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Thanks.

Brad

Winning Ways With Number Sense

Simply & Effective Strategies to Foster Success

$3{,}652 \div 88 \approx 40$

$\frac{5}{11} > \frac{3}{7}$

Mental Math

Estimation

$31 \times 23 = 713$

By Brad Fulton
Educator of the Year, 2005
brad@tttpress.com www.tttpress.com

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Brad Fulton
Educator of the Year

- ♦ **Consultant**
- ♦ **Educator**
- ♦ **Author**
- ♦ **Keynote presenter**
- ♦ **Teacher trainer**
- ♦ **Conference speaker**

PO Box 233, Millville, CA 96062
(530) 547-4687
brad@tttpress.com

Known throughout the country for motivating and engaging teachers and students, Brad has co-authored over a dozen books that provide easy-to-teach yet mathematically rich activities for busy teachers while teaching full time for over 30 years. In addition, he has co-authored over 40 teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

Seminar leader and trainer of mathematics teachers

- ♦ 2005 California League of Middle Schools Educator of the Year
- ♦ California Math Council and NCTM national featured presenter
- ♦ Lead trainer for summer teacher training institutes
- ♦ Trainer/consultant for district, county, regional, and national workshops

Author and co-author of mathematics curriculum

- ♦ Simply Great Math Activities series: six books covering all major strands
- ♦ Angle On Geometry Program: over 400 pages of research-based geometry instruction
- ♦ Math Discoveries series: bringing math alive for students in middle schools
- ♦ Teacher training seminar materials handbooks for elementary, middle, and secondary school

Available for workshops, keynote addresses, and conferences

All workshops provide participants with complete, ready-to-use activities that require minimal preparation and give clear and specific directions. Participants also receive journal prompts, homework suggestions, and ideas for extensions and assessment.

Brad's math activities are the best I've seen in 38 years of teaching!
Wayne Dequer, 7th grade math teacher, Arcadia, CA

"I can't begin to tell you how much you have inspired me!"
Sue Bonesteel, Math Dept. Chair, Phoenix, AZ

"Your entire audience was fully involved in math!! When they chatted, they chatted math. Real thinking!"
Brenda McGaffigan, principal, Santa Ana, CA

"Absolutely engaging. I can teach algebra to second graders!"
Lisa Fellers, teacher

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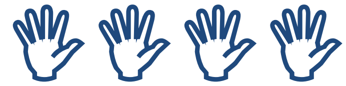
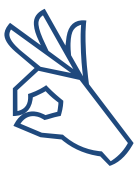


Like me even more? Then please don't make copies for

your colleagues. I know it's tempting when they say, "Wow! Groovy activity! Can I have a copy?" But this is how I make my money, and why are they still saying "groovy" anyway?

If we make copies for our friends, can we honestly tell our students not to copy or take things that don't belong to them? (Ouch!)



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Thanks, and happy teaching,

Brad 

Developing Number Sense

- ☑ Promotes the eight Common Core Mathematical Practices
- ☑ Builds the foundation for all mathematical learning and procedures
- ☑ Helps students develop proficiency with numbers in all domains
- ☑ Develops real-world mathematical thinking
- ☑ Promotes student discourse
- ☑ Fosters an understanding of numbers

Winning Ways with Number Sense

Overview:

Required Materials:

☒ None

Optional Materials:

☐ Calculators

Number sense is the foundation of mathematics. Students with a strong sense of how numbers behave are not only more likely to succeed in a mathematics course, they will be better able to estimate, perform mental calculations, and have a stronger sense of number magnitude.

Number sense is also critical in our daily living. Because of the easy availability of calculators, spreadsheets, and computers, proficiency with procedural math using pencil and paper is no longer as marketable a skill as it once was. Businesses much prefer the person who has a deep conceptual understanding of numbers.

However, two key problems emerge that prevent students from building and maintaining a strong foundation in number sense. First of all, though number sense is a key component of mathematics instruction in primary grades, its presence diminishes from third grade and on. I know of no textbook beyond elementary grades that has a chapter devoted to number sense. As students begin to learn procedures for regrouping in addition and subtraction and certainly by the time they are learning to multiply two-digit numbers, procedural fluency begins to take precedence over conceptual understanding. As students progress from the upper elementary grades into middle school and certainly high school, many of them who once voiced a love for math have lost that magic.

> ***algorithm*** – a procedure for doing mathematics. For example, our algorithm for dividing fractions requires multiplying by the reciprocal of the second fraction. Algorithms typically are efficient ways to get right answers without depending upon an understanding of concepts. Thus, they do not foster number sense.

Students begin to be separated into mythical camps of those who are "good at math" and those who simply aren't. Those who can master procedures excel because the algorithms they learn simply work whether they understand them or not. But even the student who is good at a procedural approach to mathematics may ultimately struggle in advanced classes like algebra, trigonometry, and calculus where multi-step problems are encountered. The brain is simply not very good at memorizing large quantities of steps when there is no foundation of understanding.

So just as number sense is directly taught in primary grades, we must continue this throughout all grades even into high school. How is a student supposed to understand calculus unless they have had opportunities to see that patterns of numbers approach limits as they tend toward infinity? Long before formal calculus instruction, students can look at the series of fractions in which the numerator is one less than the denominator:

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots \frac{n-1}{n}$$

By the time a student is in middle school, new content is presented as a proof or extension of existing mathematical procedures rather than built upon a conceptual base. Consider how a nebulous concept such as the zero power of a number is typically taught. Often, we see a proof like this in middle school textbooks.

$$\frac{7^3}{7^3} = 1$$

$$\frac{7^3}{7^3} = 7^{3-3} = 7^0$$

Therefore, $7^0 = 1$

This example will not help the student understand *why* a number to the zero power is one unless they *already understand the laws of exponents*, an equally challenging concept. Also, it actually does not prove that *any* number to the zero power is equal to one. It only demonstrates that it works with a base of seven. Later we will look at a more conceptual and concrete way of making sense of the zero power of a number. (See page 27.)

Thus, number sense can no longer simply be the responsibility of the teachers in primary grades. It is every math teacher's responsibility. This handout will show how we can foster rich number sense in our students through simple, engaging, and proven strategies.

> "For students to become highly skilled at estimation, it had to be incorporated into their regular instruction over several years."
>
> *Research Ideas for the Classroom: Middle Grades Mathematics*
> *Douglas T. Owens, 1993, NCTM*

The second problem we encounter in teaching number sense is that mathematicians don't even have a solid definition of what constitutes number sense. We have to teach it, but we aren't sure what it is.

Though this seems like an impossibility, it really isn't. Consider the teacher of literature or film. What makes a book good? What makes it a classic? We all have our own ideas on this, but still, we know a good book from hack prose. Like mathematics, we have a *sense* for it.

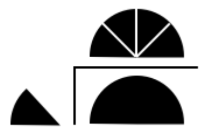
Now, for some really good news. Though we don't know how to define number sense in a specific and unarguable way, we *do* know the components that make up number sense. As we read the research on this subject, certain common ideas begin to emerge. I have isolated five that show up in most discussions of number sense:

1. **Estimation** – Students can make an estimate of an answer without formal computation. They are able to analyze their answer to a problem at the end of a computation and tell if it is reasonable. As an example, 33 x 18 should be about 600 since 30 x 20 = 600.
2. **Mental computation** – Students can precisely calculate some operations mentally. They will use mental procedures instead of pencil and paper strategies when the mental approach is faster. This differs from the previous skill in that here a student is seeking an exact answer but doing so mentally. For example, a student might decompose (14)(15) into (2 x 7)(5 x 3) and then see that it is equal to (2 x 5)(7 x 3) = (10)(21) = 210.
3. **Mathematical properties** – In the previous example, the student used the associative property of multiplication to rearrange the factors knowing that a different association does not result in a different product. Students are often asked to *memorize* the properties using variables instead of learning to apply them. For example, a student may say that they don't understand what you mean when you say the commutative property states that $a+b=b+a$, but they know that the price of a hamburger and an order of fries is the same as an order of fries and a hamburger. Students with strong number sense apply the properties fluently, even though they may not be able to recite them. In fact, they may discover these properties on their own years before formal instruction.
4. **Effects of operations** –What effect does multiplying have on a number? Does it always make it greater? When does it not make it greater? When does it make it lesser?
   Students who have number sense will understand that multiplication is a quantity of equal sets or a group of groups. They might see it as an array. Students who lack this skill will see 46 x 71 as a procedure. Students with number sense will see division as a sharing or cutting procedure. As an example, I once asked a sixth-

grade teacher whether her students could calculate *and* understand the following problem.

$$\frac{1}{2} \div \frac{1}{8} = 4$$

She admitted that many of them would make errors in calculation and that none of them could explain what the problem means. I then showed her this image without saying a word.



Her face lit up, and I could see the moment that she comprehended the meaning of division of fractions.

5. **Number magnitude** – Students who lack a grasp of number magnitude tend to make major errors in calculations as well as simply lack an idea of the size of numbers they encounter. They may think that 0.0099 is much greater than 0.1. They think that 1 is a long way from a million, but a billion is a bit larger than a million. I once tested my class by asking them to think what they would do with a million dollars. One student said that he'd buy his favorite NBA team. I asked him why that was his favorite team, and he said it was because his favorite player was on that team. Then I asked him how much that player made. He wasn't bothered that one player's salary far eclipsed what he was willing to pay for the entire franchise.

So, fostering these five characteristics will go a long way in ensuring that your students become fluent in the language of mathematics. In addition to these five *components*, there are five *strategies* that we can practice that will engage students in these characteristics. They are:

1. **Playing with numbers** – Students need time to explore what happens with numbers. In primary grades this is common. What do you get when you add two odd numbers? What do you get when you add two even numbers? What about an even number and an odd number?
   In higher grades, we still can explore with number properties that are grade-level appropriate. What happens when you multiply a whole number by a fraction less than one? What happens when you cut one in half over and over? Will it ever get to zero? What fractions create repeating decimals? Why?
2. **Solving problems in multiple ways** – In many cases, our traditional algorithms are highly efficient. However, there are times when other strategies are not only easier, *the connections between the two fosters more powerful understanding*. Consider how we typically solve this problem:
   $$\begin{array}{r} 10{,}002 \\ \underline{-\ \ 4{,}865} \end{array}$$
   Typically, we start crossing out numbers and writing a bunch of nines. In some nations, the problem is approached with what is called the *compensation method*.
   $$\begin{array}{r} 9{,}999 \\ \underline{-\ \ 4{,}862} \end{array}$$
   Three was subtracted from each number, but their *difference* remains constant.
3. **Creative practice** – Unfortunately, textbook practice sets are generated *randomly*. Students won't be able to connect how one problem relates to the next and develop conceptual understanding. Converting $^1/_3$ and $^2/_7$ to decimals won't develop the number sense that converting $^1/_7$, $^2/_7$, and $^3/_7$ will. As students solve *carefully and intentionally designed practice problems*, they will begin to synthesize their learning. The practice masters in this activity are designed with this in mind.

