although many students will think the answer should be zero, others will realize that when you have zero folds in your paper, you have only one layer! Thus $2^0 = 1$. They can further justify the logic of this by saying that half of eight is four, half of four is two, and half of two is one.

9. You can even get the students to explore negative exponents by cutting one in half repeatedly to get the final chart.

$$\begin{array}{l}
 2^{-3} = 1/8 \\
 2^{-2} = 1/4 \\
 2^{-1} = 1/2 \\
 2^0 = 1 \\
 2^1 = 2 \\
 2^2 = 4
 \end{array}$$



Journal Prompts:



- How could you increase the number of folds you could achieve?
- Why does $2^0 = 1$?
- How does 2^5 compare to 5^2 ?

Homework:



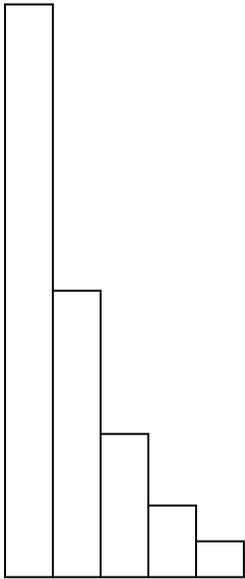
Ask the students to go home and try to break the record by folding different materials. They should record their results using a t-chart as shown on the homework master. They may wish to try using larger paper or thinner paper. Encourage them to use other materials such as aluminum foil or plastic wrap. As of the date of this book’s publication, the record is twelve folds by Britney Gallivan. Your students can check out how she did it at:

www.osb.net/pomona/12times.html

Good Tip!



As an additional assignment, ask students to find how many times they can cut a long strip of paper in half. The pieces can be taped side by side to create a bar graph showing exponential decay.



Taking a Closer Look:



Students can explore what happens when you triple or quadruple the number of layers each time you fold. This is difficult to do with paper, but students will find the t-charts helpful. Scientific calculators can be used to explore these patterns. For example, if you could fold a paper in half ten times, you would have 2^{10} layers of paper. On a calculator with an exponent key, the student should push “2”, “x^y”, “10”, “=”. The calculator shows 1,024. Ask them to find n so that $2^n = 100$. They will see that 2^6 is only 64, and $2^7 = 128$. They will have to use a decimal value for the exponent.

Assessment:



Check to see that students are using the notation correctly in their t-charts. That is one of the main purposes of this lesson. The other is to see that $n^0 = 1$ for all non-zero values of n . Ask them what the value of 3^0 would be.

Try to break your folding record from school. You may fold any material you wish. Before each attempt, write the name of the material. Then estimate the number of folds. Record your results in the t-charts.

1. Material:	fold	layers
_____	2^0	
Estimate: _____	2^1	
	2^2	
	2^3	

2. Material:	fold	layers
_____	2^0	
Estimate: _____	2^1	
	2^2	
	2^3	

3. Material:	fold	layers
_____	2^0	
Estimate: _____	2^1	
	2^2	
	2^3	

4. Material:	fold	layers
_____	2^0	
Estimate: _____	2^1	
	2^2	
	2^3	

A Weigh We Go!

Name _____

Date _____ Class _____

Can you weigh each sack of grain using **only one each** of the given weights? Put a 1 in the column if you need the weight and a 0 if you don't need it. Three examples have been done. Look for patterns as you work.

					
 1 =	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>1</u>
 2 =	_____	_____	_____	_____	_____
 3 =	_____	_____	_____	_____	_____
 4 =	_____	_____	_____	_____	_____
 5 =	_____	_____	_____	_____	_____
 6 =	_____	_____	_____	_____	_____
 7 =	_____	_____	_____	_____	_____
 8 =	_____	_____	_____	_____	_____
 9 =	_____	_____	_____	_____	_____
 10 =	_____	_____	_____	_____	_____
 11 =	_____	_____	_____	_____	_____
 12 =	_____	_____	_____	_____	_____
 13 =	<u>0</u>	<u>1</u>	<u>1</u>	<u>0</u>	<u>1</u>
 14 =	_____	_____	_____	_____	_____
 15 =	_____	_____	_____	_____	_____
 16 =	_____	_____	_____	_____	_____

					
 17 =	_____	_____	_____	_____	_____
 18 =	_____	_____	_____	_____	_____
 19 =	<u>1</u>	<u>0</u>	<u>0</u>	<u>1</u>	<u>1</u>
 20 =	_____	_____	_____	_____	_____
 21 =	_____	_____	_____	_____	_____
 22 =	_____	_____	_____	_____	_____
 23 =	_____	_____	_____	_____	_____
 24 =	_____	_____	_____	_____	_____
 25 =	_____	_____	_____	_____	_____
 26 =	_____	_____	_____	_____	_____
 27 =	_____	_____	_____	_____	_____
 28 =	_____	_____	_____	_____	_____
 29 =	_____	_____	_____	_____	_____
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 31 =	_____	_____	_____	_____	_____

A Weigh We Go!

