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Zach's Eraser

Linear Functions and Line of Fit

- ✓ Graphing
- ✓ T-tables
- ✓ Slope-intercept form
- ✓ Point-slope form
- ✓ Solving equations
- ✓ Independent and dependent variables

By Brad Fulton
Educator of the Year, 2005
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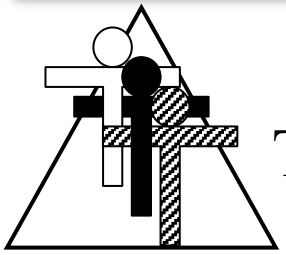

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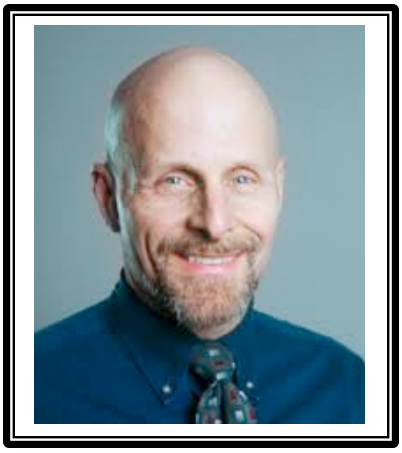
Brad Fulton

Educator of the Year



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- ♦ Consultant
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Known throughout the country for motivating and engaging teachers and students, Brad has co-authored over a dozen books that provide easy-to-teach yet mathematically rich activities for busy teachers while teaching full time for over 30 years. In addition, he has co-authored over 40 teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

Seminar leader and trainer of mathematics teachers

- ♦ 2005 California League of Middle Schools Educator of the Year
- ♦ California Math Council and NCTM national featured presenter
- ♦ Lead trainer for summer teacher training institutes
- ♦ Trainer/consultant for district, county, regional, and national workshops

Author and co-author of mathematics curriculum

- ♦ Simply Great Math Activities series: six books covering all major strands
- ♦ Angle On Geometry Program: over 400 pages of research-based geometry instruction
- ♦ Math Discoveries series: bringing math alive for students in middle schools
- ♦ Teacher training seminar materials handbooks for elementary, middle, and secondary school

Available for workshops, keynote addresses, and conferences

All workshops provide participants with complete, ready-to-use activities that require minimal preparation and give clear and specific directions. Participants also receive journal prompts, homework suggestions, and ideas for extensions and assessment.

Brad's math activities are the best I've seen in 38 years of teaching!
Wayne Dequer, 7th grade math teacher, Arcadia, CA

"I can't begin to tell you how much you have inspired me!"
Sue Bonesteel, Math Dept. Chair, Phoenix, AZ

"Your <u>entire</u> audience was fully involved in math!! When they chatted, they chatted math. Real thinking!"
Brenda McGaffigan, principal, Santa Ana, CA

"Absolutely engaging. I can teach algebra to second graders!"
Lisa Fellers, teacher

References available upon request

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Like me even more? Then please don't make copies for your colleagues. I know it's tempting when they say, "Wow! Groovy activity! Can I have a copy?" But this is how I make my money, and why are they still saying "groovy" anyway?

If we make copies for our friends, can we honestly tell our students not to copy or take things that don't belong to them? (Ouch!)

Half priced site licensed copies are available on the TPT website. Please encourage them to take advantage of this affordable option. Okay?

Thanks and happy teaching,

Brad 

Zach's Eraser

Linear Functions and Line of Fit

Overview:

This is an effective and thought-provoking way to helps students apply functions to real situations. The resulting function shows a negative slope. Though it is a simple linear function, the data is not perfect, so a line of fit must be employed. Because it is based on real data, even students who have only a beginning experience with functions can understand the concepts and mathematics.

Required Materials:

☒ Graph paper (included)

☒ Rulers

Optional Materials:

☐ Calculators

Introduction:

At the beginning of the school year, I noticed that my student, Zach, had one of those large prank erasers often sold in joke shops. I asked him if he thought it would last through the year, and he said he'd had it since he was in the seventh grade the previous year. That got me to wondering how long it would really last, so I asked him to weigh it on one of my scales. When he finished, I wrote the weight on the board under the title "Zach's Eraser" and said nothing to the class. Periodically throughout the year I asked Zach to weigh his eraser, and we recorded the data on the board. That one section of the board never got erased. During the final week of school, Zach took his last measurement, and the class had learned the necessary skills to do the task I had in mind.

Procedure:

1. Show the students the data chart on the right as you explain the context of the story from the paragraph above. Then ask them to consider these questions:
    a. How long will Zach's eraser last?
    b. Does Zach use his eraser fairly consistently?
    c. What did Zach's eraser weigh on the first day of school?
    d. When will Zach's eraser weigh 50 grams?
    e. What would the data look like if we graphed it?