4. **Thinking, talking, and writing about numbers** – Regardless of the subject being taught, the brain processes information linguistically by and large. Thus, to *listen* to mathematics instruction can never be as powerful as talking about, writing about, and thinking about mathematics. The following activities promote student use of language. I urge you to guide your students in discussing or even writing about the questions that are included. A more extensive treatment of the use of language in the mathematics classroom along with guidance in implementing math discussions and math journals can be found in my 108-page e-book, *The Language of Math* available in my Teachers Pay Teachers store.

   Here's my store:

5. **Exploring patterns** – In addition to being a language processor, the brain is also an incredible pattern detector. It finds, recognizes, evaluates, and extends patterns constantly. In fact, every brain that I've ever met from pre-kindergarten on up, regardless of skill level, loves to find patterns. A student who practices a random set of multiplication facts will at best improve on their multiplication facts. But consider what else they gain when the practice page looks like this:

6x6=	5x7=
10x10=	9x11=
4x4=	3x5=
3x3=	2x4=
7x7=	6x8=

As they progress through the practice set, they will not only study their multiplication facts, they will also compare squaring a number to multiplying one less than the number by one more than the number:

$(n)(n)=$ $\quad\quad$ $(n-1)(n+1)=$

The square is always one greater. This leads to what is called the *difference of two squares* in algebra:

$$(n-1)(n+1)=n^2-1$$

At the end of the section that deals with the five *components* of number sense, some practice worksheets are offered that not only allow students to practice skills in addition of different types of numbers, they illustrate how we can easily implement the five *strategies* for teaching number sense.

Moreover, there are added benefits to helping our students develop number sense. The strategies in this activity will build the eight Common Core Mathematical Practices:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

The problems that follow are examples that I have used in middle school classrooms. If you teach younger or older students, you will of course want to design problems best suited to your students. But these will still serve as an example of how you can do that.

Notice that many of the problems could be done at lower grade levels with pencil and paper. The first problem for example, 0.52 x 789, could be solved by an elementary student, but to *mentally estimate* it without pencil, paper, or calculator may require more sophisticated skill.

Why number sense matters:

Let's consider an interesting problem. I often ask my 8th grade students and adult audiences that I meet, "Which is greater: 86x38 or 88x36? Why"

I'll admit, I was tricked by the problem. Since I knew about the commutative property, I quickly assumed that they were equal. I even stopped thinking about it in my race to get a right answer. Then I saw a video of a student considering the problem without pencil and paper. He said, "86 x 38 is 36 groups of 86 and two more groups of 86." He was thinking of it like this: 86 x 38 = 86(36+2), though he didn't write it that way or name the distributive property.

Then he went on to explain, "88 x 36 is also 86 groups of 36 with 2 more groups of 36." He was verbalizing: 88 x 36 = (86 + 2)36."

He summarized by saying, "Both have 36 x 86, but one has two more 36s and the other has two more 88s."

I was impressed by his number sense…but embarrassed by the fact that he was a third grader! He had been educated in a school where students were given consistent opportunities to practice the types of activities explained in this unit. That is what I wanted my students to learn as well.

The Five Components of Number Sense

- **Estimation**
- Mental Computation
- Mathematical properties
- Effects of operations
- Number magnitude

In this first problem set, ask students to try to get a *close* answer quickly. Tell them not to use a pencil, paper, or calculator, but to do all the work in their head. Assure them that an exact answer is not necessary, but an approximation will be fine.

To estimate, students must be able to round off or approximate a number into a format simple enough to then be handled mentally.

Each problem can turn into a five to 20-minute lesson. Don't try to do more than one of these per day. I typically use such a question as a warmup with my middle schoolers and limit the investment in time to no more than 10 minutes. The rest of the period is devoted to my regular instruction.

Give students about a minute of *uninterrupted quiet time* to think about their estimate. Then begin the discussion. You can start by having them share as a pair or in their table groups. Then move to a whole class discussion.

By consistently providing opportunities for students to estimate answers, you will begin to see them using this skill in daily work.

> "Estimation relies on two fundamental skills: approximation and mental computation."
>
> *R. Case*
> *Intellectual Development, 1985*

After they have come up with an estimate, ask them to then calculate the exact answer. I have them do this with calculators. Initially students are disappointed that their answers are "wrong". I have to explain that their goal wasn't to get the exact answer; that's why we have calculators.

Then I show them how to calculate their accuracy. For example, students typically estimate the first problem is around 400. They notice that 0.52 is about one half, and 789 is about 800. Thus, half of 800 is 400. Then I have them ***divide their estimate by the exact answer***.

$400 \div 410.28 \approx 0.975$ or 97.5% accurate. Not bad for mental math!

Estimate these problems:

1. 0.52×789

2. 40×26.7

3. 43×52

4. $4953 \div 68$

5. $0.4 \times 58.6 \times 5 \times 3$

6. Write $\frac{715}{1866}$ as a percent.

Answers

1. 410.28
2. 1,068
3. 2,236
4. Approximately 72.8
5. 351.6
6. Approximately 38.3%

The Five Components of Number Sense

- Estimation
- **Mental Computation**
- Mathematical properties
- Effects of operations
- Number magnitude

Now we move from estimation to exact answers. Still, students must do all the work mentally; no pencils, paper, calculators or phones are allowed. This moves students away from traditional procedural models and into alternative strategies rich in number sense.

The discussion is presented the same way. I also allow students to verify their answers *after* the discussion using their calculators.

I am amazed at the ways that students solve these types of problems once they are freed of pencil and paper dependency. In the first example, calculating 16x15, I have had middle school students verbalize each of these strategies.

- "I knew 15 x 15 was 225, so I added 15 more."
- "I multiplied 10 x 16 and then added half of that again." (10 + 5)16
- "I thought about one and a half 16s and then put a zero on the end."
- "I knew 16 dimes was $1.60, so 16 nickels would be $.80. Then I added them."
- "I took the 15 apart into 3 x 5. Then I took the 16 apart into 2 x 8. Then I put them back together to get 2 x 5 and 3 x 8."
- "10 x 16 is 160, so 20 x 16 would be twice that: 320. I tried to find the number halfway between them."
- "I used F.O.I.L." (10 + 6)(10 + 5) = (10 x 10) + (10 x 5) + (6 x 10) + (6 x 5)."
- I knew 15 x 10 was 150, and 15 x 6 was the same as 30 x 3 which is 90. Then I added."

It was refreshing to watch their confidence grow as they were able to find so many ways to do this problem in their head. Even students who made errors typically made minor ones and were able to find their errors.

On the fourth problem, I once had a student find a number between the two fractions by adding the numerators and adding the denominators.

$$\frac{11}{18} < \frac{11+5}{18+7} < \frac{5}{7}$$

I explained that you can't add denominators, and he replied, "But it works!" It turned out that not only did it work in this case, *it can be proven mathematically to work in every case!*

Solve these problems mentally if possible.

1. 16×15

2. 45% of 250

3. $3467 - 1588$

4. Find a fraction between $\frac{11}{18}$ and $\frac{5}{7}$.

5. Find a fraction between $\frac{5}{7}$ and $\frac{13}{18}$.

6. $5 \times 2{,}379$

Answers:

1. 240
2. 112.5
3. 1,879
4. 2/3 is one example
5. 18/25 is one example
6. 11,895

The Five Components of Number Sense

- Estimation
- Mental Computation
- **Mathematical properties**
- Effects of operations
- Number magnitude

Many text books and assessments require students to memorize mathematical properties stated in general terms without knowing how to understand or apply them. For example, a student may recognize that a+b=b+a is the commutative property of addition but not be able to realize that 34+89=89+34 or that y=3x+7 is the same equation as y=7+3x.

People with number sense have learned to use the properties of math to facilitate their problem solving. We saw this in the 3rd grade student cited on page 14. Many of our common algorithms typically not only aren't reliant on number sense, they often impede it. Most of them, such as our addition algorithm, start with the ones place in the righthand column and move to the left toward the more significant columns. Yet people who have strong number sense typically move from left to right. If you and I were pooling our money, we'd start by adding the larger denominations first and count the pennies last.

I once observed a student who had strong number sense when he entered kindergarten. In primary grades, he was recognized as a strong math student. Yet by 6th grade, I watched him calculate 15 minus 8 this way.

$$\begin{array}{r} \overset{0}{\cancel{1}}\,\overset{1}{5} \\ \underline{-8} \end{array}$$

Sadly, he had learned well. Fortunately, students can rekindle their number sense. Each of the following problems can be solved easily if mathematical properties are applied.

Use mathematical properties to solve each problem. Explain how the properties work.

1. 16×52
2. $37+256+13+50$
3. $9(4{,}555)$
4. $10{,}002-4{,}566$
5. $18 \div 1.5$
6. $1.5 \div \frac{2}{3}$

Answers:

1. 832. The distributive property can be applied. You could also cut 16 in half while doubling 52. This gives us 16x52=8x104=4x208=2x416. Continuing one more step you will get 1x832. This is an application of the associative property of multiplication.
2. 356. The associative property allows us to rearrange the addends this way:

$$37+256+13+50=$$
$$(37+13+50)+256=$$
$$100+256=$$
$$356$$

3. 40,995. The distributive property can be used two ways:

$$9(4{,}555)=$$
$$9(4{,}000+500+50+5)=$$
$$36{,}000+4{,}500+450+45$$
or
$$9(4{,}555)=$$
$$(10-1)(4{,}555)=$$
$$45{,}550-4{,}555=$$
$$45{,}550-4{,}000-500-5--5$$

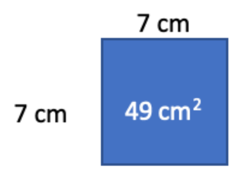
4. 5,436. 10,002–4,566=9,999–4,563. This is called the compensation method of subtraction.
5. 12. 18÷1.5=36÷3. If we think of the division problem as a fraction: $\frac{18}{1.5}$, then we have multiplied it by one in the form of $\frac{2}{2}$.
6. 2.25. Using the method above, we can multiply $\frac{1.5}{\frac{2}{3}}$ by $\frac{3}{3}$ to get $\frac{4.5}{2}$. Half of that would be 2.25.

The Five Components of Number Sense

- Estimation
- Mental Computation
- Mathematical properties
- **Effects of operations**
- Number magnitude

Students have some erroneous assumptions about how numbers behave, and through rich experiences such as those found here, they can refine their thinking. For example, it would seem natural for younger students to assume that addition always results in a sum that is higher than the addends, but this is not true when adding zero or a negative number. Likewise, multiplication can shrink numbers when fractions less than one are involved.