Name _____

Date _____ Class _____

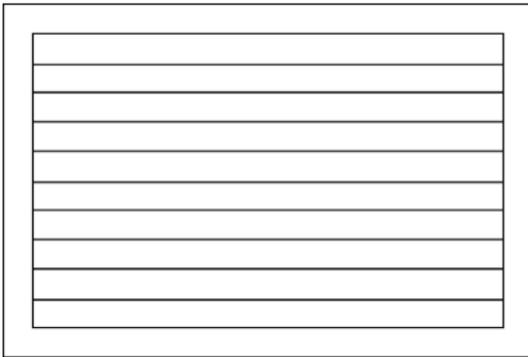
Can you weigh each sack of grain using **only one each** of the given weights? Put a 1 in the column if you need the weight and a 0 if you don't need it. Three examples have been done. Look for patterns as you work.

					
1 = 	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>1</u>
2 = 	<u>0</u>	<u>0</u>	<u>0</u>	<u>1</u>	<u>0</u>
3 = 	<u>0</u>	<u>0</u>	<u>0</u>	<u>1</u>	<u>1</u>
4 = 	<u>0</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>0</u>
5 = 	<u>0</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>1</u>
6 = 	<u>0</u>	<u>0</u>	<u>1</u>	<u>1</u>	<u>0</u>
7 = 	<u>0</u>	<u>0</u>	<u>1</u>	<u>1</u>	<u>1</u>
8 = 	<u>0</u>	<u>1</u>	<u>0</u>	<u>0</u>	<u>0</u>
9 = 	<u>0</u>	<u>1</u>	<u>0</u>	<u>0</u>	<u>1</u>
10 = 	<u>0</u>	<u>1</u>	<u>0</u>	<u>1</u>	<u>0</u>
11 = 	<u>0</u>	<u>1</u>	<u>0</u>	<u>1</u>	<u>1</u>
12 = 	<u>0</u>	<u>1</u>	<u>1</u>	<u>0</u>	<u>0</u>
13 = 	<u>0</u>	<u>1</u>	<u>1</u>	<u>0</u>	<u>1</u>
14 = 	<u>0</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>0</u>
15 = 	<u>0</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>
16 = 	<u>1</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>

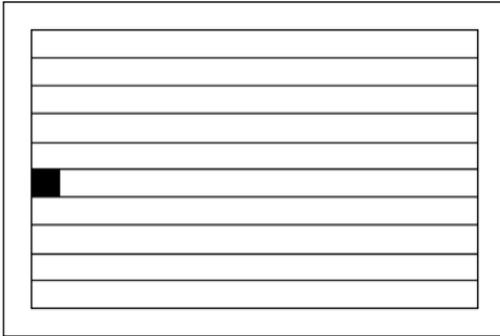
					
17 = 	<u>1</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>1</u>
18 = 	<u>1</u>	<u>0</u>	<u>0</u>	<u>1</u>	<u>0</u>
19 = 	<u>1</u>	<u>0</u>	<u>0</u>	<u>1</u>	<u>1</u>
20 = 	<u>1</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>0</u>
21 = 	<u>1</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>1</u>
22 = 	<u>1</u>	<u>0</u>	<u>1</u>	<u>1</u>	<u>0</u>
23 = 	<u>1</u>	<u>0</u>	<u>1</u>	<u>1</u>	<u>1</u>
24 = 	<u>1</u>	<u>1</u>	<u>0</u>	<u>0</u>	<u>0</u>
25 = 	<u>1</u>	<u>1</u>	<u>0</u>	<u>0</u>	<u>1</u>
26 = 	<u>1</u>	<u>1</u>	<u>0</u>	<u>1</u>	<u>0</u>
27 = 	<u>1</u>	<u>1</u>	<u>0</u>	<u>1</u>	<u>1</u>
28 = 	<u>1</u>	<u>1</u>	<u>1</u>	<u>0</u>	<u>0</u>
29 = 	<u>1</u>	<u>1</u>	<u>1</u>	<u>0</u>	<u>1</u>
30 = 	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>0</u>
31 = 	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u>



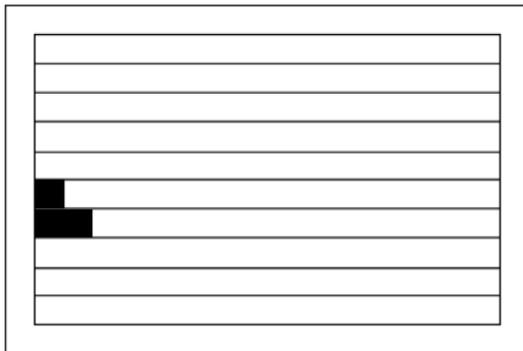
Begin with the paper in the landscape orientation.
Draw a frame that measures 10" tall by 16" wide.