    You may wish to add grade-level appropriate questions of your own. The students agreed that the graph would have a negative slope. Some said it would not be linear since the weight column does not go down at a constant rate. However others pointed out that the weights were not recorded at consistent intervals either.

Zach's Eraser

Date	Weight
Oct. 15	91.5 g
Nov. 4	91 g
Jan. 8	89.7 g
Mar. 18	89.3 g
Apr. 29	88.9 g
Jun. 2	88.1 g

2. Have the students put the data in a t-table. First they should decide how to count on the left column of the table. Some of my students simply began with January 1st, but they were soon corrected by students who realized that the school year didn't start with the calendar year. They agreed to count only school days beginning with the first day of the school year as zero. They assumed Zach would not use his eraser significantly during the summer or on weekends. We used the school calendar to determine the school day that corresponded to each date. The first data point on October 15th was the 41st day of school. This resulted in the t-table shown.

Day	Weight (g)
0	
41	91.5
54	91
86	89.7
132	89.3
155	88.9
180	88.1

3. Have the students find the slopes at various intervals on the table. For example, from day 41 to day 54, the slope is $^{-.5}/_{13}$ or approximately –0.038. That is, the weight dropped .5 grams in 13 days. Between the next two points he slope is $^{-1.3}/_{32}$ or approximately –0.041. Have them find the slopes between each pair of data points.

4. Next have them calculate the slope between the initial and final data points. There is an overall decrease of 3.4 grams in the eraser's weight over a period of 139 days. This slope of $^{-3.4}/_{139}$ is approximately –0.024.

5. Have the students graph the data. First they will need to establish scales on the two axes. My students decided to count the horizontal axis of the graph so that each unit represented 10 school days. On the vertical axis, each mark represents ¼ of a gram. However, they had to break the axis as shown in the answer key to make this work.

6. Next I had the students draw a line of fit passing through the first and final points on the graph. This resulted in two of the other points falling above the line and two falling below which was nice.

7. The graph answers three of the questions posed earlier, one of which was to determine what the function would look like when graphed of course (question e). It also shows that the use of the eraser was fairly constant (in answer to question b). Many of the students did not expect this. One time when I asked Zach to weigh his eraser, he told me that Steven had bitten off a piece. By studying the graph, the students were able to tell that this occurred between October 15th and November 4th when the graph took a steeper plunge.

8. By extending the line to the left until it intersected the vertical axis, the students were able to identify the y-intercept and answer question c. The eraser weighed approximately 92.5 grams on the first day of school.

9. To determine how long the eraser will last, the students will use the graph and t-table to find a formula in slope-intercept form: y=mx+b. Since the slope is approximately –0.024, and the eraser weighed 92.5 grams on the first day of school, the equation is:

$$y = -0.024x + 92.5$$

In this equation, x represents the number of school days, and y stands for the weight of the eraser in grams.

10. You could also use the point-slope formula if you wished: $y–y_1=m(x–x_1)$. This gives us:

$$y – 91.5 = –0.024(x – 41)$$

$$y – 91.5 = –0.024x + .984$$

$$y = –0.024 + 92.484$$

This is only 16 one thousandths of a gram different from the formula generated by the slope-intercept method.

11. By substituting zero for y, the students can calculate when the eraser's weight drops to zero to answer question a. This give us:

$$0 = –0.024x + 92.5$$

$$–92.5 = –0.024x$$

$$3{,}854 \text{ days} \approx x$$

12. Since 180 of these days had already been used in the year, the students subtracted that from the total to get 3,674 more days of use. Dividing this by 180 days of school per year told them that Zach's eraser would last over 20 years–well beyond high school and college even if he decided to get his doctorate!

13. Finally, by substituting 50 grams for y, the students answered question d:

$$50 = –0.024x + 92.5$$

$$–42.5 = –0.024x$$

$$1{,}771 \text{ days} \approx x$$

14. Students can also come to understand the interrelationships between the independent variable (time) and the dependent variable (the eraser's weight). As time varies, the weight of the eraser *depends* upon it. If time changes, we should expect to see the eraser's weight change accordingly. However, time does *not* depend on the eraser's weight. When Steven bit the eraser, it didn't make time elapse any faster.

The Common Core Connection:

CCSS.MATH.CONTENT.7.EE.B.3
Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically.