Memorizing squares or square roots of numbers is not as critical as understanding the concepts of squaring and square rooting numbers. I'm always pleasantly surprised when a mathematically advanced student who has mastered squares of numbers realizes for the first time that the term "squaring" a number refers to the area of the number's square.



Likewise, the light may dawn when a student realizes that second degree equations are called quadratics because they deal with quadrilaterals that are two-dimensional shapes.

Students can explore the effects and patterns of operations and gain a richer number sense. This can lead into the idea of limits in calculus as in the last example on the next page. For example, if we start with ½ and keep adding one to the numerator and denominator, we get this series of fractions:

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \cdots \frac{n}{n+1}$$

This series converges toward one as *n* approaches infinity. However, if we start with the reciprocal of ½, what happens?

$$\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \cdots$$

These are great explorations for students in middle school. They not only develop concepts with fractions, they help lay the foundation for "calculus sense".

Explain your answer to each question and give examples when necessary.

1. If two numbers are added, will the sum always be greater?
2. If you subtract one number from another will the result be smaller than the first number?
3. If numbers are multiplied is the product always greater?
4. When you square a number, is the answer always greater?
5. If divide one number by another, will the quotient always be smaller?
6. If you keep adding one to both the numerator and denominator of $\frac{1}{2}$, how big will the number eventually be?

Answers:

1. No, if you add zero or a negative number, the sum is not greater.
2. No, if you subtract zero or a negative number, the difference is greater.
3. If you multiply by one or less, the product is equal or less.
4. The square will be greater unless the value of the number is between zero and one.
5. If you divide by a number x such that $0<x\leq 1$, the quotient is not greater.
6. Eventually the fraction approaches 1 as a limit.

The Five Components of Number Sense

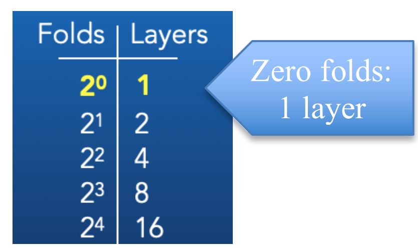
- Estimation
- Mental Computation
- Mathematical properties
- Effects of operations
- **Number magnitude**

Most students and even many adults lack an intuitive ability to conceptualize the magnitudes of very large numbers. We can picture a single millimeter and a single meter that is 1,000 times longer. But we will typically grossly underestimate the length of a kilometer even though the scaling factor is the same.

Similarly, it takes 11 days to live one million seconds, but when asked how many days are equal to a billion seconds, few people will realize it is over 30 years, or that no one could ever live a trillion seconds as it's over 30,000 years. Likewise, when we hear that our deficit is in the trillions, we tend to think it has gotten a bit worse than when it was in the billions.

The same inability to imagine smaller numbers causes students to make errors with fractions and decimals. A student may multiply 0.002x0.003 to get 0.006 instead of the much smaller answer which is in the millionths.

When asked how many times a paper can repeatedly be folded in half, most people who have no prior experience will overestimate. This is due to our inability to imagine the incredible growth rate of exponential functions. This activity can also be used to demonstrate why the zero power of two is one as shown here



I once asked my students what they would do with a million dollars. One student responded that he'd buy a particular NBA team. I asked him why he would buy that team, and he said his favorite player was on it. I asked him if he knew the player's salary, and he did. He was not bothered by the great discrepancy between one player's salary and his imagined price for the entire franchise.

To help students conceptualize the vast dimensions of space, I once took them outside to the soccer field and showed them the yellow number one ball from a pool table. I set it on one goal line. I explained that if we used this as a scaled model of the sun, Mercury was one meter away. Venus was about another meter, and earth was 3.5 meters from the ball. Mars was about another meter. We had only walked a bit over four meters. Pluto, on the other hand, was on the other goal line, 100 meters away. Using the same scale, our nearest neighboring star is over 300 *miles* away! They realized that space is mostly, well, simply space. This activity, *A Walk Through the Solar System*, is also available in my Teachers Pay Teachers store.

1. How long will it take to live a million seconds?

2. How long will it take to live a billion seconds?

3. How long will it take to live a trillion seconds?

4. How tall is a stack of one million dollar bills?

5. If you cut your paper in half twenty times, how big would it be?

6. How many times will you blink in your life?

Answers:

1. About 11.6 days
2. 31.7 years
3. 31,700 years
4. Over 650 feet tall. Each dollar is 1/125” thick.
5. Less than one ten-thousandth of a square inch, or about 1/100” by 1/100”
6. Between 300 billion and 500 billion times.

Strategies that Develop Number Sense

Earlier, I mentioned that in addition to the five *components* of number sense, there are five *strategies* that we can employ that will help foster number sense while providing necessary skills practice. These will also help develop the eight mathematical practices in the Common Core Math Standards. These five strategies are:

1. Playing with numbers
2. Solving problems in multiple ways
3. Creative practice
4. Thinking, talking, and writing about numbers
5. Exploring patterns

We will now look at one way we can employ all five of these strategies in a skills practice format. Unlike typical textbook practice sets in which problems are generated randomly, this activity uses problems that build upon one another and foster the number sense that we seek.

Pyramid Math

Where Skill Practice and Number Sense Combine

Overview:

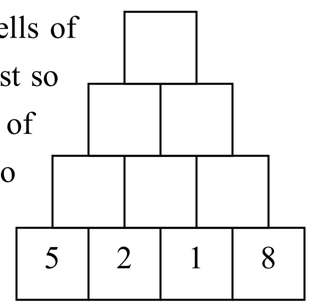
This creative activity facilitates discovery of number patterns and develops number sense while providing critical skills practice. It works great with both positive and negative numbers and decimals, fractions, and even variable expressions in algebra. Because it can be designed to be self-checking, it is easy for the teacher and engaging for the students.

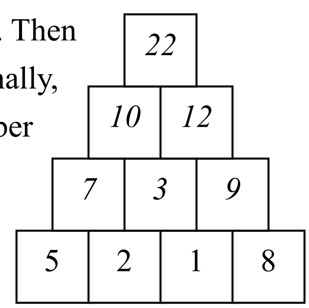
Required Materials:
- ☒ Paper
- ☒ Blank master

Optional Materials:
- ☐ activity master
- ☐ calculators

Procedure:

1 Display the transparency master and enter four numbers in the cells of the bottom row as shown. Use single-digit whole numbers at first so students can focus on the structure of the problem instead of struggling with the computation at this point. Then, transition to more grade-level appropriate numbers such as decimals, integers, or even binomials. To solve the pyramid, an adjacent pair of numbers is added. The sum is written in the box above the number pair. This is repeated for the other number pairs in the bottom row. Then this process is repeated for the second row to fill the third row. Finally, the number pair in the third row is added to get the final top number as shown.



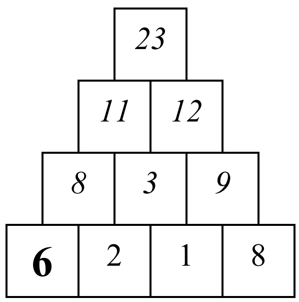


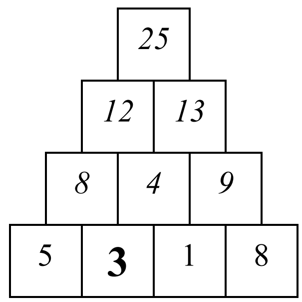
2 Since each sum is based on the sums below, all students should get the same answer in the top cell. **Thus, they only need to check the top answer.** If that is correct, all other cells are likely correct too. I provide an answer bank of the top numbers so students can self-assess.

3 Now try another pyramid using new numbers. Students will catch on to the process quickly and will be eager to check their answer with those of their classmates. (No more correcting papers!)

4 As students understand how the problems work, introduce appropriate numbers. If you are studying decimals, throw in a few decimal points. If you have covered integers, use some negative numbers. Fractions make these problems much more difficult. Try one yourself before asking the students to do so. I suggest beginning with like denominators. Or you could use fractions that have a fairly small common denominator. For example, ¼, ½, ½, and ¾ can all use fourths for a common denominator.

5 Here is where the number sense comes in. Make slight variations in the arrangement and values of the numbers. For example, in the first problem if we increase the five by one, making it a six, the top number also increases by one, but if we change the two to a three, the top number increases by three.





Is this always true when we add one to a cell? What would happen if we added two to the first or second cell of the bottom row? What happens when we do this to a five-row pyramid? As students answer these questions they will develop number sense.

6 Ask students to change the order of the numbers in the bottom row of a pyramid. How does this affect the top cell? Is the result always the same? How does the commutative property affect this result?

7 If everyone puts the same number in the top cell of a blank pyramid, will everyone get the same bottom row by working backwards? Why or why not?

8 Introduce subtraction by using Pyramid Math 6 in which other cells are filled in. You can create one of your own easily, or have students create them for their classmates to solve.

9 Explore what happens when all odd numbers or all even numbers are used. What patterns occur when all four cells in the bottom row contain the same number?

Journal Prompts:

If you rearrange the numbers on the bottom row of a pyramid, will you always get the same numbers on top? Why or why not?

What can you predict about the number on the top of a four-row pyramid if all the starting numbers are equal? Does the number of rows in the pyramid affect this? In what way?

Homework:

Use one of the accompanying activity masters or tailor one to your students' needs using one of the blank masters.

Taking a Closer Look:

There is a way of predicting the top of the pyramid without solving all the rows. This leads students into the algebra involved in the process. For example, let's assume that we are going to solve a four-row pyramid. The bottom cells contain four numbers called a, b, c, and d as shown. It follows that the second row contains three sums which are a + b, b + c, and c + d respectively. The third row contains these two sums:

a+3b+3c+d			
a+2b+c	b+2c+d		
a+b	b+c	c+d	
a	b	c	d

$$(a + b) + (b + c) \text{ and } (b + c) \text{ and } (c + d)$$

These simplify into a + 2b + c and b + 2b + c. Adding these to get the top row gives a + 3b + 3c + d.

Now let's start with four numbers: a = 3, b = 6, c = 5, and d = 8. Using the formula, the top answer should be:

44			
20	24		
9	11	13	
3	6	5	8

$$a + 3b + 3c + d =$$
$$3 + 3(6) + 3(5) + 8 =$$
$$3 + 18 + 15 + 8 = 44$$

Assessment: ☑

These activities can be made self-assessing by writing the answers randomly at the bottom of the page. As students solve each pyramid, they can cross off the answers. If they get an answer that is not listed, they know they have made a mistake and will try again.