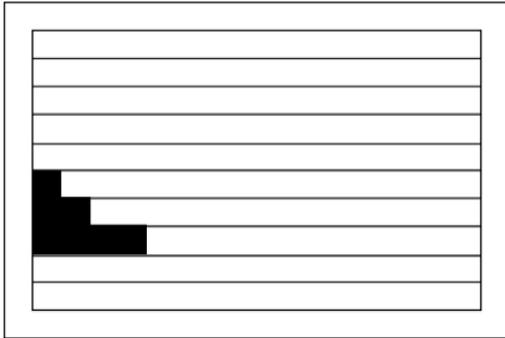
Draw horizontal lines every inch inside the frame creating ten rectangles measuring 1" by 16".



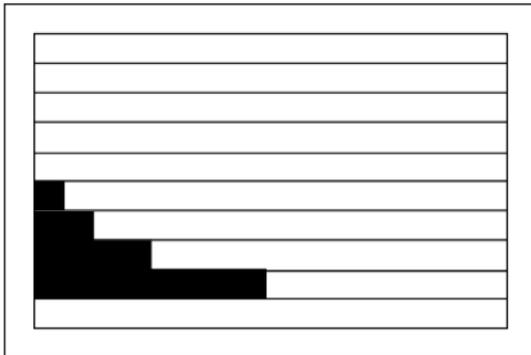
In the sixth rectangle, measure one inch from the left side and create a square.



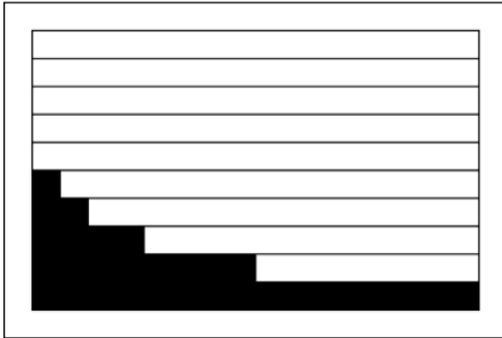
Double this on the seventh line creating a rectangle measuring 1' by 2".



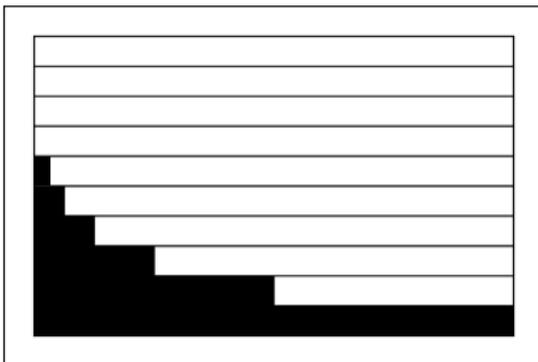
Double it again on the next line creating a rectangle that is 4' long.



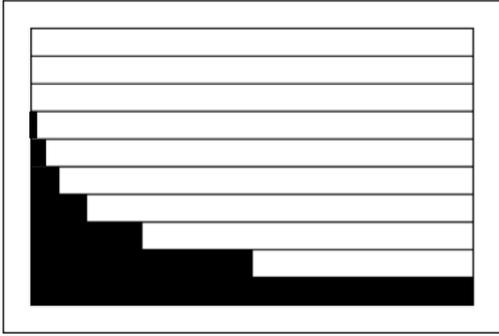
Double it again on the ninth line to make a rectangle 8" in length.



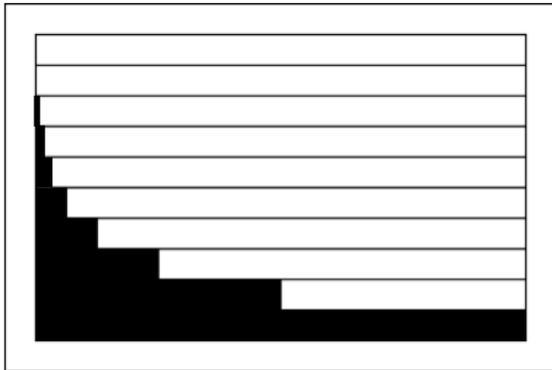
The tenth line is already double that size and measures 16" long.