CSS.MATH.CONTENT.7.EE.B.4.A
Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

CCSS.MATH.CONTENT.8.F.A.1
Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

CCSS.MATH.CONTENT.8.F.A.3
Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

CCSS.MATH.CONTENT.8.F.B.4
Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

CCSS.MATH.CONTENT.8.F.B.5
Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

CSS.MATH.CONTENT.HSF.IF.A.1
Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range.

CCSS.MATH.CONTENT.HSF.IF.A.2
Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

CCSS.MATH.CONTENT.HSF.IF.B.4
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*[*]

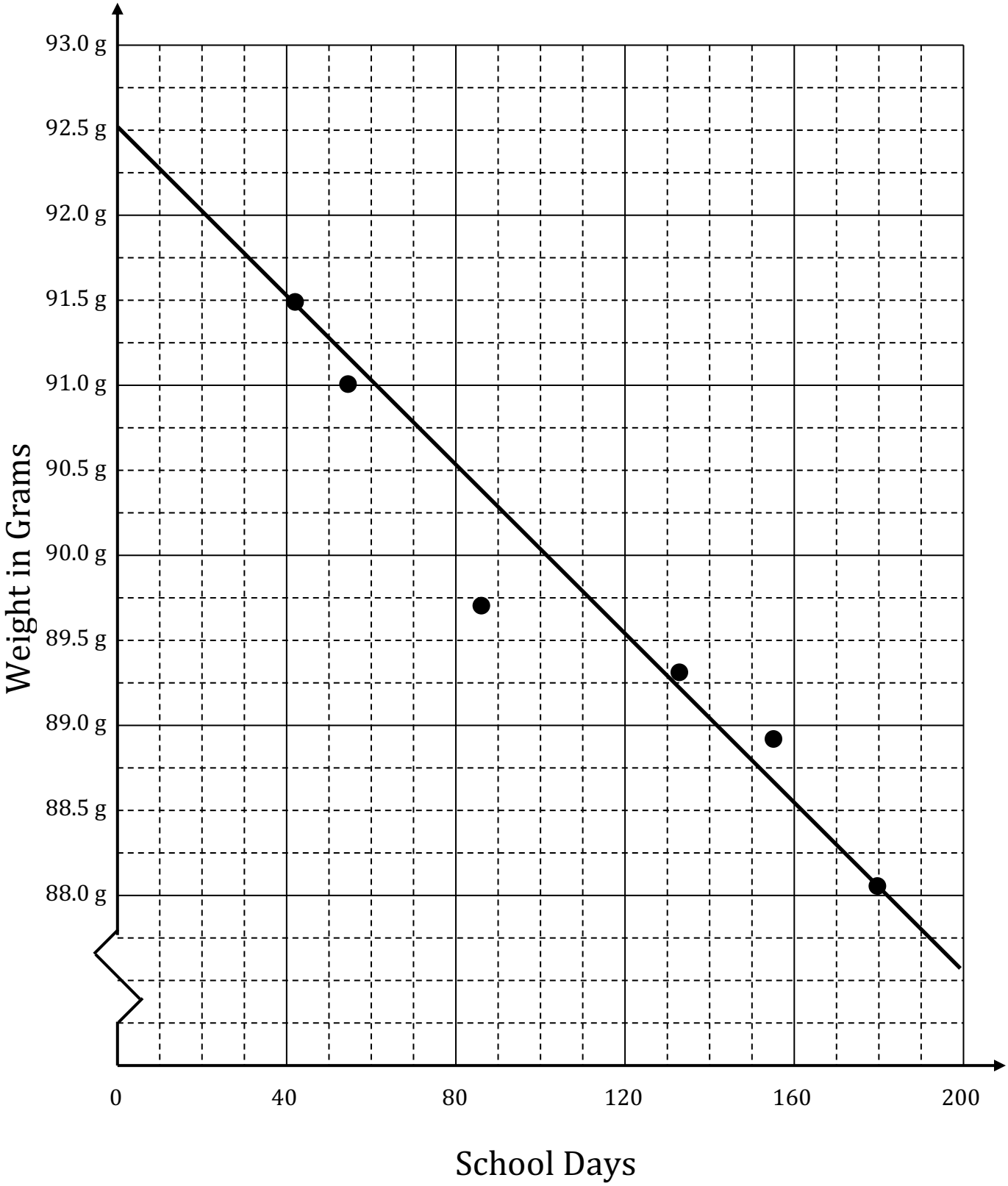
CCSS.MATH.CONTENT.HSF.IF.B.5
Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

CCSS.MATH.CONTENT.HSF.IF.B.6
Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

CCSS.MATH.CONTENT.HSF.BF.A.1.A
Determine an explicit expression, a recursive process, or steps for calculation from a context.