Answer Key:

	Pyramid Math Worksheet Number				
Problem	1	2	3	4	5
1	33	16.5	21	267	$4\frac{2}{5}$
2	24	10.2	-21	224	$3\frac{3}{7}$
3	40	4	11	377	
4	32	3.2	-20	264	
5	60	2.05	-8	312	
6	68	23.86	0	332	
7	80	6.84	24	222	
8	23	16	-34	308	
9	23	21.81	0	332	
10	100	3.08	100	363	
11	106	1	1	528	
12	188	1.76	9	2000	

Great Tip!

For students who are preparing for algebra, put variable expressions in the bottom cells. They can then practice combining like terms as shown. Or put them in some of the upper cells so they can subtract binomials by working downward.

		-9x+3	
	1	-9x+2	
3x+2	-3x–1	-6x+3	
x+3	2x–1	-5x	-x+3

For sets six through eight that involve , these are the answers to the bottom row of each pyramid.

Problem	Pyramid Math Worksheet Number: 6	7	8
1	8, 3, 2, 2	.3, .1, .7, .6	6, -4, 5, -2
2	3, 4, 4, 2	.5, .6, .2, .5	8, -12, 6, -8
3	11, 3, 2, 10	.6, .2, .5, .5	8, 6, -12, -8
4	1, 0, 11, 23	3, .6, .9, 1	-3, -6, -9, -14
5	7, 12, 10, 11	.7, .9, .9, 2	-8, 0, 19, -20
6	19, 0, 6, 26	.9, 0, .5, .9	-7, -6, 16, -16
7	27, 0, 0, 27	1.2, 6.3, 2.7, 3.1	21, -9, -19, -8
8	8, 19, 10, 9	17, 1.9, .8, 6.4	11, 30, -16, -14
9	31, 26, 19, 17	8.7, 7.6, 6.5, 5.4	11, -30, -16, 14
10	18, 2, 0, 27	.29, .16, .5, 1.2	-17, -3, 3, 17
11	5, 38, 19, 31	7.1, .99, .09, .5	-15, -5, 6, 13
12	13, 6, 34, 9	7.7, .77, .55, 5	18, -9, 6, -3

The Common Core Connection

In addition to providing skills practice in addition of all number types, these pyramid problems can also help students think mathematically and develop number sense. They are a great way to develop many of the **eight mathematical practices** of the Common Core Math Standards:

1. Make sense of problems and persevere in solving them

This practice applies when students are developing an understanding of the problem and its underlying mathematics. Students also practice this when they make conjectures about what will happen in a problem. That is why it is important to pose questions to the students such as, “What do you think will happen if we rearrange the numbers?” “If the bottom numbers in a pyramid are all odd, will the top number be even or odd?”

2. Reason abstractly and quantitatively

As students move past the arithmetic of a problem and begin to think about the fact that the numbers can vary and what happens when they do, they are thinking abstractly. That is why it is important to help your students to move toward exploring what happens when the bottom row contains variables instead of specific numbers.

3. Construct viable arguments and critique the reasoning of others

When students are asked to make conjectures, we should also ask them to explain their reasoning. “Why do you think that the top number will be even?” Students will need to learn the skills of explaining their thinking and of disagreeing with the conjectures of others in an appropriate manner.

4. Model with mathematics

This practice involves using mathematics to represent the problems they see in the real world. At first it might seem not to apply to the pyramid problems because they are skills practice. However, using a formula as a model to explain why the pyramids behave the way they do is an example of this practice.

5. Use appropriate tools strategically

In this activity students might employ pencil and paper as their primary tools. However, the teacher may wish to allow calculators on some of the more challenging problems. A good guide in knowing when to switch from paper to calculator is this: **when the arithmetic is impeding the mathematical thinking, students are only getting skills practice and are not likely to develop number sense**.

Students in a computer class could also create a spreadsheet that would solve pyramid problems. Designing such a spreadsheet would require the students to know the *how and why* behind the mathematics that governs the pyramid problems.

6. Attend to precision

When students self-assess either by comparing their answers to others or by checking their solution against an answer bank, they are more likely to amend their errors (Marcy). Working separately, they often lack the number sense to know if the result of their calculation is right or even reasonable.

7. Look for and make use of structure

As students explore and come to understand how the structure of these problems affects the number at the top of the pyramid, they are developing this skill. That is why it is important not only to provide them with random numbers in practice problems, but also with examples that illustrate what happens when the same numbers are rearranged or changed slightly.

8. Look for and express regularity in repeated reasoning

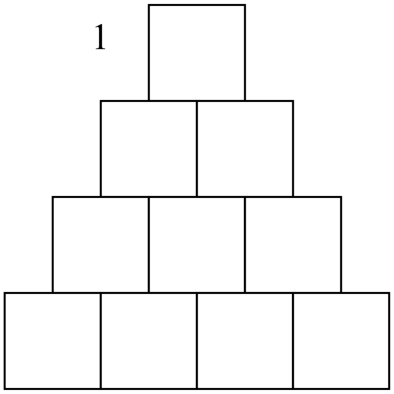
As students notice the effects of these changes in the bottom numbers, they begin to notice shortcuts. For example, adding one to either of the two squares on the outside of the bottom row *always* increases the top box by one, but adding one to one of the interior cells of the bottom row *always* increases the top number by three.

Activity master

Pyramid Math

Add pairs of adjacent numbers and write their sums above them. Keep going until you reach the top of the pyramid.

1



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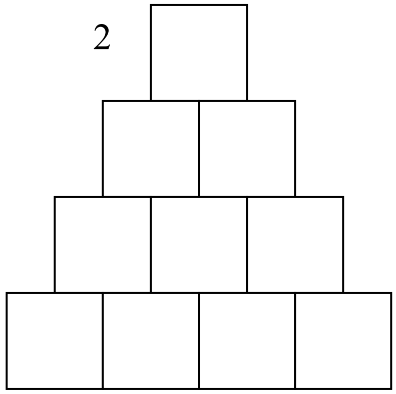

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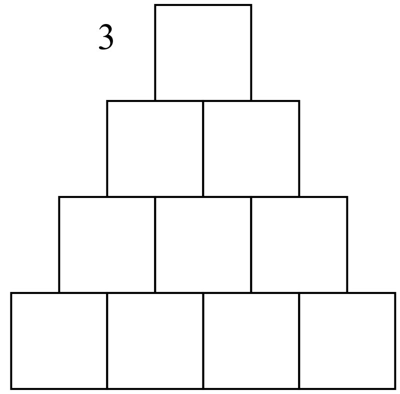

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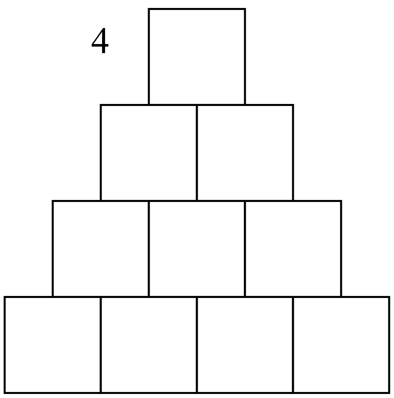

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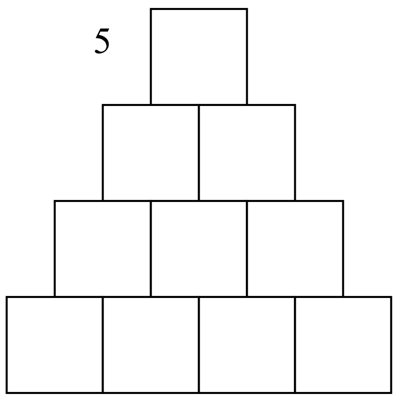

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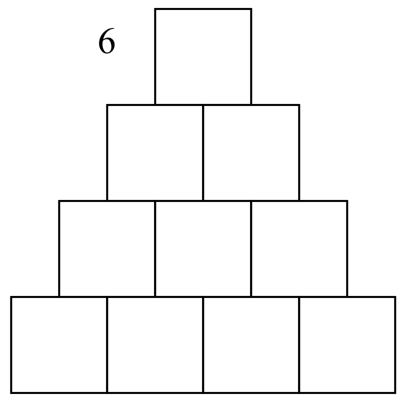

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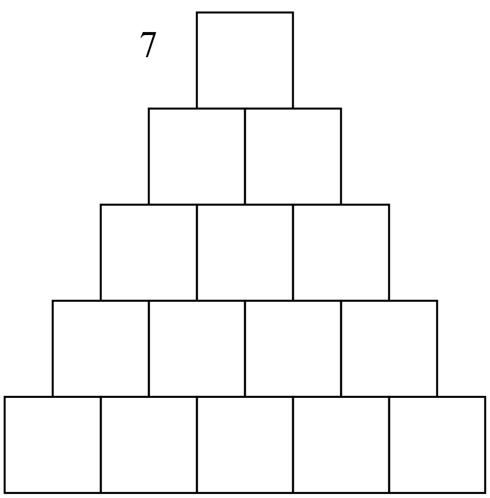

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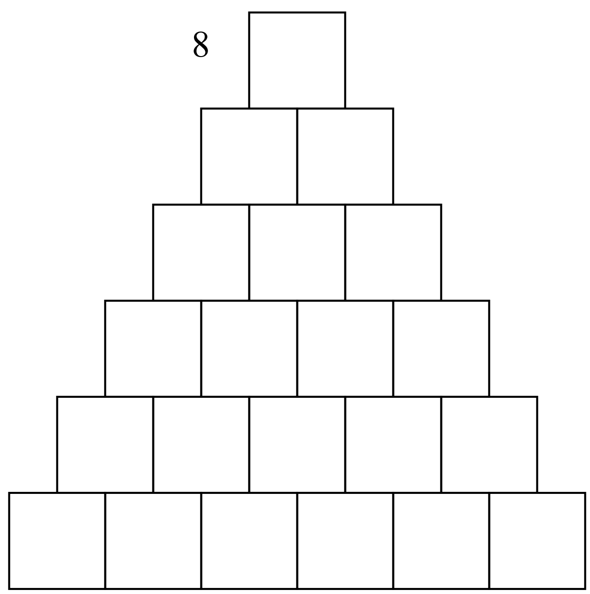
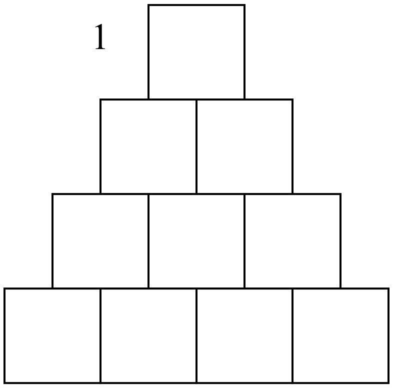


Pyramid Math

Name______________________________

Add pairs of adjacent numbers and write their sums in the box above them. Keep going until you reach the top of the pyramid.