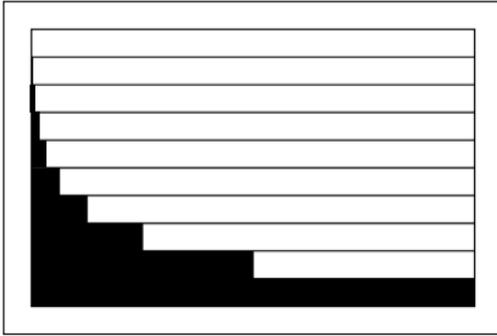
Reverse the process moving upward. The fourth row from the top has a rectangle half as long as the one below it. It is $\frac{1}{2}$ " long.



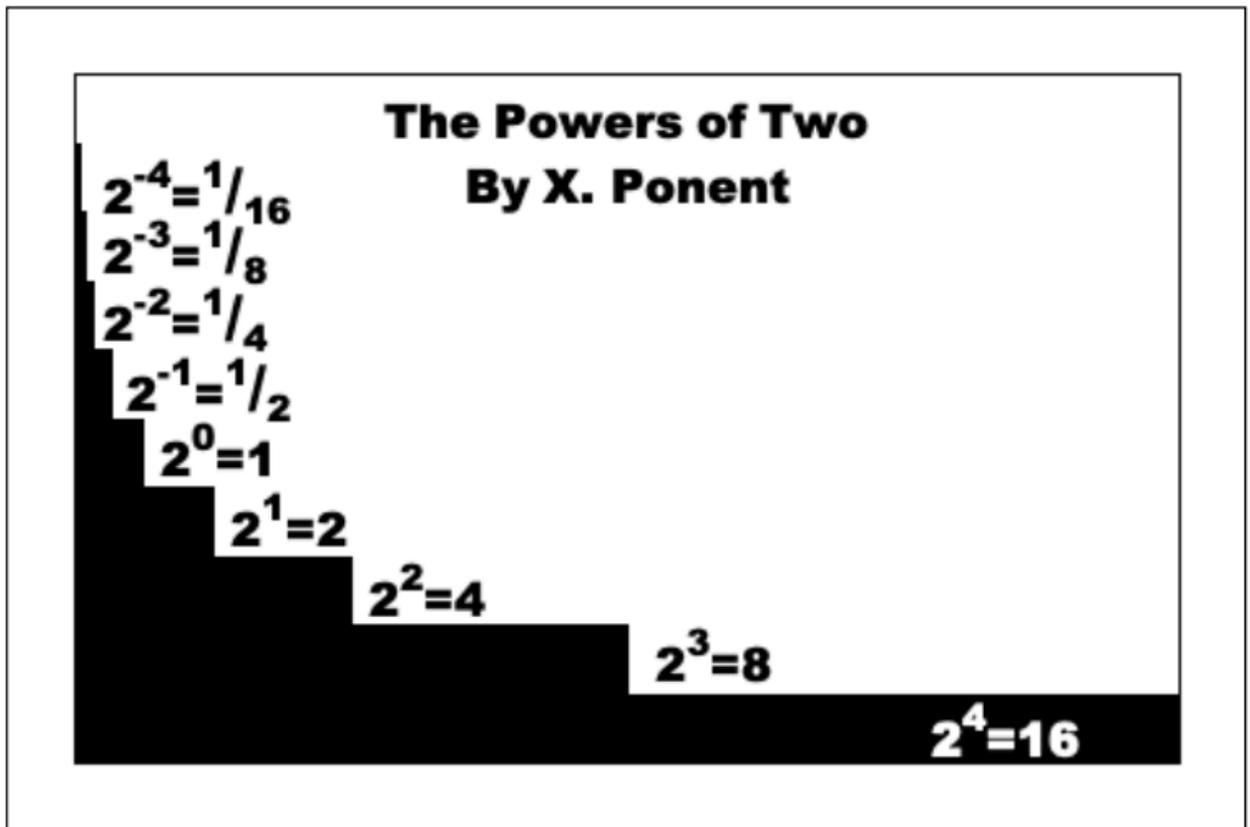
The one above that is $\frac{1}{4}$ " long.



The third row from the top has a rectangle $\frac{1}{8}$ " long.



The second row has a rectangle measuring 1" tall by $\frac{1}{16}$ " in length. The top line is reserved for the title.



Lastly, label the project as shown and color it as you wish.

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Feel free to contact me if you have questions or comments or would like to discuss a staff development training or keynote address at your site.

Happy teaching,

Brad