CCSS.MATH.CONTENT.HSF.LE.A.1.B
Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Answer Key

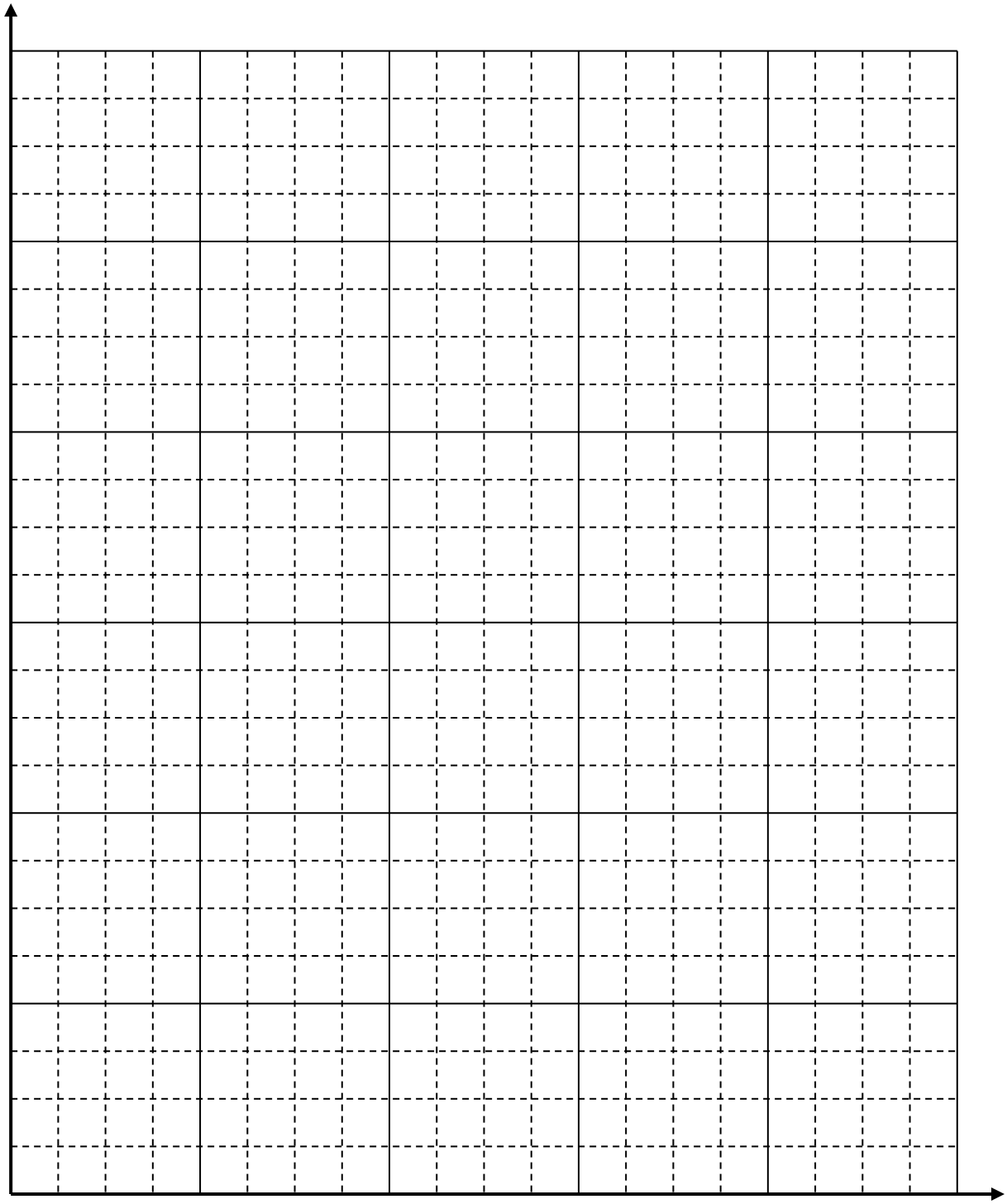


Zach's Eraser

Name _____

Date _____ Class _____

Zach has a BIG eraser. Every so often he recorded its weight. How long will his eraser last? How old will he be when the eraser weighs 50 grams?



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You can also find many free and inexpensive resources on my personal website, www.tttpress.com. Be sure to subscribe to receive monthly newsletters, blogs, and activities.

Similar activities include:

- *Losing Your Marbles (linear functions)*
- *Milk Carton Apartments (linear functions)*
- *The Sounds of Summer (linear functions)*
- *The Great Yo-yo Festival (slope-intercept form)*
- *Guessing the Gifts (systems of equations)*

Feel free to contact me if you have questions or comments or would like to discuss a staff development training or keynote address at your site.

Happy teaching,

Brad