1



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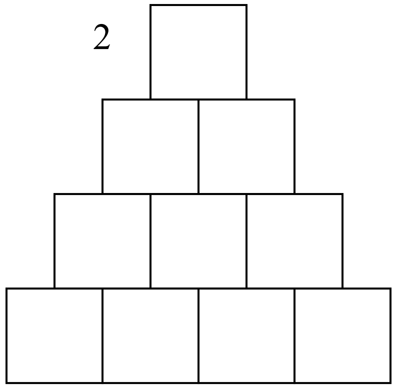

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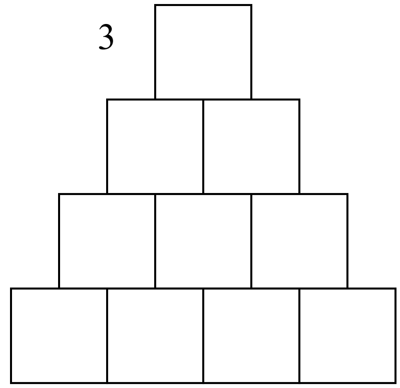

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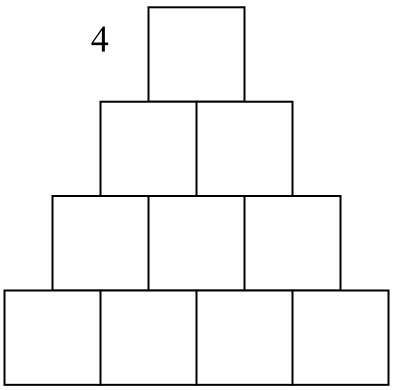

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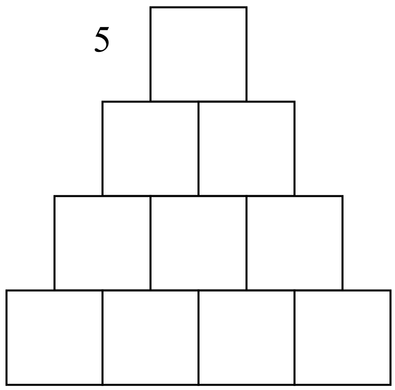

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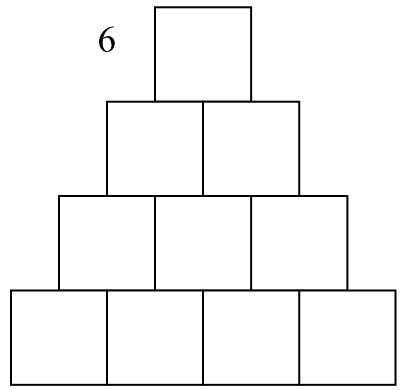

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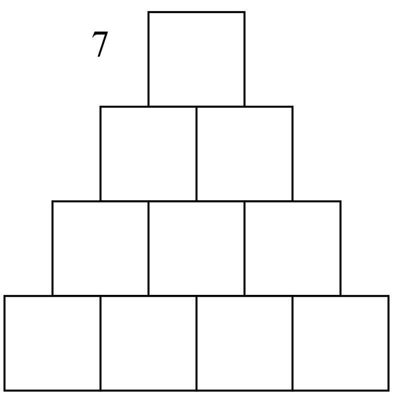

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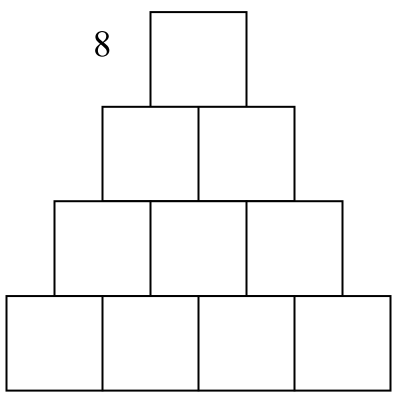

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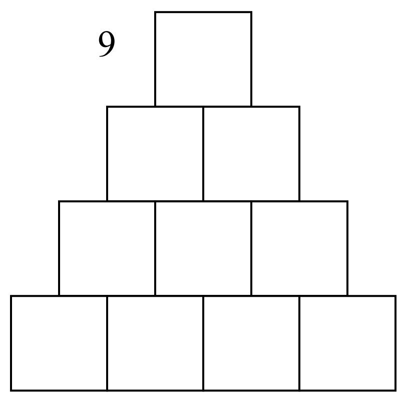

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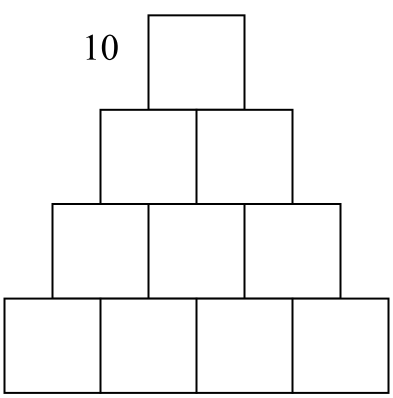

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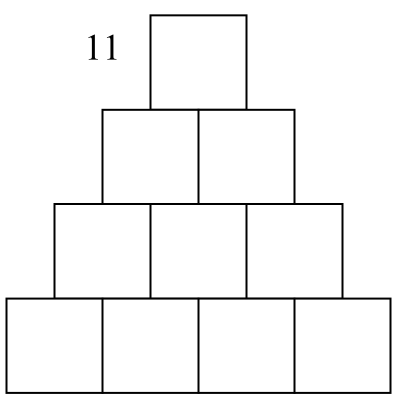

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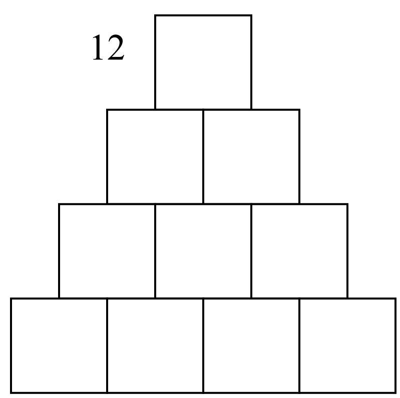


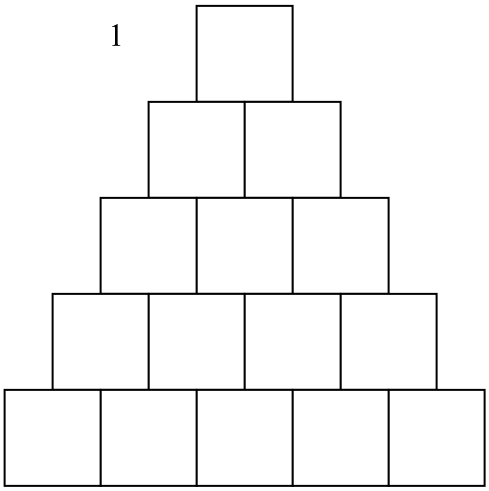
Activity Master 2

Pyramid Math

Name_______________________________

Add pairs of adjacent numbers and write their sums in the box above them. Keep going until you reach the top of the pyramid.

1



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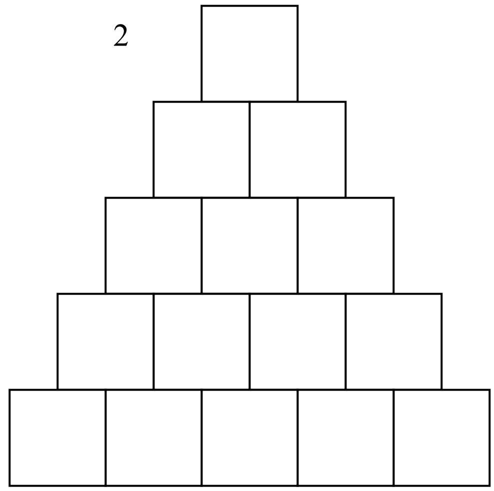

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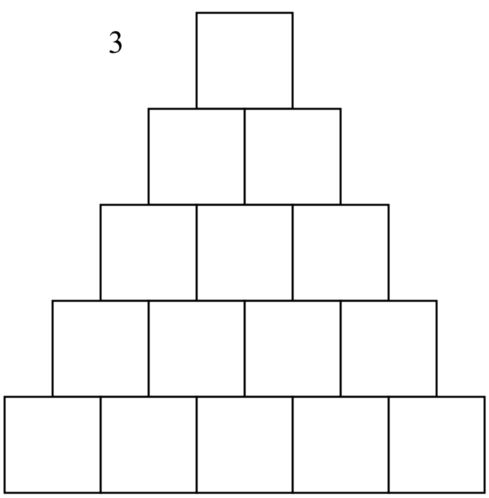

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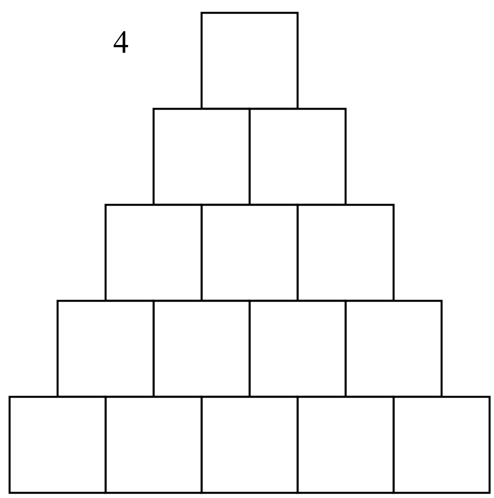


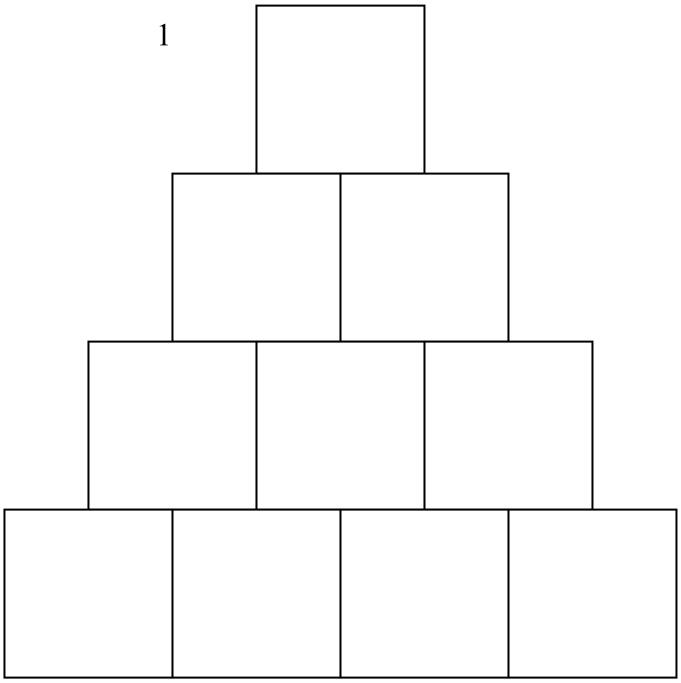
Activity Master 3

Pyramid Math

Name______________________________

Add pairs of adjacent numbers and write their sums in the box above them. Keep going until you reach the top of the pyramid.

1



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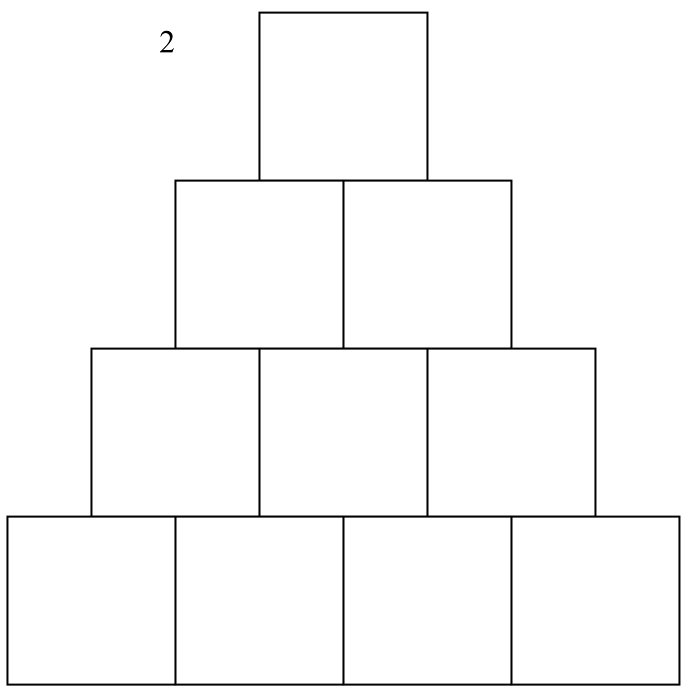


Pyramid Math 1

Name______________________________

Add pairs of adjacent numbers and write their sums in the box above them as in the first example. Keep going until you reach the top of the pyramid.

1

		9	
8	4	2	7

2

6	0	3	9

3

2	4	6	8

4

1	3	5	7

5

3	6	9	12

6

4	7	10	13

7

4	8	12	16

8

8	0	5	0

9

0	5	0	8

10

20	9	12	17

11

11	14	14	11

12

22	23	24	25

Activity Master

Pyramid Math 2

Name______________________________

Add pairs of adjacent numbers and write their sums in the box above them as in the first example. Keep going until you reach the top of the pyramid.

1

		.9	
.2	5	.2	.7

2

.7	.16	3	.02

3

.2	.4	.6	.8

4

.1	.3	.5	.7

5

.25	.11	.09	1.2

6

.06	7	.5	1.3

7

2	.06	1.5	.16

8

.01	5	0	.99

9

8.4	.67	3.2	1.8

10

.22	.33	.44	.55

11

.06	.19	.06	.19

12

.22	.22	.22	.22

Activity Master

Pyramid Math 3

Name______________________________

Add pairs of adjacent numbers and write their sums in the box above them as in the first example. Keep going until you reach the top of the pyramid.

1

		-2	
-4	7	3	-5

2

4	-7	-3	5

3

4	7	-3	-5

4

-1	-2	-3	-4

5

-1	2	-3	-4

6

1	-2	3	-4

7

8	-6	15	-11

8

-13	-5	0	-6

9

11	-21	31	-41

10

40	-15	40	-15

11

8	2	-14	29

12

9	3	-13	30

Activity Master

Pyramid Math 4

Name________________________________

Add pairs of adjacent numbers and write their sums in the box above them as in the first example. Keep going until you reach the top of the pyramid.

1

		54	
55	42	16	38

2

27	19	31	47

3

58	65	34	22

4

18	28	38	48

5

54	44	34	24

6

37	37	46	46

7

59	29	9	49

8

43	34	43	34

9

82	28	28	82

10

63	37	45	54

11

18	82	82	18

12

100	200	300	400

Activity Master

Pyramid Math 5

Name______________________________

Add pairs of adjacent numbers and write their sums in the box above them. Keep going until you reach the top of the pyramid.

1

$\frac{1}{5}$ $\frac{2}{5}$ $\frac{4}{5}$ $\frac{3}{5}$

2

$\frac{6}{7}$ $\frac{4}{7}$ $\frac{1}{7}$ $\frac{3}{7}$

Activity Master

Pyramid Math 6

Name_______________________________

Each number is the sum of the two numbers below it. Work backward to fill in the bottom row.

1

25			
16			
11			
8			

2

29			
15			
	8		
	4		

3

36			
19			
		12	
	3		

4

57			
12			
	11		
		11	

5

41			
19			
7			11

6

25	38		
	6		
		6	

7

54			
27			
		27	
27			

8

104			
	48		
27			
			9

9

183			
102			
		36	
31			

10

51			
	29		
20			
	2		

11

207			
43			
	38	19	

12

	83		
		43	
13			9

Activity Master

Pyramid Math 7

Name________________________________

Each number is the sum of the two numbers below it. Work backward to fill in the bottom row.

1

3.3			
1.2			
.4			
.3			

2

3.4			
1.9			
	.8		
	.6		

3

3.2			
1.5			
		1	
	.2		

4

8.5			
5.1			
	1.5		
		.9	

5

3.4			
1.6			
.7			2

6

1.4	1.9		
	.5		
		.5	

7

31.3			
16.5			
		5.8	
1.2			

8

31.5			
	9.9		
18.9			
			6.4

9

56.4			
30.4			
		11.9	
8.7			

10

3.47			
	2.36		
.45			
	.16		

11

10.84			
8.09			
	.99	.09	

12

	6.87		
		5.55	
7.7			5

Activity Master

Pyramid Math 8

Name________________________________

Each number is the sum of the two numbers below it. Work backward to fill in the bottom row.

1

7			
3			
2			
6			

2

-18			
-10			
	-6		
	-12		

3

-18			
8			
		-20	
	6		

4

-62			
-24			
	-15		
		-9	

5

11			
-8			
-8			-20

6

-3	10		
	10		
		16	

7

-71			
-16			
		-27	
21			

8

39			
	-16		
41			
			-14

9

-113			
-65			
		-2	
11			

10

0			
	20		
-20			
	-3		

11

1			
-20			
	-5	6	

12

	0		
		3	
18			-3

Notice that in the Pyramid Math activity described above we addressed all five strategies for practicing number sense:

1. Playing with numbers
2. Solving problems in multiple ways
3. Creative practice
4. Thinking, talking, and writing about numbers
5. Exploring patterns

We also addressed many of the eight mathematical practices in the Common Core Math Standards. However, this is true only if it is presented as an exploration instead of simply as a worksheet that is given to students without discussion.

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Characteristics of Number Sense

We began by noting that like the characteristics of a good book or a good movie, number sense is difficult to pin down. Since then, we explored the five components that make up number sense and five strategies that we can employ in practice work. As we employ the techniques in this activity, we will begin to recognize when we are teaching mathematics in a way that builds number sense. We find that teaching number sense tends to be:

- **Nonalgorithmic** – Algorithms have a purpose, and they are effective in what they are designed to do, but they tend to diminish rather than build number sense

- **Tends to be complex** – When students are building their number sense, they are thinking at high cognitive levels. This does not necessarily mean that this is hard; in fact, number sense often makes math easier. But thinking about numbers requires a higher depth of knowledge than following a procedure such as we encounter with long division.

- **Involves meaning** – Number sense never strays far from concepts and understanding.

- **Relies on judgment** – Students must attend to their thinking when developing number sense. They must rely on metacognition and think about their thinking. How did you get that answer? How do you know it's right?

- **Is "effortful"** – While not a recognized word, I think "effortful" captures the thought process involved in developing number sense. It is the opposite of "effortless". This does not mean that it will require an inordinate addition of work for the teacher. In fact, as you work with this in your classroom, it will become more natural to incorporate these strategies into your normal teaching style.

> Number sense is something that "unfolds" rather than something that is "taught" directly.
>
> *P. R. Trafton*
> *Establishing foundations for research on number sense and related topics, 1989*

- **Relies on self-regulation of the thinking process** – Students must pay attention to their work and their thinking much more than they do with an algorithmic approach.

- **May involve uncertainty** – Like exploring a wilderness, there is not a predetermined path or series of steps as you would use in an algorithm.

- **May yield multiple solutions or multiple solution paths** – Students often find many creative ways to get to an answer when they rely on number sense instead of algorithms. I have often been amazed and surprised by the discoveries that students have made in solving problems. Sometimes, they even teach me.

Tips for Fostering Great Mathematical Discourse

We will close this activity with a few pages showing some discussion tips and accompanying examples of the types of questions you can use in your classroom to build rich experiences with number sense. These pages come from my 108-page e-book, *The Language of Math*, available in my Teachers Pay Teachers store. The examples can be used with students of many ages, but they also serve as models to give you an idea of the types of questions you can ask to foster number sense in your own students.

If you want to improve the quality and depth of math talk in your classroom, try posing some of these quick and simple questions. They are written to elicit a higher level of thinking than the typical questions asked in a math classes of the past.

- How did you solve the problem?
- Did anyone else solve the problem this way?
- Can you solve the problem a different way?
- Who solved it a different way?
- Is this problem similar to another you solved?
- Can you write a rule or formula for your problem?
- What patterns do you notice?
- Does that always work?
- How did you know that?
- Why does that work?
- Why is that true?
- Can you think of a situation in which that wouldn't work?
- Why did you decide to do it that way?
- What patterns do you see?
- Do you agree or disagree with what that student said? Why?
- How would you convince me that you are right?
- Does that seem like a reasonable answer?
- Can you paraphrase what that student said?
- What information is important in this problem?

Mr. Infinity's math class is so popular everyone is trying to get in. Here is his seating chart. If you were student 100, explain how you could find the row and column of your seat.

Column	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*
Row 1	1	2	3	4	5	6	7	8
Row 2	9	10	11	12	13	14	15	16
Row 3	17	18	19	20	21	22	23	24
Row 4	25	26	27	28	29	30	31	32

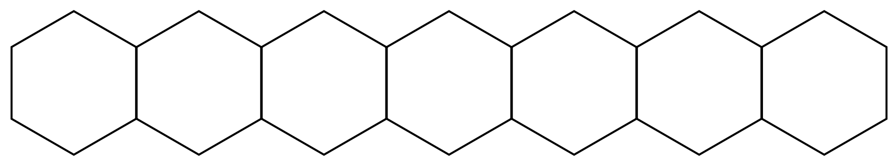
Five friends meet on the street and shake hands. How many handshakes will be required? Explain how you got your answer.

☺

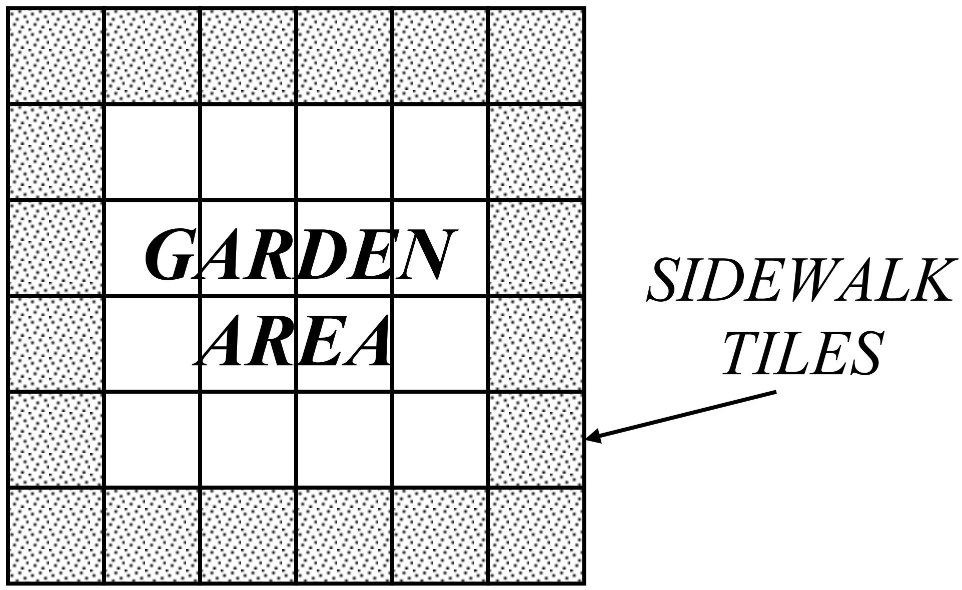
☺ ☺

☺ ☺

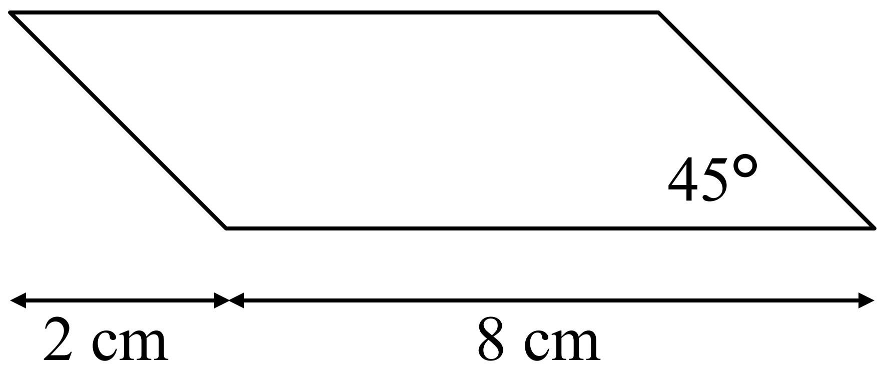
How many people can be seated at this arrangement of tables? People sit on each side and both ends. What if 17 tables were used? What if a total of 100 tables were used?



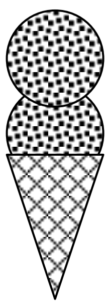
How many gray sidewalk tiles does it take to surround this garden? If the garden remains in a square shape but each side of the *white* garden area has a side length of seven, how many *gray* tiles are needed to surround it? What if the garden measures 100 by 100 squares?



Write instructions to a friend explaining how to draw this exact shape.

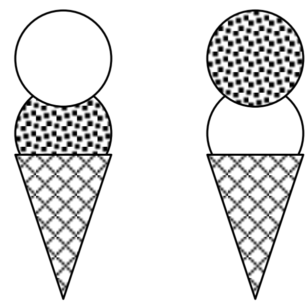


An ice cream shop sells five different flavors of ice cream. How many different types of double scoop cones can they create? It is acceptable to use the same flavor in a cone, but switching two flavors is not a new type.

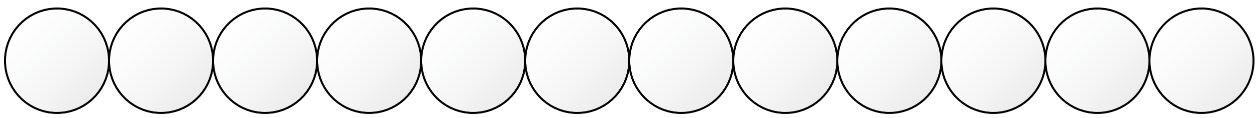


This is allowed.

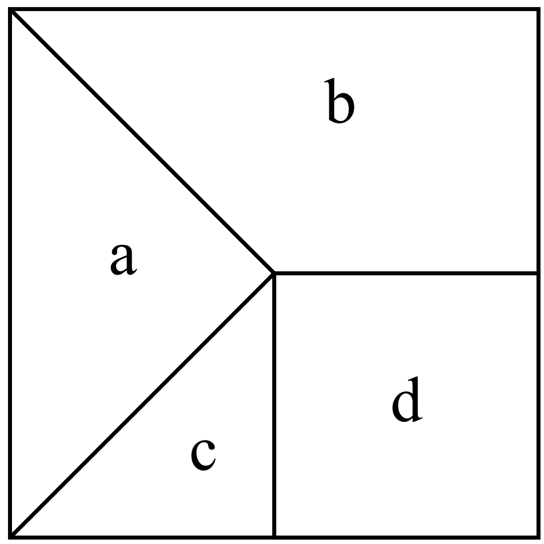
These are the same type.



Twelve marbles are in a row. One third of them are yellow. There is an equal amount of red and blue. There are two less green marbles than yellow. Explain how to find out how many of each color of marble are in the row.



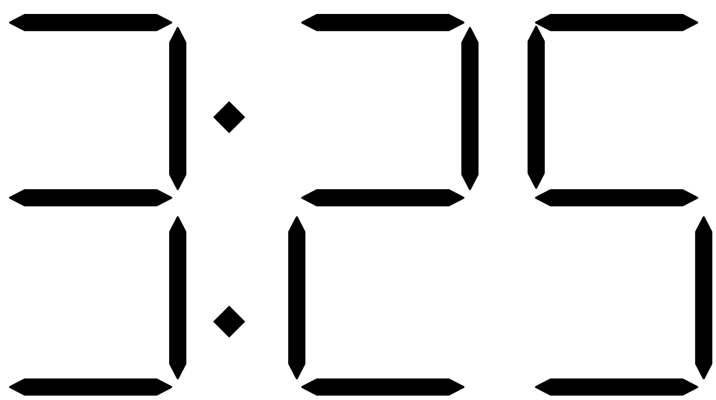
This tile costs $2 when it is whole. Explain how to find a reasonable price for each of the pieces.



Which of these numbers does not belong? Explain your reasoning. Can you find a reason why a different number might not belong?

2002	2020
2202	0202

The digital clock shown here displays the time by lighting up bars to make the numbers. The more bars needed to make a number, the brighter the time appears. For example, a 3 produces more light than a 1. At what time will the clock produce the most light?



Explain how you could show that this statement is true.

$$\frac{3}{4} > \frac{2}{3}$$

Arrange these fractions in order from least to greatest. Explain how you know you are correct.

$$\frac{2}{5} \quad \frac{3}{4} \quad \frac{1}{5} \quad \frac{3}{8} \quad \frac{1}{3}$$

Explain a way to solve this problem in your head.

$$52 + 36 + 2 + 18 + 12 =$$

Juan uses this trick to add. "I begin by adding the hundreds, then the tens, and finally the ones. Then I add those answers together." Explain why the method works.

$$\begin{array}{r} 358 \\ 267 \\ +\ 649 \\ \hline 1100 \\ 150 \\ +\ 24 \\ \hline 1274 \end{array}$$

Here are some number patterns. Create a number pattern of your own and write instructions so a person could understand the pattern and recreate it. Can you write a rule for your pattern or predict the 14^{th} term of the pattern?

1, 3, 6, 10, 15, …

83, 77, 71, 65, 59, …

These four digits go in the boxes. Where will you place them to create the greatest possible product? Explain how you know this will yield the greatest product.

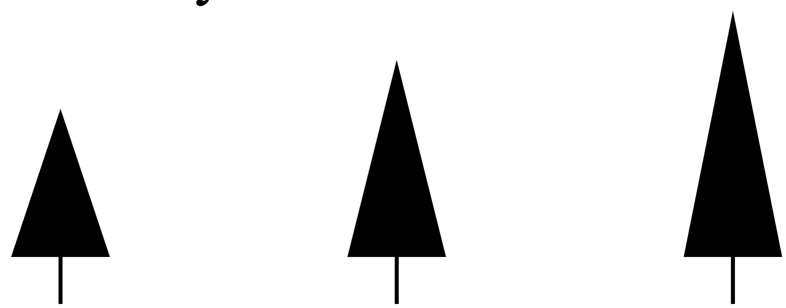
5, 3, 2, 8

$$\begin{array}{r} \square\,\square \\ \times\ \square\,\square \\ \hline \end{array}$$

Here are test scores for Celia. Explain to the student what overall grade you would give to her.

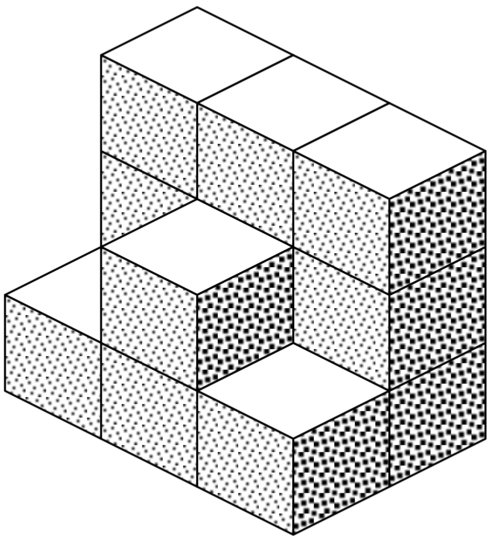
88%, 97%, 52%, 81%, 100%, 85%, 91%, 97%, 93%

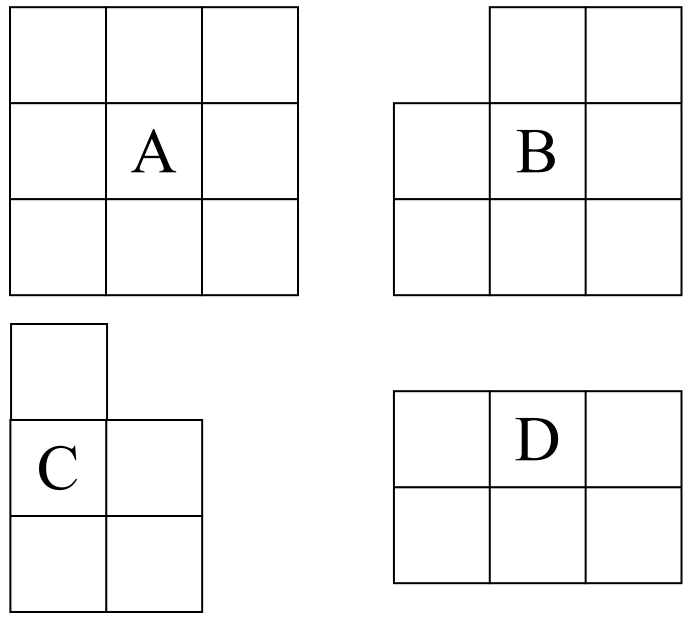
Here are the heights of a tree over a period of three years. Estimate how tall the tree will be when it is five years old. Explain why you think your estimate is correct.



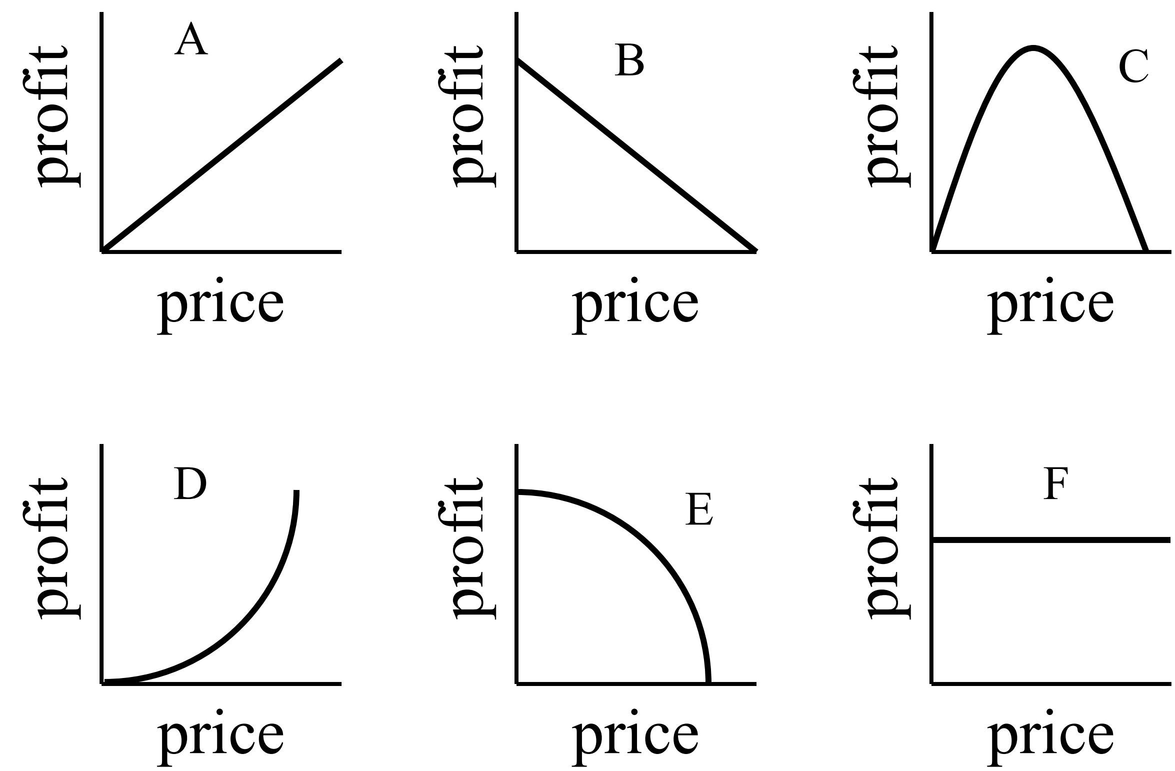
12’	16’	21’
Year 1	Year 2	Year 3

Which of these is *not* a view of this building? Explain how you know this.

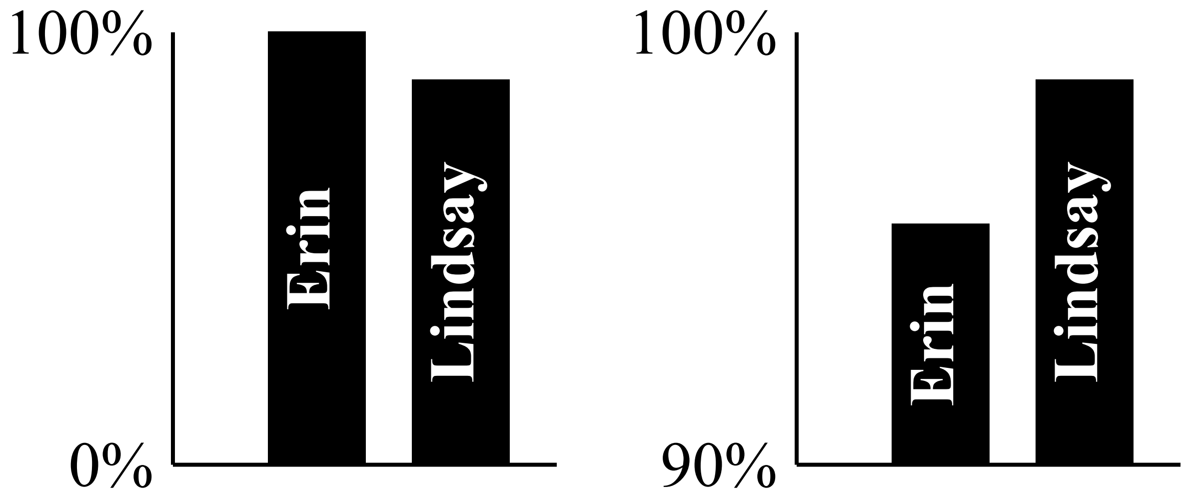




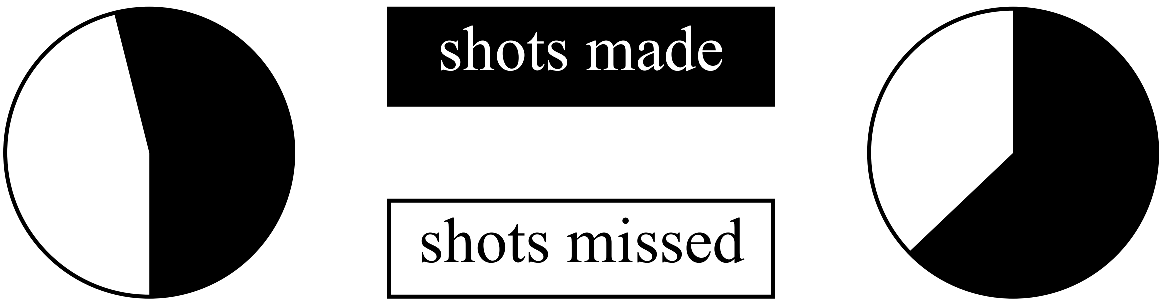
Which of these graphs best represents the relationship between the price a store charges for a candy bar and how much profit it makes? Explain your answer.



“I did better than you on the first test,” said Erin.
“Yes, but I did a *lot* better than you on the second test,” said Lindsay. “I did better overall.”
Explain who is correct and why.

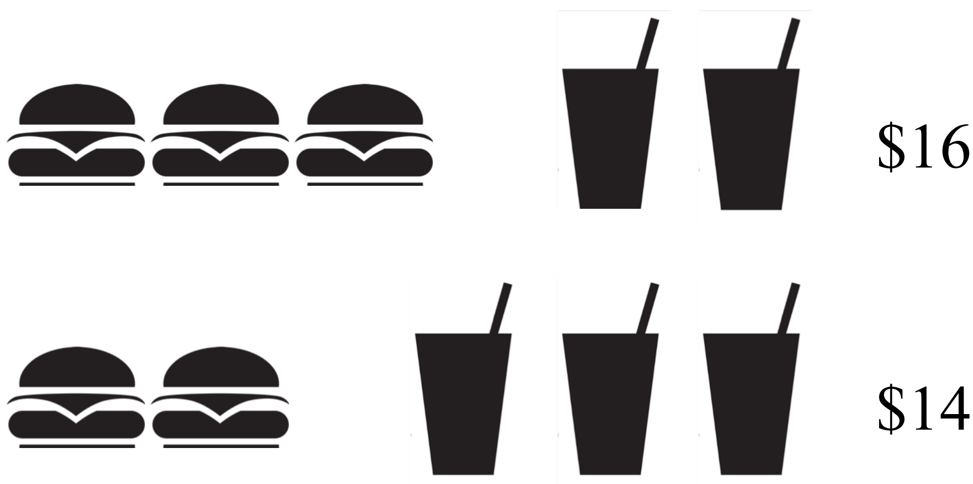


Explain which basketball player is better based on these graphs. Which one scored more points? Which one would be most likely to make his or her next shot?



Player A Player B

Sasha and Josh are at the food court in the mall. They noticed these prices. Explain how to find the price for the burger and for the drink.



Answer Key:

Page:	Answer:
53.	Row 13, column 4
54.	10
55.	30, 70, 402
56.	20, 32, 404
57.	Answers will vary.
58.	15
59.	4 yellow, 3 red, 3 blue, 2 green
60.	a: \$.50, b: \$.75, c: \$.25, d: \$.50
61.	Answers will vary.
62.	10:08
63.	Answers will vary.
64.	$^{1}/_{5}$, $^{1}/_{3}$, $^{3}/_{8}$, $^{2}/_{5}$, $^{3}/_{4}$
65.	Answers will vary.
66.	Answers will vary.
67.	Answers will vary.
68.	82 x 53 or 53 x 82. Explanations will vary
69.	Answers will vary.
70.	Answers will vary.
71.	B, explanations will vary.
72.	Graph C is correct. If the price is too low, there is no profit. If it is too high, there will be no sales and no profit.
73.	It isn't possible to determine who is correct as the second graph is truncated; it starts at 90% making the difference in the two scores appear to be more significant. Likely, the two students performed about the same.
74.	We can't determine from the graphs which player is better as that takes many other skills besides shooting into account. Nor can we tell how many points they scored unless we knew the total number of shots attempted. Player B seems to score more consistently than Player A, but we would need to know the sample size to determine if the data is reliable.
75.	Burgers cost \$4, and sodas are \$2. Explanations will vary.

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Feel free to contact me if you have questions or comments or would like to discuss a staff development training or keynote address at your site.

Happy teaching,

Brad